Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/30-1.1.4.2-c-x-^m-a-x^j+b-x^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [454]. This is test number [30].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (454)	0.00 (0)
Mathematica	100.00 (454)	0.00 (0)
Maple	85.02 (386)	14.98 (68)
Fricas	71.59 (325)	28.41 (129)
Giac	57.49 (261)	42.51 (193)
Mupad	42.51 (193)	57.49 (261)
Maxima	33.70 (153)	66.30 (301)
Sympy	27.53 (125)	72.47 (329)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

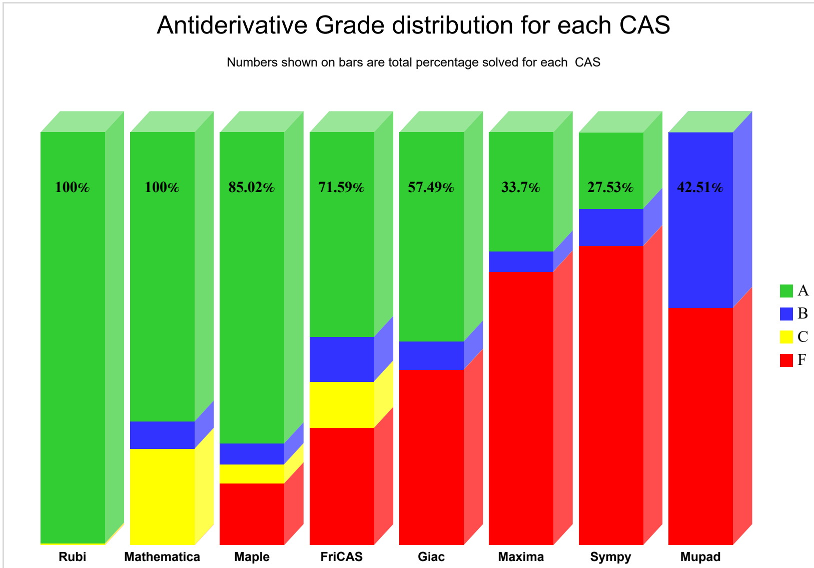
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

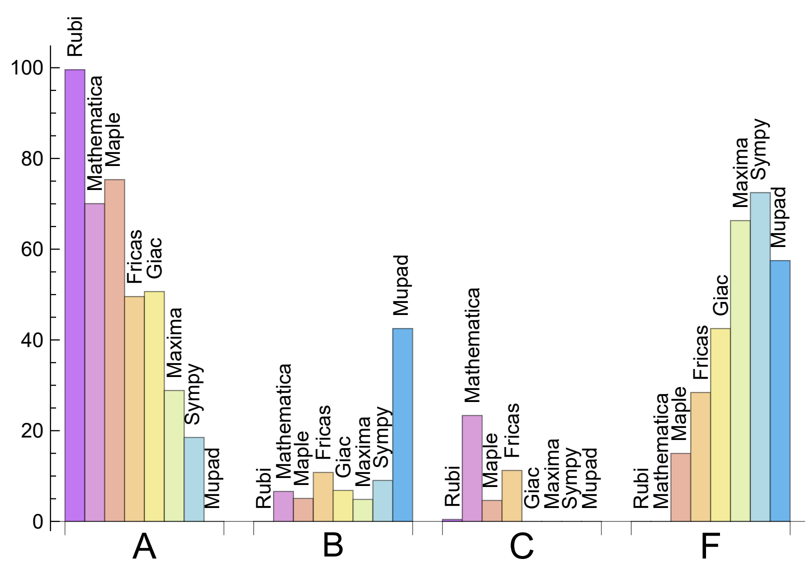
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.559	0.000	0.441	0.000
Maple	75.330	5.066	4.626	14.978
Mathematica	70.044	6.608	23.348	0.000
Giac	50.661	6.828	0.000	42.511
Fricas	49.559	10.793	11.233	28.414
Maxima	28.855	4.846	0.000	66.300
Sympy	18.502	9.031	0.000	72.467
Mupad	0.000	42.511	0.000	57.489

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	68	100.00	0.00	0.00
Fricas	129	39.53	25.58	34.88
Giac	193	97.41	0.52	2.07
Mupad	261	0.00	100.00	0.00
Maxima	301	100.00	0.00	0.00
Sympy	329	92.10	7.60	0.30

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.22
Rubi	0.28
Giac	0.30
Sympy	1.44
Maple	2.25
Mathematica	2.41
Mupad	5.95
Fricas	8.92

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	42.06	1.64	27.00	0.89
Mupad	46.89	1.45	37.00	0.87
Sympy	65.68	2.60	31.00	0.93
Mathematica	67.24	1.14	59.00	0.92
Maple	122.80	1.21	58.50	0.87
Rubi	125.29	1.04	76.50	1.00
Fricas	126.86	1.80	57.00	1.00
Giac	274.17	8.09	53.00	0.96

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

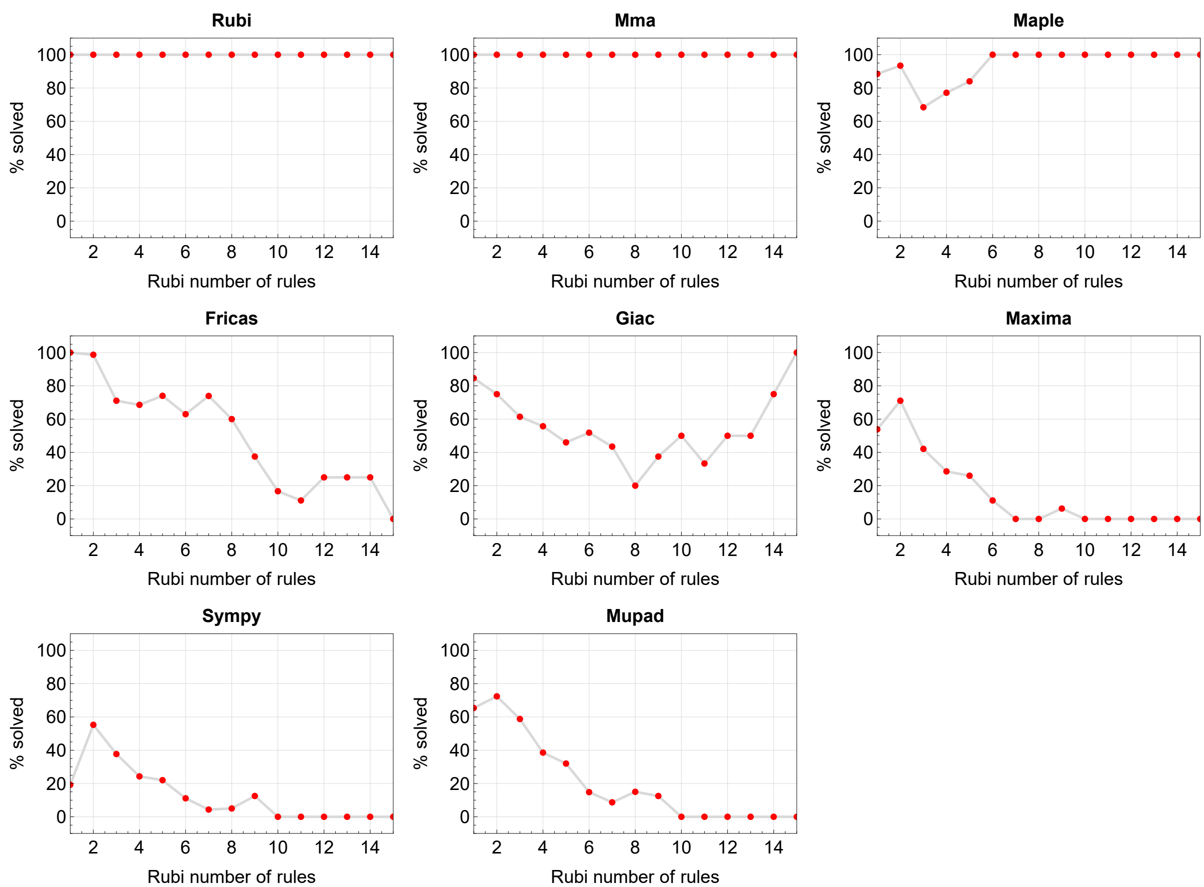


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

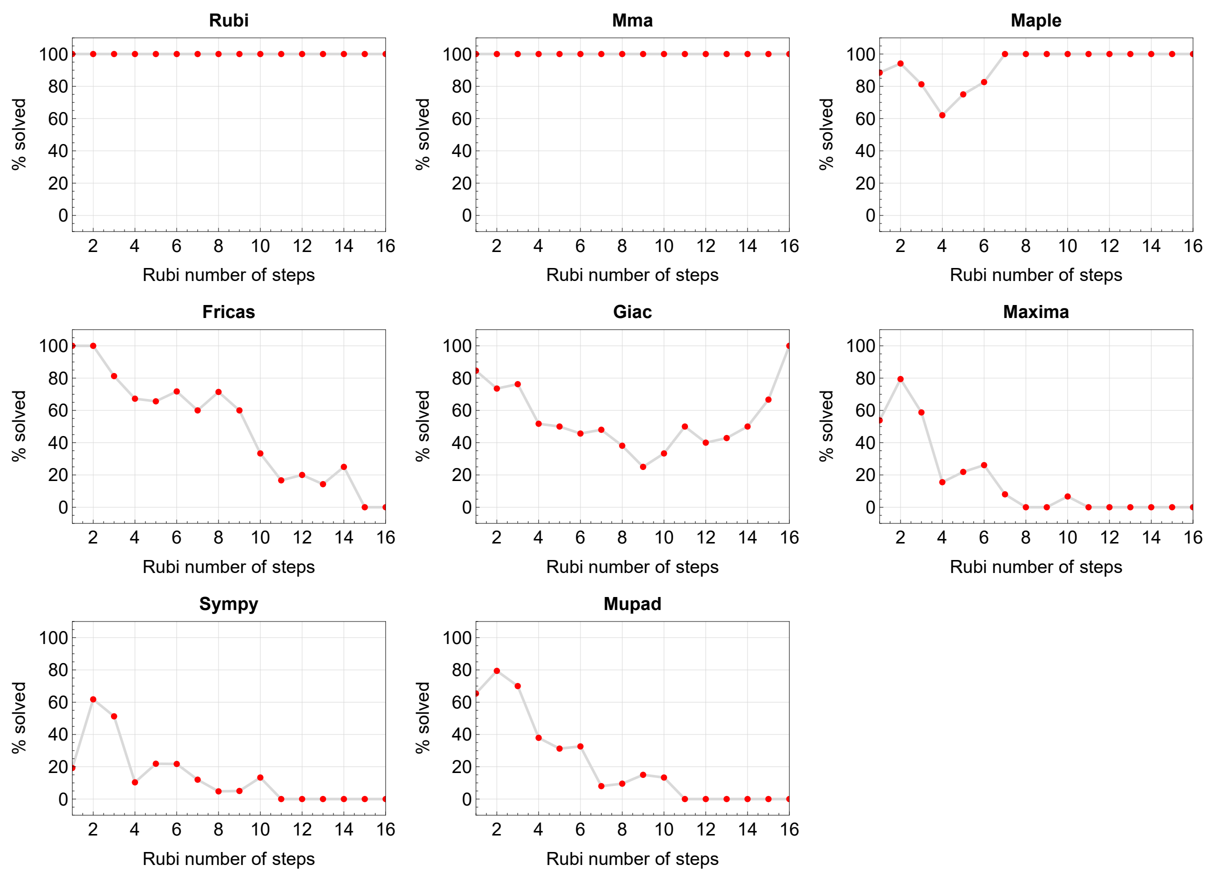


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

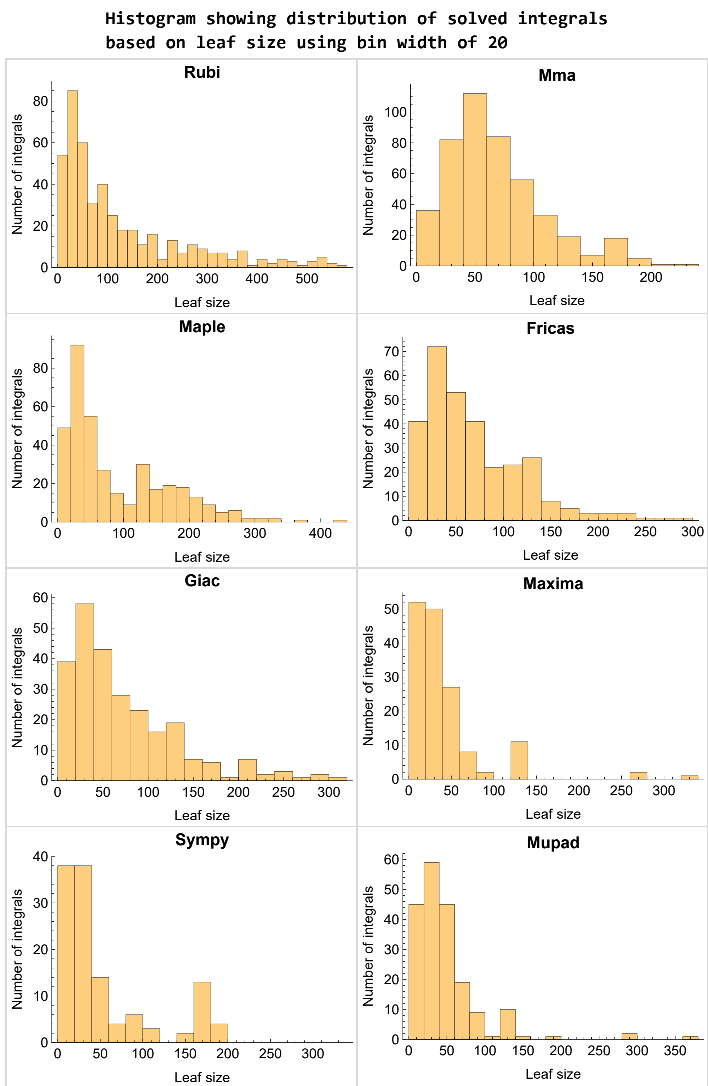


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

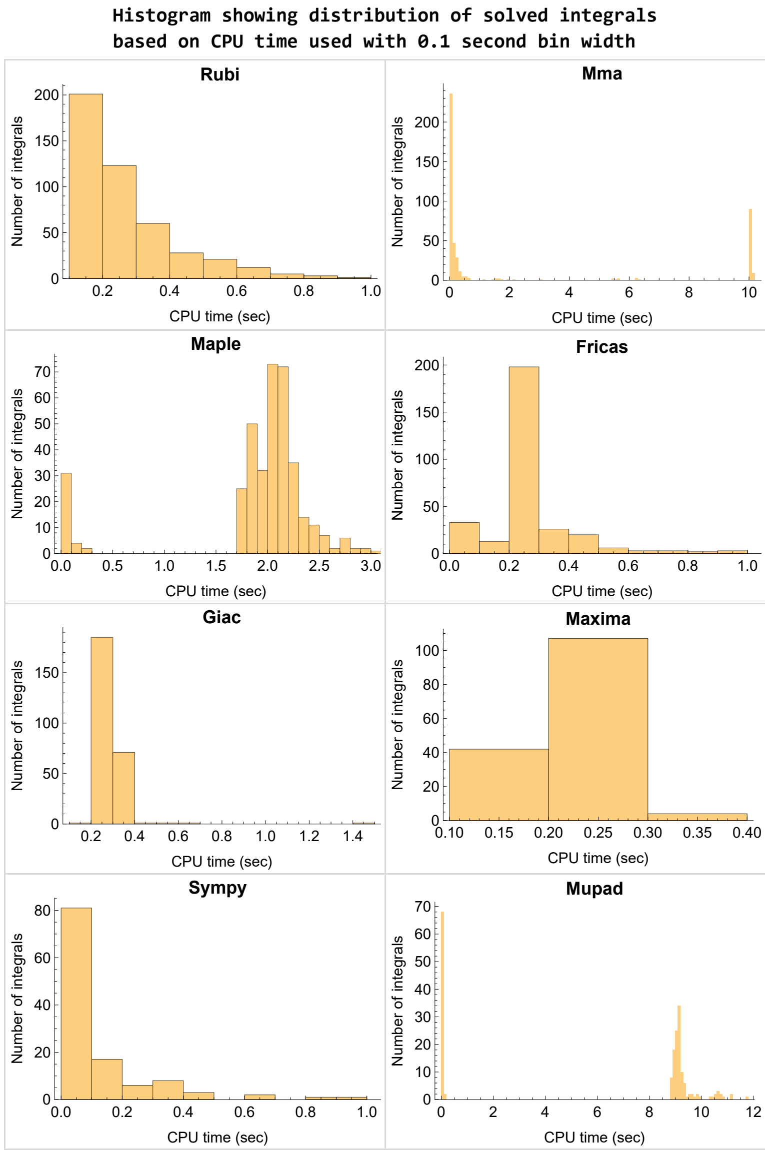


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

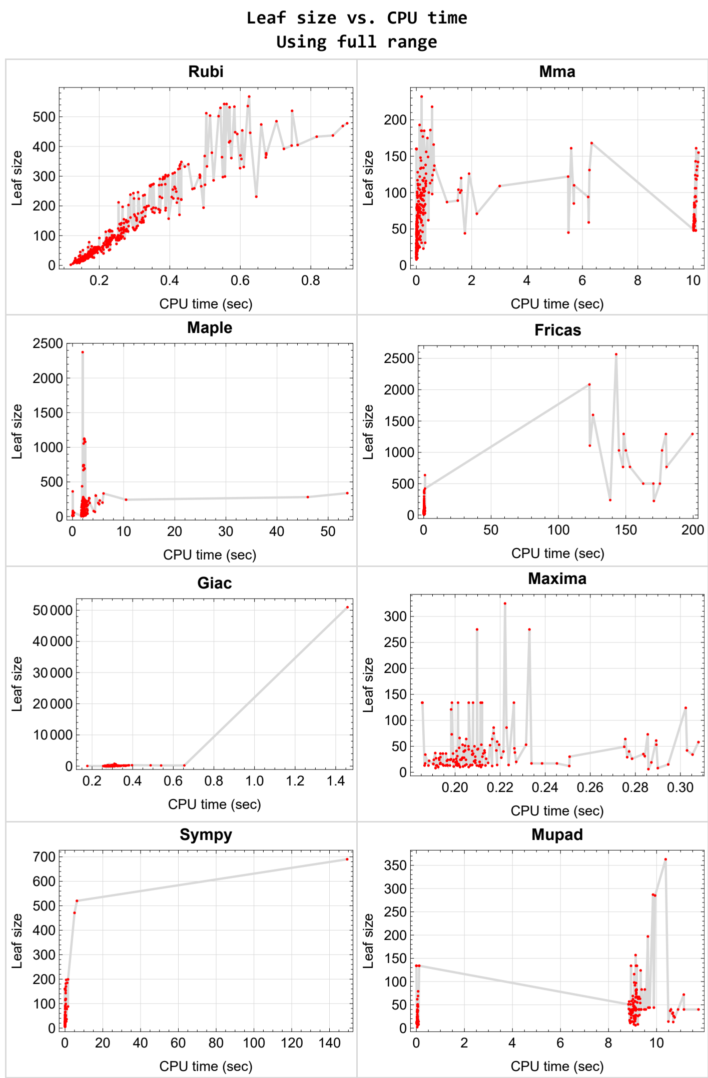


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

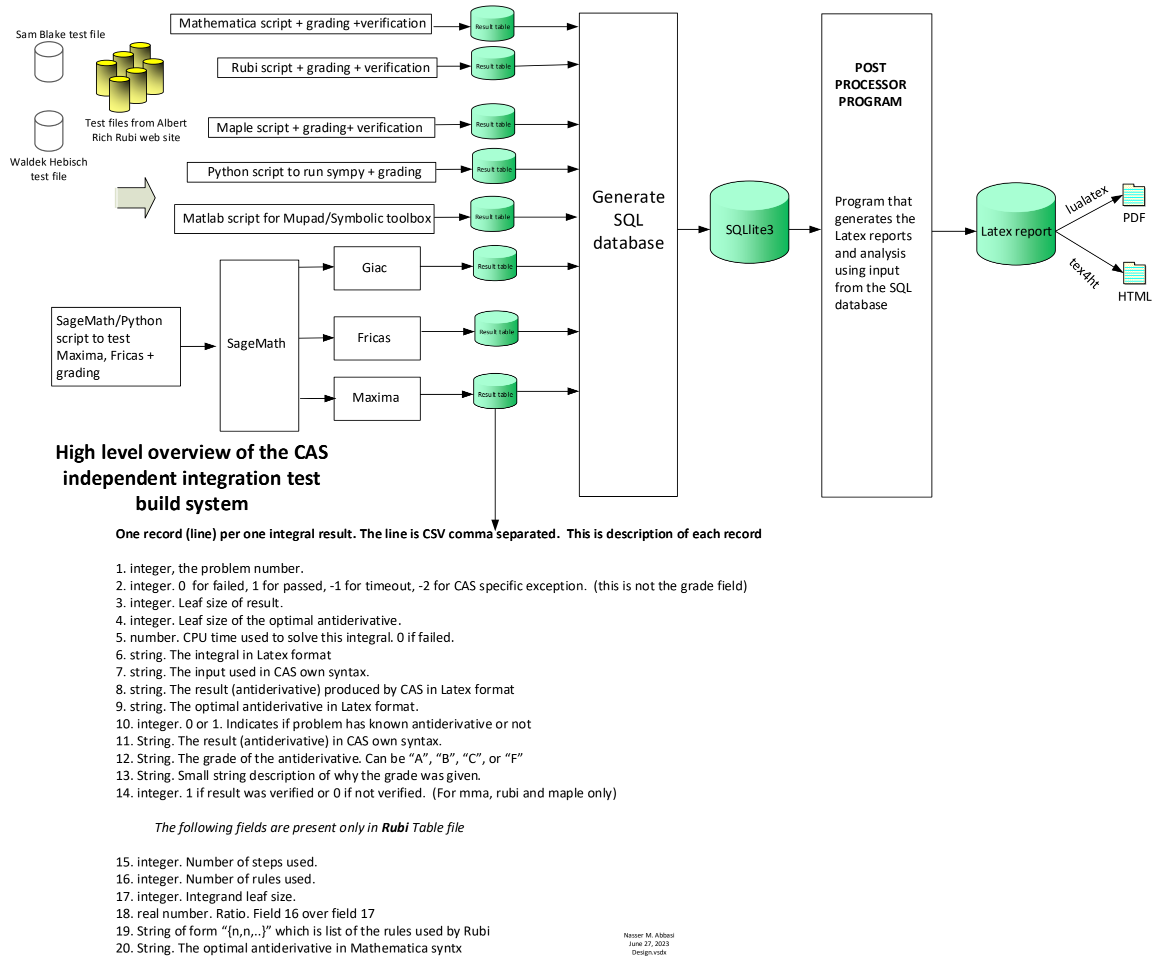
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { }

C grade { 437, 438 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { 26, 28, 30, 32, 34, 328, 329, 330, 332, 333, 334, 335, 336, 348, 349, 350, 351, 352, 353, 354, 392, 410, 414, 415, 416, 417, 418, 419, 420, 421 }

C grade { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 181, 182, 183, 184, 200, 201, 202, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 411, 412, 413, 437, 438 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 388, 389, 394, 402, 408, 412, 431, 432, 433, 434, 435, 436, 448, 449, 450 }

B grade { 86, 112, 113, 126, 272, 300, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 350, 351, 353, 387, 407, 411, 454 }

C grade { 26, 28, 32, 34, 97, 98, 99, 100, 101, 102, 243, 295, 296, 298, 299, 301, 302, 304, 305, 307, 347 }

F normal fail { 277, 278, 279, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 338, 339, 340, 344, 345, 346, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 387, 388, 389, 394, 402, 407, 408, 409, 410, 411, 412, 413, 414, 416, 419, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { 28, 30, 32, 34, 79, 84, 127, 167, 168, 169, 170, 171, 176, 177, 178, 179, 185, 186, 187, 188, 189, 194, 195, 196, 197, 283, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354 }

C grade { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 98, 99, 102, 289, 290, 291, 292, 293, 294, 301, 302, 304, 305, 307 }

F normal fail { 97, 100, 101, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 277, 278, 279, 295, 296, 298, 299, 439, 440, 441, 442, 443 }

F(-1) timedout fail { 103, 104, 105, 110, 111, 112, 117, 118, 119, 120, 124, 125, 126, 172, 173, 174, 175, 180, 181, 182, 183, 184, 190, 191, 192, 193, 198, 199, 200, 201, 202, 437, 438 }

F(-2) exception fail { 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 415, 417, 418, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 34, 35, 36, 37, 94, 95, 96, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 260, 261, 262, 263, 284, 285, 286, 303, 306, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 338, 339, 340, 344, 345, 346, 347, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 383, 387, 394, 402, 433, 434, 435, 436, 448, 449, 450 }

B grade { 28, 30, 32, 283, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 348, 350, 351, 353 }

C grade { }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 352, 354, 360, 361, 382, 384, 386, 387, 388, 408, 412, 432, 433, 434, 435, 436 }

B grade { 26, 28, 30, 32, 34, 176, 177, 178, 179, 235, 240, 241, 242, 243, 244, 245, 328, 329, 330, 331, 332, 333, 334, 336, 348, 350, 351, 353, 362, 383, 431 }

C grade { }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 99, 100, 101, 102, 114, 115, 116, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 325, 326, 327, 337, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

F(-1) timedout fail { 335 }

F(-2) exception fail { 407, 409, 411, 413 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 48, 61, 72, 94, 95, 96, 98, 105, 113, 134, 142, 153, 162, 170, 178, 189, 198, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 258, 260, 261, 262, 263, 266, 268, 280, 281, 282, 283, 284, 285, 286, 291, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 371, 382, 383, 384, 386, 387, 388, 392, 415, 416, 417, 418, 419, 420, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 444 }

C grade { }

F normal fail { }

F(-1) timedout fail { 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 235, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 259, 264, 265, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 364, 365, 366, 367, 368, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 421, 422, 423, 424, 425, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 103, 104, 105, 117, 118, 119, 120, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 308, 309, 319, 320, 321, 322, 324, 338, 339, 340, 344, 345, 346, 347, 360, 362, 363, 369, 377, 394, 433 }

B grade { 9, 13, 15, 17, 19, 21, 26, 28, 30, 32, 283, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 341, 342, 343, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 402 }

C grade { }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 364, 365, 366, 367, 368, 370, 371, 372, 373, 375, 376, 378, 379, 381, 382, 383, 385, 386, 387, 389, 390, 391, 392, 393, 395, 396, 397, 398, 400, 401, 403, 404, 406, 407, 408, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 447, 451, 452, 453 }

F(-1) timedout fail { 75, 76, 77, 78, 79, 80, 81, 82, 83, 148, 184, 374, 380, 384, 388, 399, 405, 409, 413, 434, 446, 448, 449, 450, 454 }

F(-2) exception fail { 337 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.143	0.002	0.204	0.195	0.253	0.019	0.271	0.024

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.146	0.002	0.100	0.213	0.261	0.016	0.285	0.023

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.142	0.000	0.104	0.196	0.250	0.016	0.278	0.020

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.141	0.001	0.044	0.212	0.279	0.017	0.261	0.018

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	14	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.08	0.85
time (sec)	N/A	0.142	0.002	0.044	0.204	0.262	0.031	0.255	0.024

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.156	0.002	1.963	0.205	0.398	0.017	0.263	0.042

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.167	0.001	1.987	0.198	0.381	0.017	0.269	0.034

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.171	0.006	1.998	0.203	0.343	0.017	0.261	0.032

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	24	24	24	24	24
N.S.	1	1.00	1.00	0.94	1.50	1.50	1.50	1.50	1.50
time (sec)	N/A	0.136	0.005	1.973	0.191	0.370	0.019	0.277	0.035

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.159	0.002	1.975	0.208	0.383	0.020	0.295	0.032

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	36	42	36	36
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.91	0.78	0.78
time (sec)	N/A	0.187	0.005	2.121	0.212	0.238	0.023	0.269	0.043

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.166	0.007	2.036	0.209	0.258	0.067	0.277	0.042

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.150	0.011	2.192	0.279	0.263	0.064	0.278	10.752

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.137	0.002	2.117	0.197	0.256	0.046	0.270	10.693

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.140	0.005	2.130	0.295	0.256	0.055	0.276	0.048

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	20	18	15	24	18
N.S.	1	1.18	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.158	0.006	2.088	0.211	0.263	0.093	0.278	0.065

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.144	0.015	2.102	0.276	0.264	0.083	0.264	10.709

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	36	35	32	31	33	31	43	31
N.S.	1	1.03	1.00	0.91	0.89	0.94	0.89	1.23	0.89
time (sec)	N/A	0.172	0.008	2.059	0.201	0.245	0.130	0.263	10.658

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	51	43	39	40	106	87	40	37
N.S.	1	1.19	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.161	0.024	2.076	0.277	0.253	0.102	0.269	10.567

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	50	49	44	44	45	42	57	46
N.S.	1	1.02	1.00	0.90	0.90	0.92	0.86	1.16	0.94
time (sec)	N/A	0.181	0.009	2.138	0.210	0.246	0.149	0.265	0.067

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.150	0.028	2.074	0.284	0.415	0.101	0.272	10.680

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	39	33	35	34	47	34	47	34
N.S.	1	1.03	0.87	0.92	0.89	1.24	0.89	1.24	0.89
time (sec)	N/A	0.177	0.019	2.081	0.208	0.368	0.136	0.268	0.048

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	61	54	45	49	136	92	47	44
N.S.	1	1.07	0.95	0.79	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.183	0.043	2.144	0.275	0.437	0.138	0.274	9.655

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	52	41	52	50	73	51	51	51
N.S.	1	1.06	0.84	1.06	1.02	1.49	1.04	1.04	1.04
time (sec)	N/A	0.193	0.040	2.487	0.209	0.804	0.188	0.273	0.058

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	78	67	55	64	172	114	59	58
N.S.	1	1.15	0.99	0.81	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.179	0.045	2.834	0.276	0.256	0.182	0.276	8.961

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	29	21	21	21	19	23	11
N.S.	1	1.00	2.23	1.62	1.62	1.62	1.46	1.77	0.85
time (sec)	N/A	0.152	0.007	2.153	0.199	0.249	0.039	0.270	8.903

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	18	14	14	15	14
N.S.	1	1.00	0.90	0.75	0.90	0.70	0.70	0.75	0.70
time (sec)	N/A	0.158	0.008	2.194	0.209	0.244	0.032	0.278	0.038

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	22	14	16	16	14	18	6
N.S.	1	1.00	3.67	2.33	2.67	2.67	2.33	3.00	1.00
time (sec)	N/A	0.129	0.004	2.121	0.190	0.259	0.043	0.275	0.060

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	13	8	8	15	8
N.S.	1	1.00	1.00	0.75	1.08	0.67	0.67	1.25	0.67
time (sec)	N/A	0.133	0.002	2.098	0.204	0.264	0.039	0.278	0.030

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.119	0.003	2.086	0.188	0.425	0.051	0.276	0.035

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	19	15	12	15	11	10	16	11
N.S.	1	1.27	1.00	0.80	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.151	0.005	2.160	0.205	0.472	0.048	0.268	0.051

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	24	16	18	20	15	20	8
N.S.	1	1.00	3.00	2.00	2.25	2.50	1.88	2.50	1.00
time (sec)	N/A	0.131	0.005	2.063	0.194	0.350	0.048	0.259	0.034

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	24	22	17	20	24	17	26	16
N.S.	1	1.09	1.00	0.77	0.91	1.09	0.77	1.18	0.73
time (sec)	N/A	0.158	0.007	2.333	0.227	0.752	0.043	0.265	0.034

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	31	22	25	30	24	27	13
N.S.	1	1.00	2.07	1.47	1.67	2.00	1.60	1.80	0.87
time (sec)	N/A	0.138	0.006	2.210	0.198	0.259	0.059	0.270	9.045

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	31	29	23	27	30	22	33	23
N.S.	1	1.07	1.00	0.79	0.93	1.03	0.76	1.14	0.79
time (sec)	N/A	0.162	0.006	2.267	0.192	0.252	0.055	0.276	0.034

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	19	15	14	13	13	12	18	14
N.S.	1	1.27	1.00	0.93	0.87	0.87	0.80	1.20	0.93
time (sec)	N/A	0.149	0.005	2.244	0.207	0.265	0.060	0.279	10.491

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	20	18	16	15	15	12	18	16
N.S.	1	1.11	1.00	0.89	0.83	0.83	0.67	1.00	0.89
time (sec)	N/A	0.158	0.009	2.250	0.194	0.253	0.069	0.278	0.056

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	174	95	158	0	59	0	0	0
N.S.	1	1.07	0.58	0.97	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.325	10.081	2.388	0.000	0.087	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	300	80	197	0	57	0	0	0
N.S.	1	1.07	0.28	0.70	0.00	0.20	0.00	0.00	0.00
time (sec)	N/A	0.425	10.039	2.420	0.000	0.089	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	142	79	146	0	49	0	0	0
N.S.	1	1.04	0.58	1.07	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.269	0.031	2.357	0.000	0.157	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	268	51	175	0	43	0	0	40
N.S.	1	1.05	0.20	0.69	0.00	0.17	0.00	0.00	0.16
time (sec)	N/A	0.352	0.015	2.223	0.000	0.138	0.000	0.000	11.139

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	48	124	0	34	0	0	0
N.S.	1	1.00	0.42	1.10	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.233	0.014	2.182	0.000	0.094	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	266	51	177	0	40	0	0	0
N.S.	1	1.07	0.21	0.71	0.00	0.16	0.00	0.00	0.00
time (sec)	N/A	0.371	10.017	2.127	0.000	0.083	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	53	123	0	36	0	0	0
N.S.	1	1.00	0.46	1.06	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.231	10.017	2.162	0.000	0.076	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	298	53	195	0	54	0	0	0
N.S.	1	1.05	0.19	0.69	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.413	10.021	2.190	0.000	0.164	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	200	94	169	0	70	0	0	0
N.S.	1	1.08	0.51	0.91	0.00	0.38	0.00	0.00	0.00
time (sec)	N/A	0.369	10.077	2.109	0.000	0.197	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	326	84	210	0	68	0	0	0
N.S.	1	1.07	0.28	0.69	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.444	10.047	2.131	0.000	0.256	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	166	83	158	0	60	0	0	40
N.S.	1	1.05	0.53	1.00	0.00	0.38	0.00	0.00	0.25
time (sec)	N/A	0.308	10.036	2.060	0.000	0.087	0.000	0.000	8.985

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	291	52	195	0	58	0	0	0
N.S.	1	1.06	0.19	0.71	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.396	10.019	2.114	0.000	0.089	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	139	49	143	0	47	0	0	0
N.S.	1	1.04	0.37	1.07	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.282	0.019	2.129	0.000	0.138	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	290	52	188	0	52	0	0	0
N.S.	1	1.06	0.19	0.69	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.403	10.018	2.126	0.000	0.176	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	137	54	139	0	45	0	0	0
N.S.	1	1.02	0.40	1.04	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.276	10.019	2.148	0.000	0.267	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	292	54	189	0	51	0	0	0
N.S.	1	1.05	0.19	0.68	0.00	0.18	0.00	0.00	0.00
time (sec)	N/A	0.408	10.019	2.192	0.000	0.086	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	142	54	139	0	44	0	0	0
N.S.	1	1.04	0.39	1.01	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.277	10.022	2.245	0.000	0.092	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	324	54	210	0	67	0	0	0
N.S.	1	1.06	0.18	0.69	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.456	10.020	2.307	0.000	0.090	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	171	54	160	0	59	0	0	0
N.S.	1	1.05	0.33	0.98	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.328	10.021	2.283	0.000	0.127	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	148	80	147	0	48	0	0	0
N.S.	1	1.06	0.57	1.05	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.292	10.051	2.162	0.000	0.146	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	274	66	178	0	43	0	0	0
N.S.	1	1.06	0.26	0.69	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.370	10.031	2.268	0.000	0.088	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	64	127	0	34	0	0	0
N.S.	1	1.00	0.55	1.09	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.241	10.033	2.122	0.000	0.091	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	246	53	158	0	22	0	0	0
N.S.	1	1.07	0.23	0.69	0.00	0.10	0.00	0.00	0.00
time (sec)	N/A	0.332	10.020	2.037	0.000	0.093	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	49	108	0	14	0	0	40
N.S.	1	1.00	0.53	1.17	0.00	0.15	0.00	0.00	0.43
time (sec)	N/A	0.203	10.027	2.037	0.000	0.163	0.000	0.000	10.598

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	272	48	182	0	42	0	0	0
N.S.	1	1.08	0.19	0.72	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.378	10.017	2.193	0.000	0.139	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	53	129	0	36	0	0	0
N.S.	1	1.00	0.45	1.08	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.239	10.019	2.188	0.000	0.085	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	304	53	195	0	56	0	0	0
N.S.	1	1.06	0.19	0.68	0.00	0.20	0.00	0.00	0.00
time (sec)	N/A	0.415	10.019	2.252	0.000	0.081	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	177	80	172	0	80	0	0	0
N.S.	1	1.10	0.50	1.07	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.338	10.051	2.709	0.000	0.087	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	303	68	200	0	76	0	0	0
N.S.	1	1.09	0.24	0.72	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.418	10.041	2.731	0.000	0.121	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	145	67	147	0	68	0	0	0
N.S.	1	1.06	0.49	1.07	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.276	10.032	2.590	0.000	0.206	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	271	57	182	0	61	0	0	0
N.S.	1	1.07	0.23	0.72	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.376	10.026	2.118	0.000	0.219	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	54	130	0	51	0	0	0
N.S.	1	1.00	0.47	1.13	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.245	10.029	2.105	0.000	0.083	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	270	56	184	0	60	0	0	0
N.S.	1	1.06	0.22	0.72	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.377	10.019	2.112	0.000	0.085	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	54	132	0	51	0	0	0
N.S.	1	1.00	0.47	1.16	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.239	0.027	2.056	0.000	0.138	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	297	51	206	0	72	0	0	40
N.S.	1	1.09	0.19	0.75	0.00	0.26	0.00	0.00	0.15
time (sec)	N/A	0.412	10.017	2.790	0.000	0.146	0.000	0.000	9.136

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	147	56	150	0	68	0	0	0
N.S.	1	1.06	0.40	1.08	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.290	10.020	2.797	0.000	0.343	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	332	56	228	0	90	0	0	0
N.S.	1	1.08	0.18	0.75	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.463	10.020	2.627	0.000	0.085	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	189	128	212	0	376	0	100	0
N.S.	1	1.19	0.81	1.33	0.00	2.36	0.00	0.63	0.00
time (sec)	N/A	0.393	0.048	2.110	0.000	0.294	0.000	0.312	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	146	68	70	0	108	0	80	0
N.S.	1	1.16	0.54	0.56	0.00	0.86	0.00	0.63	0.00
time (sec)	N/A	0.330	0.023	2.062	0.000	0.271	0.000	0.279	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	145	117	198	0	348	0	86	0
N.S.	1	1.12	0.90	1.52	0.00	2.68	0.00	0.66	0.00
time (sec)	N/A	0.342	0.041	2.079	0.000	0.421	0.000	0.303	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	117	64	59	0	97	0	64	0
N.S.	1	1.16	0.63	0.58	0.00	0.96	0.00	0.63	0.00
time (sec)	N/A	0.287	0.023	2.003	0.000	0.421	0.000	0.276	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	61	0	17	0
N.S.	1	1.00	1.00	1.08	0.00	2.44	0.00	0.68	0.00
time (sec)	N/A	0.157	0.018	1.983	0.000	0.483	0.000	0.302	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	84	46	48	0	86	0	50	0
N.S.	1	1.11	0.61	0.63	0.00	1.13	0.00	0.66	0.00
time (sec)	N/A	0.234	0.020	2.028	0.000	0.572	0.000	0.294	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	38	37	0	76	0	29	0
N.S.	1	1.00	0.75	0.73	0.00	1.49	0.00	0.57	0.00
time (sec)	N/A	0.189	0.017	2.048	0.000	0.281	0.000	0.329	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	0	75	0	33	0
N.S.	1	1.00	0.69	0.73	0.00	1.47	0.00	0.65	0.00
time (sec)	N/A	0.188	0.017	2.152	0.000	0.282	0.000	0.280	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	84	56	48	0	87	0	43	0
N.S.	1	1.11	0.74	0.63	0.00	1.14	0.00	0.57	0.00
time (sec)	N/A	0.229	0.020	2.190	0.000	0.279	0.000	0.300	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	63	0	23	0
N.S.	1	1.00	1.00	1.08	0.00	2.52	0.00	0.92	0.00
time (sec)	N/A	0.151	0.016	2.164	0.000	0.266	0.000	0.295	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	117	58	59	0	95	0	55	0
N.S.	1	1.16	0.57	0.58	0.00	0.94	0.00	0.54	0.00
time (sec)	N/A	0.268	0.022	2.176	0.000	0.272	0.000	0.283	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	145	106	217	0	360	0	114	0
N.S.	1	1.12	0.82	1.67	0.00	2.77	0.00	0.88	0.00
time (sec)	N/A	0.340	0.069	2.155	0.000	0.280	0.000	0.289	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	145	68	70	0	110	0	90	0
N.S.	1	1.15	0.54	0.56	0.00	0.87	0.00	0.71	0.00
time (sec)	N/A	0.332	0.020	2.044	0.000	0.415	0.000	0.312	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	188	122	234	0	396	0	104	0
N.S.	1	1.18	0.77	1.47	0.00	2.49	0.00	0.65	0.00
time (sec)	N/A	0.398	0.123	2.164	0.000	0.385	0.000	0.289	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	177	79	81	0	121	0	147	0
N.S.	1	1.16	0.52	0.53	0.00	0.80	0.00	0.97	0.00
time (sec)	N/A	0.373	0.021	2.090	0.000	0.667	0.000	0.310	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	222	134	247	0	422	0	138	0
N.S.	1	1.17	0.71	1.31	0.00	2.23	0.00	0.73	0.00
time (sec)	N/A	0.450	0.257	2.124	0.000	0.952	0.000	0.312	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	211	90	92	0	132	0	202	0
N.S.	1	1.17	0.50	0.51	0.00	0.73	0.00	1.12	0.00
time (sec)	N/A	0.445	0.210	2.063	0.000	0.374	0.000	0.324	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	82	45	0	133	0	45	0
N.S.	1	1.00	1.49	0.82	0.00	2.42	0.00	0.82	0.00
time (sec)	N/A	0.198	0.289	2.417	0.000	0.361	0.000	0.306	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	62	25	0	94	0	23	0
N.S.	1	1.00	1.94	0.78	0.00	2.94	0.00	0.72	0.00
time (sec)	N/A	0.163	0.433	2.199	0.000	0.341	0.000	0.294	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	19	0	14	19
N.S.	1	1.00	1.00	0.87	1.13	0.83	0.00	0.61	0.83
time (sec)	N/A	0.154	0.254	2.409	0.203	0.265	0.000	0.275	9.113

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	31	28	38	29	0	30	27
N.S.	1	1.00	0.65	0.58	0.79	0.60	0.00	0.62	0.56
time (sec)	N/A	0.189	0.320	2.588	0.227	0.267	0.000	0.281	9.123

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	44	41	50	40	0	47	40
N.S.	1	1.08	0.59	0.55	0.68	0.54	0.00	0.64	0.54
time (sec)	N/A	0.225	1.754	2.934	0.206	0.265	0.000	0.274	9.550

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	64	688	0	0	0	0	0
N.S.	1	1.00	0.29	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	10.034	2.342	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	49	671	0	16	0	39	40
N.S.	1	1.00	0.25	3.41	0.00	0.08	0.00	0.20	0.20
time (sec)	N/A	0.292	10.016	2.119	0.000	0.073	0.000	0.177	9.257

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	53	696	0	43	0	0	0
N.S.	1	1.00	0.24	3.09	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.324	10.019	2.360	0.000	0.077	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	534	66	1079	0	0	0	0	0
N.S.	1	1.06	0.13	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.624	10.031	2.491	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	504	53	1054	0	0	0	0	0
N.S.	1	1.06	0.11	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	10.022	2.170	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	530	48	1083	0	24	0	0	0
N.S.	1	1.07	0.10	2.18	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.575	10.017	2.368	0.000	0.075	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	195	113	151	0	0	182	109	0
N.S.	1	1.12	0.65	0.87	0.00	0.00	1.05	0.63	0.00
time (sec)	N/A	0.349	0.333	2.215	0.000	0.000	0.422	0.325	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	125	89	99	0	0	153	81	0
N.S.	1	1.08	0.77	0.85	0.00	0.00	1.32	0.70	0.00
time (sec)	N/A	0.277	0.262	2.207	0.000	0.000	0.329	0.324	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	62	50	0	0	119	52	72
N.S.	1	1.00	1.11	0.89	0.00	0.00	2.12	0.93	1.29
time (sec)	N/A	0.212	0.190	2.156	0.000	0.000	0.278	0.324	11.133

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	0	19	0	25	0
N.S.	1	1.00	1.00	0.80	0.00	0.76	0.00	1.00	0.00
time (sec)	N/A	0.153	0.122	2.220	0.000	0.320	0.000	0.301	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	90	48	67	0	42	0	84	0
N.S.	1	1.07	0.57	0.80	0.00	0.50	0.00	1.00	0.00
time (sec)	N/A	0.230	0.144	2.225	0.000	0.314	0.000	0.299	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	160	72	119	0	64	0	146	0
N.S.	1	1.13	0.51	0.84	0.00	0.45	0.00	1.03	0.00
time (sec)	N/A	0.324	0.152	2.219	0.000	0.333	0.000	0.307	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	230	96	171	0	86	0	208	0
N.S.	1	1.15	0.48	0.86	0.00	0.43	0.00	1.04	0.00
time (sec)	N/A	0.433	0.201	2.195	0.000	0.342	0.000	0.294	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	231	137	227	0	0	0	148	0
N.S.	1	1.17	0.70	1.15	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.687	0.650	2.191	0.000	0.000	0.000	0.345	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	157	113	175	0	0	0	120	0
N.S.	1	1.13	0.81	1.26	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.415	0.499	2.155	0.000	0.000	0.000	0.339	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	87	84	124	0	0	0	92	0
N.S.	1	1.13	1.09	1.61	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.240	0.319	2.292	0.000	0.000	0.000	0.327	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	31	45	0	36	0	34	40
N.S.	1	1.00	1.24	1.80	0.00	1.44	0.00	1.36	1.60
time (sec)	N/A	0.153	0.191	2.291	0.000	0.506	0.000	0.285	9.488

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	85	55	46	0	63	0	0	0
N.S.	1	1.08	0.70	0.58	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.235	0.232	2.232	0.000	0.945	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	155	81	98	0	87	0	0	0
N.S.	1	1.13	0.59	0.72	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.329	0.253	2.183	0.000	0.315	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	225	105	150	0	109	0	0	0
N.S.	1	1.15	0.54	0.77	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.430	0.255	2.185	0.000	0.314	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	231	126	177	0	0	199	123	0
N.S.	1	1.13	0.62	0.87	0.00	0.00	0.98	0.60	0.00
time (sec)	N/A	0.396	0.349	2.180	0.000	0.000	1.605	0.332	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	161	102	125	0	0	170	95	0
N.S.	1	1.10	0.70	0.86	0.00	0.00	1.16	0.65	0.00
time (sec)	N/A	0.309	0.308	2.284	0.000	0.000	0.487	0.335	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	93	80	74	0	0	143	67	0
N.S.	1	1.07	0.92	0.85	0.00	0.00	1.64	0.77	0.00
time (sec)	N/A	0.233	0.258	2.410	0.000	0.000	0.326	0.332	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	40	32	0	0	99	35	0
N.S.	1	1.00	1.18	0.94	0.00	0.00	2.91	1.03	0.00
time (sec)	N/A	0.176	0.150	2.158	0.000	0.000	0.398	0.339	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	35	41	0	29	0	53	0
N.S.	1	1.00	0.65	0.76	0.00	0.54	0.00	0.98	0.00
time (sec)	N/A	0.187	0.151	2.133	0.000	0.442	0.000	0.288	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	124	59	93	0	50	0	115	0
N.S.	1	1.11	0.53	0.83	0.00	0.45	0.00	1.03	0.00
time (sec)	N/A	0.267	0.177	2.133	0.000	0.345	0.000	0.312	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	194	83	145	0	72	0	177	0
N.S.	1	1.14	0.49	0.85	0.00	0.42	0.00	1.04	0.00
time (sec)	N/A	0.365	0.192	2.159	0.000	0.368	0.000	0.300	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	194	124	201	0	0	0	134	0
N.S.	1	1.13	0.73	1.18	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.527	0.584	2.138	0.000	0.000	0.000	0.344	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	119	100	148	0	0	0	106	0
N.S.	1	1.05	0.88	1.31	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.316	0.451	2.141	0.000	0.000	0.000	0.356	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	62	72	101	0	0	0	71	0
N.S.	1	1.03	1.20	1.68	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.211	0.277	2.136	0.000	0.000	0.000	0.365	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	25	0	54	0	26	0
N.S.	1	1.00	1.53	0.83	0.00	1.80	0.00	0.87	0.00
time (sec)	N/A	0.169	0.224	2.184	0.000	0.459	0.000	0.289	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	119	70	72	0	79	0	0	0
N.S.	1	1.11	0.65	0.67	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.276	0.251	2.166	0.000	0.492	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	189	96	124	0	101	0	0	0
N.S.	1	1.15	0.58	0.75	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.368	0.275	2.174	0.000	0.459	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	259	120	176	0	123	0	0	0
N.S.	1	1.16	0.54	0.79	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.489	0.290	2.214	0.000	0.423	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	364	155	264	0	0	0	0	0
N.S.	1	1.21	0.51	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	10.191	2.062	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	448	136	273	0	0	0	0	0
N.S.	1	1.09	0.33	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.609	10.149	2.096	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	258	118	198	0	0	0	0	0
N.S.	1	1.21	0.55	0.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	10.119	2.078	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	340	94	207	0	0	0	0	40
N.S.	1	1.05	0.29	0.64	0.00	0.00	0.00	0.00	0.12
time (sec)	N/A	0.446	10.062	2.035	0.000	0.000	0.000	0.000	9.151

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	155	54	132	0	0	0	0	0
N.S.	1	1.26	0.44	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	10.048	2.046	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	340	59	213	0	0	0	0	0
N.S.	1	1.05	0.18	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	10.051	1.982	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	227	59	179	0	0	0	0	0
N.S.	1	1.21	0.31	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	10.053	2.093	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	446	59	281	0	0	0	0	0
N.S.	1	1.08	0.14	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.640	10.053	2.079	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	333	59	245	0	0	0	0	0
N.S.	1	1.21	0.21	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	10.055	2.370	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	358	142	196	0	0	0	0	0
N.S.	1	1.20	0.48	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	10.176	3.363	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	442	123	261	0	0	0	0	0
N.S.	1	1.08	0.30	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	10.151	2.222	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	248	106	163	0	0	0	0	40
N.S.	1	1.19	0.51	0.78	0.00	0.00	0.00	0.00	0.19
time (sec)	N/A	0.410	10.100	2.043	0.000	0.000	0.000	0.000	9.089

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	335	60	205	0	0	0	0	0
N.S.	1	1.05	0.19	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	10.053	2.051	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	181	60	130	0	0	0	0	0
N.S.	1	1.26	0.42	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	10.057	1.996	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	368	62	235	0	0	0	0	0
N.S.	1	1.05	0.18	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	10.054	2.018	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	255	62	168	0	0	0	0	0
N.S.	1	1.20	0.29	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	10.060	3.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	474	62	301	0	0	0	0	0
N.S.	1	1.08	0.14	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.693	10.056	4.563	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	361	62	201	0	0	0	0	0
N.S.	1	1.20	0.21	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.617	10.065	5.863	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	370	161	196	0	0	0	0	0
N.S.	1	1.22	0.53	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.629	10.107	5.022	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	454	143	261	0	0	0	0	0
N.S.	1	1.10	0.35	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.627	10.087	3.283	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	264	124	163	0	0	0	0	0
N.S.	1	1.22	0.57	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	10.076	2.109	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	348	106	210	0	0	0	0	0
N.S.	1	1.07	0.33	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	10.069	1.996	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	152	80	127	0	0	0	0	40
N.S.	1	1.21	0.63	1.01	0.00	0.00	0.00	0.00	0.32
time (sec)	N/A	0.294	10.043	2.011	0.000	0.000	0.000	0.000	9.264

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	312	54	195	0	0	0	0	0
N.S.	1	1.06	0.18	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	10.052	2.002	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	199	59	142	0	0	0	0	0
N.S.	1	1.22	0.36	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	10.048	2.022	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	418	59	262	0	0	0	0	0
N.S.	1	1.08	0.15	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.580	10.053	3.265	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	305	59	179	0	0	0	0	0
N.S.	1	1.22	0.24	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	10.049	5.210	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	485	131	303	0	0	0	0	0
N.S.	1	1.11	0.30	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	10.125	4.530	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	295	119	228	0	0	0	0	0
N.S.	1	1.23	0.50	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	10.095	3.109	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	379	94	237	0	0	0	0	0
N.S.	1	1.09	0.27	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	10.088	2.148	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	187	82	160	0	0	0	0	0
N.S.	1	1.26	0.55	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	10.074	2.075	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	309	62	197	0	0	0	0	40
N.S.	1	1.04	0.21	0.67	0.00	0.00	0.00	0.00	0.14
time (sec)	N/A	0.416	10.033	2.031	0.000	0.000	0.000	0.000	9.284

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	195	62	162	0	0	0	0	0
N.S.	1	1.23	0.39	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	10.065	2.076	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	414	64	267	0	0	0	0	0
N.S.	1	1.08	0.17	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.594	10.061	3.246	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	301	64	231	0	0	0	0	0
N.S.	1	1.22	0.26	0.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	10.059	5.330	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	520	64	333	0	0	0	0	0
N.S.	1	1.10	0.14	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	10.057	6.066	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	437	185	156	0	1293	0	396	0
N.S.	1	1.18	0.50	0.42	0.00	3.49	0.00	1.07	0.00
time (sec)	N/A	0.881	0.179	2.084	0.000	179.802	0.000	0.299	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	331	133	123	0	1031	0	312	0
N.S.	1	1.17	0.47	0.43	0.00	3.64	0.00	1.10	0.00
time (sec)	N/A	0.619	0.147	2.072	0.000	150.030	0.000	0.297	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	225	96	90	0	767	0	228	0
N.S.	1	1.15	0.49	0.46	0.00	3.93	0.00	1.17	0.00
time (sec)	N/A	0.431	0.109	2.037	0.000	147.950	0.000	0.297	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	121	74	57	0	501	0	143	40
N.S.	1	1.11	0.68	0.52	0.00	4.60	0.00	1.31	0.37
time (sec)	N/A	0.268	0.073	2.065	0.000	162.933	0.000	0.292	11.743

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	0	224	0	23	0
N.S.	1	1.00	1.00	1.17	0.00	9.74	0.00	1.00	0.00
time (sec)	N/A	0.153	0.063	2.051	0.000	170.839	0.000	0.275	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	89	76	79	0	0	0	72	0
N.S.	1	0.99	0.84	0.88	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.244	0.257	2.054	0.000	0.000	0.000	0.314	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	193	112	125	0	0	0	126	0
N.S.	1	1.08	0.63	0.70	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.394	0.340	2.029	0.000	0.000	0.000	0.324	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	299	149	167	0	0	0	177	0
N.S.	1	1.12	0.56	0.63	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.581	0.422	2.011	0.000	0.000	0.000	0.369	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	405	186	209	0	0	0	228	0
N.S.	1	1.14	0.53	0.59	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.785	0.506	2.098	0.000	0.000	0.000	0.377	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	403	168	145	0	1293	0	770	0
N.S.	1	1.17	0.49	0.42	0.00	3.77	0.00	2.24	0.00
time (sec)	N/A	0.756	6.333	2.034	0.000	199.502	0.000	0.310	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	297	131	112	0	1031	0	602	0
N.S.	1	1.16	0.51	0.44	0.00	4.04	0.00	2.36	0.00
time (sec)	N/A	0.545	6.254	2.015	0.000	144.930	0.000	0.317	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	193	94	79	0	768	0	434	40
N.S.	1	1.14	0.56	0.47	0.00	4.54	0.00	2.57	0.24
time (sec)	N/A	0.373	6.208	2.088	0.000	153.105	0.000	0.311	10.894

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	90	59	48	0	501	0	265	0
N.S.	1	1.07	0.70	0.57	0.00	5.96	0.00	3.15	0.00
time (sec)	N/A	0.235	6.227	2.048	0.000	175.324	0.000	0.305	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	80	88	67	0	0	0	83	0
N.S.	1	1.03	1.13	0.86	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.242	10.118	2.050	0.000	0.000	0.000	0.305	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	115	61	93	0	0	0	92	0
N.S.	1	1.02	0.54	0.82	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.278	10.070	1.749	0.000	0.000	0.000	0.311	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	221	61	139	0	0	0	143	0
N.S.	1	1.09	0.30	0.68	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.432	10.068	1.797	0.000	0.000	0.000	0.332	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	327	61	181	0	0	0	194	0
N.S.	1	1.12	0.21	0.62	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.623	10.074	1.846	0.000	0.000	0.000	0.540	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	433	61	223	0	0	0	245	0
N.S.	1	1.14	0.16	0.59	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.835	10.079	1.882	0.000	0.000	0.000	0.488	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	469	185	167	0	1294	0	206	0
N.S.	1	1.17	0.46	0.42	0.00	3.23	0.00	0.51	0.00
time (sec)	N/A	0.924	0.193	2.068	0.000	148.429	0.000	0.654	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	363	148	134	0	1031	0	164	0
N.S.	1	1.16	0.47	0.43	0.00	3.29	0.00	0.52	0.00
time (sec)	N/A	0.716	0.150	1.765	0.000	176.923	0.000	0.279	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	257	111	101	0	768	0	122	0
N.S.	1	1.14	0.49	0.45	0.00	3.41	0.00	0.54	0.00
time (sec)	N/A	0.473	0.129	2.077	0.000	180.269	0.000	0.292	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	151	74	68	0	502	0	80	0
N.S.	1	1.10	0.54	0.50	0.00	3.66	0.00	0.58	0.00
time (sec)	N/A	0.313	0.086	1.790	0.000	170.471	0.000	0.289	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	238	0	36	40
N.S.	1	1.00	0.77	0.77	0.00	5.06	0.00	0.77	0.85
time (sec)	N/A	0.183	0.057	1.793	0.000	138.458	0.000	0.283	9.364

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	61	0	0	0	51	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.203	0.183	1.809	0.000	0.000	0.000	0.300	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	101	123	0	0	0	109	0
N.S.	1	1.08	0.66	0.80	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.342	0.270	2.078	0.000	0.000	0.000	0.329	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	271	138	183	0	0	0	160	0
N.S.	1	1.12	0.57	0.76	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.497	0.333	2.183	0.000	0.000	0.000	0.326	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	377	175	243	0	0	0	211	0
N.S.	1	1.15	0.53	0.74	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.703	0.400	10.475	0.000	0.000	0.000	0.354	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	392	161	143	0	2566	0	214	0
N.S.	1	1.17	0.48	0.43	0.00	7.64	0.00	0.64	0.00
time (sec)	N/A	0.741	5.592	1.924	0.000	142.885	0.000	0.313	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	286	122	110	0	2083	0	163	0
N.S.	1	1.15	0.49	0.44	0.00	8.40	0.00	0.66	0.00
time (sec)	N/A	0.533	5.482	2.757	0.000	123.024	0.000	0.296	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	180	85	77	0	1598	0	112	0
N.S.	1	1.12	0.53	0.48	0.00	9.99	0.00	0.70	0.00
time (sec)	N/A	0.364	5.689	1.997	0.000	125.662	0.000	0.291	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	74	45	45	0	1107	0	60	0
N.S.	1	1.09	0.66	0.66	0.00	16.28	0.00	0.88	0.00
time (sec)	N/A	0.213	5.494	1.837	0.000	123.329	0.000	0.289	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	71	56	0	0	0	71	40
N.S.	1	1.00	1.18	0.93	0.00	0.00	0.00	1.18	0.67
time (sec)	N/A	0.192	2.186	2.286	0.000	0.000	0.000	0.286	9.343

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	160	110	88	0	0	0	105	0
N.S.	1	1.10	0.75	0.60	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.327	5.698	1.787	0.000	0.000	0.000	0.314	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	266	48	126	0	0	0	156	0
N.S.	1	1.13	0.20	0.53	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.509	10.092	2.033	0.000	0.000	0.000	0.329	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	372	48	159	0	0	0	207	0
N.S.	1	1.15	0.15	0.49	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.690	10.087	1.812	0.000	0.000	0.000	0.362	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	478	48	192	0	0	0	258	0
N.S.	1	1.16	0.12	0.47	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.925	10.096	1.991	0.000	0.000	0.000	0.382	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.153	0.003	0.038	0.198	0.279	0.017	0.281	0.023

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.151	0.001	0.034	0.192	0.289	0.016	0.281	0.021

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.148	0.000	0.030	0.186	0.271	0.016	0.279	0.021

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.148	0.001	0.027	0.202	0.277	0.018	0.284	0.021

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	14	12	11	10	10	8	10	10
N.S.	1	1.17	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.132	0.001	0.024	0.193	0.257	0.017	0.276	0.018

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.163	0.003	1.820	0.193	0.235	0.017	0.283	0.040

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.170	0.003	1.877	0.197	0.246	0.017	0.265	0.033

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.179	0.004	1.769	0.199	0.240	0.018	0.272	0.032

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.164	0.002	2.625	0.198	0.250	0.018	0.276	0.033

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.162	0.002	2.598	0.202	0.249	0.018	0.273	0.033

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	51
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89
time (sec)	N/A	0.196	0.006	2.537	0.216	0.251	0.060	0.276	8.831

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	40
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91
time (sec)	N/A	0.189	0.004	2.158	0.204	0.246	0.052	0.287	0.040

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.175	0.004	1.780	0.193	0.242	0.050	0.290	0.041

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.161	0.003	1.815	0.215	0.243	0.040	0.275	0.036

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.135	0.001	1.767	0.202	0.240	0.018	0.286	0.021

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	16	10	20	15
N.S.	1	1.00	1.00	0.89	1.00	0.89	0.56	1.11	0.83
time (sec)	N/A	0.145	0.004	2.029	0.201	0.261	0.059	0.271	8.898

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.183	0.005	1.862	0.203	0.258	0.074	0.268	0.055

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.184	0.006	1.751	0.221	0.280	0.097	0.288	0.058

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	48
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86
time (sec)	N/A	0.198	0.007	1.783	0.204	0.275	0.118	0.270	0.060

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	62	62
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.07
time (sec)	N/A	0.211	0.030	1.907	0.219	0.245	0.093	0.266	0.039

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	48	50
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.09
time (sec)	N/A	0.195	0.015	1.968	0.202	0.248	0.088	0.266	0.046

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	34	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.09
time (sec)	N/A	0.176	0.015	1.828	0.199	0.266	0.071	0.295	0.040

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.00
time (sec)	N/A	0.167	0.009	2.178	0.201	0.247	0.056	0.277	0.037

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00
time (sec)	N/A	0.137	0.002	2.295	0.210	0.246	0.053	0.283	0.032

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	0.90
time (sec)	N/A	0.173	0.012	2.312	0.204	0.277	0.095	0.284	0.046

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	41
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	0.98
time (sec)	N/A	0.191	0.043	2.358	0.210	0.260	0.126	0.280	8.845

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	64	57
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.10	0.98
time (sec)	N/A	0.204	0.052	1.974	0.205	0.248	0.146	0.278	8.858

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	73	69
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.00
time (sec)	N/A	0.227	0.062	4.385	0.199	0.261	0.156	0.286	0.069

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	86	79
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.02	0.94
time (sec)	N/A	0.231	0.050	4.141	0.217	0.269	0.171	0.281	0.074

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	117	53	32	53	62	0	131	62
N.S.	1	1.11	0.50	0.30	0.50	0.59	0.00	1.25	0.59
time (sec)	N/A	0.284	0.019	2.575	0.212	0.252	0.000	0.263	9.017

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	42	21	42	51	0	108	51
N.S.	1	1.08	0.52	0.26	0.52	0.64	0.00	1.35	0.64
time (sec)	N/A	0.228	0.013	1.864	0.218	0.263	0.000	0.273	9.026

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	13	30	39	0	81	39
N.S.	1	1.00	0.79	0.25	0.58	0.75	0.00	1.56	0.75
time (sec)	N/A	0.193	0.008	1.914	0.212	0.263	0.000	0.285	8.936

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	12	26	0	50	0
N.S.	1	1.00	0.92	1.00	0.48	1.04	0.00	2.00	0.00
time (sec)	N/A	0.157	0.008	1.890	0.211	0.264	0.000	0.290	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	36	0	111	0	67	73
N.S.	1	1.00	1.04	0.71	0.00	2.18	0.00	1.31	1.43
time (sec)	N/A	0.194	0.030	1.909	0.000	0.250	0.000	0.288	9.147

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	64	50	0	127	0	45	0
N.S.	1	1.00	1.23	0.96	0.00	2.44	0.00	0.87	0.00
time (sec)	N/A	0.189	0.026	1.971	0.000	0.256	0.000	0.294	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	82	81	61	0	149	0	72	0
N.S.	1	0.98	0.96	0.73	0.00	1.77	0.00	0.86	0.00
time (sec)	N/A	0.235	0.039	2.392	0.000	0.270	0.000	0.291	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	93	72	0	175	0	92	0
N.S.	1	1.04	0.83	0.64	0.00	1.56	0.00	0.82	0.00
time (sec)	N/A	0.284	0.040	1.998	0.000	0.271	0.000	0.316	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	185	80	32	86	95	0	282	80
N.S.	1	1.15	0.50	0.20	0.53	0.59	0.00	1.75	0.50
time (sec)	N/A	0.375	0.056	1.891	0.223	0.274	0.000	0.276	9.139

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	154	69	21	75	84	0	246	69
N.S.	1	1.13	0.51	0.15	0.55	0.62	0.00	1.81	0.51
time (sec)	N/A	0.329	0.048	1.855	0.217	0.262	0.000	0.282	9.010

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	120	58	13	64	73	0	210	58
N.S.	1	1.11	0.54	0.12	0.59	0.68	0.00	1.94	0.54
time (sec)	N/A	0.269	0.019	1.818	0.216	0.256	0.000	0.290	9.086

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	47	35	53	62	0	173	47
N.S.	1	1.08	0.59	0.44	0.66	0.78	0.00	2.16	0.59
time (sec)	N/A	0.240	0.018	1.823	0.220	0.263	0.000	0.290	9.034

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	41	50	0	136	36
N.S.	1	1.00	0.60	0.67	0.79	0.96	0.00	2.62	0.69
time (sec)	N/A	0.193	0.015	1.838	0.208	0.252	0.000	0.292	8.923

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	28	37	0	89	28
N.S.	1	1.00	0.92	1.08	1.12	1.48	0.00	3.56	1.12
time (sec)	N/A	0.159	0.010	2.421	0.221	0.254	0.000	0.277	9.287

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	76	68	61	0	130	0	85	0
N.S.	1	1.03	0.92	0.82	0.00	1.76	0.00	1.15	0.00
time (sec)	N/A	0.231	0.048	2.020	0.000	0.282	0.000	0.280	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	66	70	0	136	0	62	0
N.S.	1	1.05	0.90	0.96	0.00	1.86	0.00	0.85	0.00
time (sec)	N/A	0.230	0.045	1.888	0.000	0.284	0.000	0.299	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	80	82	67	0	154	0	70	0
N.S.	1	0.99	1.01	0.83	0.00	1.90	0.00	0.86	0.00
time (sec)	N/A	0.229	0.039	1.943	0.000	0.273	0.000	0.336	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	110	94	81	0	175	0	92	0
N.S.	1	1.01	0.86	0.74	0.00	1.61	0.00	0.84	0.00
time (sec)	N/A	0.281	0.050	2.248	0.000	0.281	0.000	0.297	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	144	104	92	0	197	0	109	0
N.S.	1	1.05	0.76	0.67	0.00	1.44	0.00	0.80	0.00
time (sec)	N/A	0.324	0.236	2.148	0.000	0.276	0.000	0.304	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	178	116	103	0	219	0	126	0
N.S.	1	1.08	0.70	0.62	0.00	1.33	0.00	0.76	0.00
time (sec)	N/A	0.379	0.252	2.873	0.000	0.277	0.000	0.313	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	115	53	52	53	51	0	64	51
N.S.	1	1.12	0.51	0.50	0.51	0.50	0.00	0.62	0.50
time (sec)	N/A	0.271	0.018	1.877	0.232	0.263	0.000	0.272	8.919

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	81	42	41	42	40	0	52	40
N.S.	1	1.08	0.56	0.55	0.56	0.53	0.00	0.69	0.53
time (sec)	N/A	0.226	0.014	1.834	0.213	0.260	0.000	0.282	8.878

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	30	30	30	28	0	38	31
N.S.	1	1.00	0.61	0.61	0.61	0.57	0.00	0.78	0.63
time (sec)	N/A	0.184	0.011	1.846	0.251	0.262	0.000	0.272	8.866

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	21	12	21	0	27	17
N.S.	1	1.00	0.91	0.91	0.52	0.91	0.00	1.17	0.74
time (sec)	N/A	0.151	0.006	2.034	0.216	0.268	0.000	0.271	8.878

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	13	0	74	0	45	0
N.S.	1	1.00	1.53	0.43	0.00	2.47	0.00	1.50	0.00
time (sec)	N/A	0.155	0.010	1.830	0.000	0.265	0.000	0.276	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	18	0	127	0	51	0
N.S.	1	1.00	1.11	0.33	0.00	2.35	0.00	0.94	0.00
time (sec)	N/A	0.190	0.021	2.158	0.000	0.259	0.000	0.312	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	88	83	36	0	153	0	73	44
N.S.	1	1.01	0.95	0.41	0.00	1.76	0.00	0.84	0.51
time (sec)	N/A	0.233	0.031	2.494	0.000	0.268	0.000	0.293	9.120

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	122	96	56	0	175	0	88	0
N.S.	1	1.06	0.83	0.49	0.00	1.52	0.00	0.77	0.00
time (sec)	N/A	0.286	0.031	2.027	0.000	0.280	0.000	0.304	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	110	50	56	41	60	0	79	57
N.S.	1	1.12	0.51	0.57	0.42	0.61	0.00	0.81	0.58
time (sec)	N/A	0.270	0.013	1.916	0.201	0.263	0.000	0.284	9.118

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	39	46	30	49	0	64	47
N.S.	1	1.08	0.54	0.64	0.42	0.68	0.00	0.89	0.65
time (sec)	N/A	0.228	0.011	1.991	0.209	0.260	0.000	0.269	9.074

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	26	34	19	38	0	48	35
N.S.	1	1.00	0.55	0.72	0.40	0.81	0.00	1.02	0.74
time (sec)	N/A	0.187	0.010	1.849	0.287	0.283	0.000	0.280	8.991

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	12	29	0	27	28
N.S.	1	1.00	0.90	1.29	0.57	1.38	0.00	1.29	1.33
time (sec)	N/A	0.151	0.005	1.885	0.251	0.251	0.000	0.286	8.936

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	54	31	0	156	0	77	0
N.S.	1	1.00	1.04	0.60	0.00	3.00	0.00	1.48	0.00
time (sec)	N/A	0.193	0.015	1.774	0.000	0.271	0.000	0.271	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	62	20	0	189	0	72	0
N.S.	1	1.07	0.83	0.27	0.00	2.52	0.00	0.96	0.00
time (sec)	N/A	0.234	0.023	1.824	0.000	0.275	0.000	0.290	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	84	13	0	219	0	92	42
N.S.	1	1.06	0.76	0.12	0.00	1.99	0.00	0.84	0.38
time (sec)	N/A	0.275	0.027	1.829	0.000	0.279	0.000	0.311	9.139

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	151	96	31	0	241	0	107	0
N.S.	1	1.09	0.70	0.22	0.00	1.75	0.00	0.78	0.00
time (sec)	N/A	0.329	0.184	1.851	0.000	0.272	0.000	0.305	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	185	106	51	0	263	0	122	44
N.S.	1	1.11	0.64	0.31	0.00	1.58	0.00	0.73	0.27
time (sec)	N/A	0.394	0.243	1.866	0.000	0.296	0.000	0.313	9.888

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	132	110	100	0	180	0	83	0
N.S.	1	1.06	0.88	0.80	0.00	1.44	0.00	0.66	0.00
time (sec)	N/A	0.289	0.061	1.936	0.000	0.280	0.000	0.303	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	96	99	89	0	159	0	71	0
N.S.	1	1.01	1.04	0.94	0.00	1.67	0.00	0.75	0.00
time (sec)	N/A	0.245	0.032	2.066	0.000	0.272	0.000	0.280	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	80	78	0	131	0	55	0
N.S.	1	1.00	1.33	1.30	0.00	2.18	0.00	0.92	0.00
time (sec)	N/A	0.200	0.032	1.836	0.000	0.277	0.000	0.291	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	53	58	0	77	0	37	0
N.S.	1	1.00	1.56	1.71	0.00	2.26	0.00	1.09	0.00
time (sec)	N/A	0.162	0.014	1.769	0.000	0.267	0.000	0.283	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	0	21	0	34	0
N.S.	1	1.00	0.92	1.00	0.00	0.84	0.00	1.36	0.00
time (sec)	N/A	0.150	0.011	1.871	0.000	0.264	0.000	0.289	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	31	31	0	29	0	54	0
N.S.	1	1.00	0.55	0.55	0.00	0.52	0.00	0.96	0.00
time (sec)	N/A	0.185	0.014	1.804	0.000	0.269	0.000	0.310	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	92	44	44	0	40	0	81	0
N.S.	1	1.07	0.51	0.51	0.00	0.47	0.00	0.94	0.00
time (sec)	N/A	0.229	0.016	1.816	0.000	0.265	0.000	0.292	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	128	55	55	0	51	0	107	0
N.S.	1	1.10	0.47	0.47	0.00	0.44	0.00	0.92	0.00
time (sec)	N/A	0.275	0.019	2.790	0.000	0.268	0.000	0.314	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	61	59	0	0	0	0	0	0
N.S.	1	1.27	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	0	38	0	0	54
N.S.	1	1.00	0.94	1.12	0.00	1.19	0.00	0.00	1.69
time (sec)	N/A	0.165	0.044	1.934	0.000	0.274	0.000	0.000	9.031

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	44	50	0	70	0	0	98
N.S.	1	1.00	0.63	0.71	0.00	1.00	0.00	0.00	1.40
time (sec)	N/A	0.221	0.070	1.921	0.000	0.272	0.000	0.000	9.068

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	114	72	84	0	111	0	0	157
N.S.	1	0.98	0.62	0.72	0.00	0.96	0.00	0.00	1.35
time (sec)	N/A	0.288	0.061	1.937	0.000	0.274	0.000	0.000	9.126

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.148	0.011	1.803	0.196	0.258	0.168	0.289	8.945

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	46	43	46	42	0	65	42
N.S.	1	1.08	0.58	0.54	0.58	0.52	0.00	0.81	0.52
time (sec)	N/A	0.246	0.017	2.506	0.226	0.270	0.000	0.283	9.090

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	34	32	34	30	0	48	33
N.S.	1	1.00	0.65	0.62	0.65	0.58	0.00	0.92	0.63
time (sec)	N/A	0.200	0.012	2.275	0.208	0.256	0.000	0.275	8.999

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	14	21	0	29	21
N.S.	1	1.00	1.00	0.88	0.56	0.84	0.00	1.16	0.84
time (sec)	N/A	0.158	0.010	2.218	0.219	0.264	0.000	0.275	9.069

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	54	43	0	75	0	47	0
N.S.	1	1.00	1.69	1.34	0.00	2.34	0.00	1.47	0.00
time (sec)	N/A	0.161	0.012	2.144	0.000	0.274	0.000	0.289	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	76	66	0	127	0	55	0
N.S.	1	1.00	1.29	1.12	0.00	2.15	0.00	0.93	0.00
time (sec)	N/A	0.200	0.025	2.311	0.000	0.275	0.000	0.291	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	68	248	0	37	0	0	0
N.S.	1	1.00	0.29	1.04	0.00	0.16	0.00	0.00	0.00
time (sec)	N/A	0.311	10.036	2.256	0.000	0.074	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	52	231	0	14	0	0	0
N.S.	1	1.00	0.25	1.09	0.00	0.07	0.00	0.00	0.00
time (sec)	N/A	0.256	10.018	1.945	0.000	0.072	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	55	248	0	38	0	0	44
N.S.	1	1.00	0.23	1.02	0.00	0.16	0.00	0.00	0.18
time (sec)	N/A	0.307	10.016	2.025	0.000	0.077	0.000	0.000	9.711

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	543	68	25	0	45	0	0	0
N.S.	1	1.06	0.13	0.05	0.00	0.09	0.00	0.00	0.00
time (sec)	N/A	0.572	10.039	1.995	0.000	0.075	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	512	55	15	0	22	0	0	0
N.S.	1	1.06	0.11	0.03	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.513	10.021	1.859	0.000	0.076	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	543	50	20	0	46	0	0	0
N.S.	1	1.06	0.10	0.04	0.00	0.09	0.00	0.00	0.00
time (sec)	N/A	0.575	10.020	1.825	0.000	0.077	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	273	86	742	0	0	0	0	0
N.S.	1	1.03	0.32	2.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.409	10.038	2.232	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	536	70	1115	0	0	0	0	0
N.S.	1	1.02	0.13	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.658	10.031	2.248	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	82	79	0	148	0	52	0
N.S.	1	1.00	1.26	1.22	0.00	2.28	0.00	0.80	0.00
time (sec)	N/A	0.215	0.034	1.872	0.000	0.361	0.000	0.313	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	70	727	0	0	0	0	0
N.S.	1	1.00	0.30	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	10.034	2.080	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	502	57	2374	0	0	0	0	0
N.S.	1	1.02	0.12	4.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	10.024	1.977	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	60	59	0	101	0	40	0
N.S.	1	1.00	1.67	1.64	0.00	2.81	0.00	1.11	0.00
time (sec)	N/A	0.177	0.012	1.780	0.000	0.353	0.000	0.299	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	55	437	0	16	0	0	0
N.S.	1	1.00	0.27	2.15	0.00	0.08	0.00	0.00	0.00
time (sec)	N/A	0.294	10.016	1.873	0.000	0.079	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	532	55	1115	0	24	0	0	0
N.S.	1	1.03	0.11	2.15	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.595	10.020	2.352	0.000	0.078	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	26	21	0	28	0
N.S.	1	1.00	1.00	1.07	0.96	0.78	0.00	1.04	0.00
time (sec)	N/A	0.156	0.012	1.804	0.214	0.254	0.000	0.300	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	57	732	0	48	0	0	0
N.S.	1	1.00	0.24	3.11	0.00	0.20	0.00	0.00	0.00
time (sec)	N/A	0.336	10.019	2.164	0.000	0.074	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	568	57	1125	0	55	0	0	0
N.S.	1	1.02	0.10	2.03	0.00	0.10	0.00	0.00	0.00
time (sec)	N/A	0.657	10.018	2.285	0.000	0.076	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	35	37	38	31	0	43	0
N.S.	1	1.00	0.62	0.66	0.68	0.55	0.00	0.77	0.00
time (sec)	N/A	0.204	0.015	1.826	0.213	0.266	0.000	0.323	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	271	57	742	0	62	0	0	0
N.S.	1	1.02	0.22	2.80	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.379	10.019	2.108	0.000	0.085	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.170	0.006	2.066	0.204	0.250	0.082	0.268	0.050

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.201	0.005	2.001	0.204	0.259	0.092	0.285	9.032

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	119	112	91	0	171	0	104	0
N.S.	1	1.06	1.00	0.81	0.00	1.53	0.00	0.93	0.00
time (sec)	N/A	0.310	0.053	2.429	0.000	0.284	0.000	0.321	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	87	101	69	0	150	0	88	0
N.S.	1	1.01	1.17	0.80	0.00	1.74	0.00	1.02	0.00
time (sec)	N/A	0.263	0.029	2.145	0.000	0.281	0.000	0.287	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	82	47	0	122	0	69	0
N.S.	1	1.00	1.46	0.84	0.00	2.18	0.00	1.23	0.00
time (sec)	N/A	0.211	0.034	2.106	0.000	0.260	0.000	0.307	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	57	25	0	74	0	47	0
N.S.	1	1.00	1.78	0.78	0.00	2.31	0.00	1.47	0.00
time (sec)	N/A	0.177	0.012	2.006	0.000	0.279	0.000	0.307	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	0	21	0	27	21
N.S.	1	1.00	0.91	0.87	0.00	0.91	0.00	1.17	0.91
time (sec)	N/A	0.154	0.008	2.001	0.000	0.262	0.000	0.296	9.076

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	26	0	29	0	53	42
N.S.	1	1.00	0.60	0.50	0.00	0.56	0.00	1.02	0.81
time (sec)	N/A	0.190	0.012	2.012	0.000	0.285	0.000	0.302	9.051

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	42	39	0	40	0	82	40
N.S.	1	1.08	0.52	0.49	0.00	0.50	0.00	1.02	0.50
time (sec)	N/A	0.230	0.012	2.078	0.000	0.270	0.000	0.302	9.088

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	120	53	50	0	51	0	111	92
N.S.	1	1.11	0.49	0.46	0.00	0.47	0.00	1.03	0.85
time (sec)	N/A	0.281	0.016	2.122	0.000	0.279	0.000	0.314	9.101

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	154	64	61	0	62	0	140	116
N.S.	1	1.13	0.47	0.45	0.00	0.46	0.00	1.03	0.85
time (sec)	N/A	0.332	0.150	2.146	0.000	0.269	0.000	0.298	9.065

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	27	26	23	22	28	22	32	22
N.S.	1	1.04	1.00	0.88	0.85	1.08	0.85	1.23	0.85
time (sec)	N/A	0.185	0.007	1.787	0.209	0.279	0.099	0.275	0.051

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	22	28	22	32	22
N.S.	1	0.96	1.00	0.85	0.81	1.04	0.81	1.19	0.81
time (sec)	N/A	0.192	0.008	1.716	0.196	0.270	0.103	0.294	9.074

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	14	8	5	15	8
N.S.	1	1.00	1.75	1.12	1.75	1.00	0.62	1.88	1.00
time (sec)	N/A	0.139	0.004	0.019	0.212	0.268	0.022	0.275	9.058

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	18	15	16	10
N.S.	1	1.00	1.00	1.10	1.60	1.80	1.50	1.60	1.00
time (sec)	N/A	0.138	0.004	0.017	0.202	0.275	0.024	0.278	0.034

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	16	26	27	16	26
N.S.	1	1.00	1.00	0.92	1.33	2.17	2.25	1.33	2.17
time (sec)	N/A	0.139	0.005	0.020	0.193	0.256	0.028	0.276	9.086

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00
time (sec)	N/A	0.135	0.001	1.754	0.206	0.248	0.022	0.282	0.027

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	21	22	32	0	19
N.S.	1	1.00	1.00	0.95	1.05	1.10	1.60	0.00	0.95
time (sec)	N/A	0.146	0.004	0.041	0.201	0.248	0.259	0.000	9.080

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	40	36	82	0	21
N.S.	1	1.00	1.00	1.05	2.00	1.80	4.10	0.00	1.05
time (sec)	N/A	0.148	0.004	0.031	0.204	0.258	0.375	0.000	9.080

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	53	52	119	0	21
N.S.	1	1.00	1.00	1.05	2.65	2.60	5.95	0.00	1.05
time (sec)	N/A	0.148	0.004	0.038	0.203	0.256	0.453	0.000	9.096

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.161	0.005	1.791	0.211	0.258	0.042	0.281	9.142

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.148	0.005	1.959	0.212	0.252	0.046	0.272	9.154

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.146	0.005	2.210	0.208	0.245	0.048	0.283	0.119

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	338	275	231	520	285	287
N.S.	1	1.00	0.89	12.52	10.19	8.56	19.26	10.56	10.63
time (sec)	N/A	0.158	0.014	53.747	0.233	0.257	6.300	0.397	9.851

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.160	0.002	1.820	0.201	0.248	0.042	0.294	0.002

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.159	0.005	1.988	0.206	0.242	0.048	0.276	9.181

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.155	0.007	2.230	0.226	0.242	0.047	0.283	0.117

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	193	281	275	205	471	0	285
N.S.	1	1.00	7.15	10.41	10.19	7.59	17.44	0.00	10.56
time (sec)	N/A	0.166	0.119	46.017	0.210	0.256	5.034	0.000	9.937

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	88	120	121	93	197	93	124
N.S.	1	1.00	3.26	4.44	4.48	3.44	7.30	3.44	4.59
time (sec)	N/A	0.174	0.075	2.236	0.198	0.260	0.634	0.288	9.336

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	39	66	82	0	0	38
N.S.	1	1.00	1.00	1.44	2.44	3.04	0.00	0.00	1.41
time (sec)	N/A	0.173	0.084	1.850	0.202	0.247	0.000	0.000	9.132

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.157	0.003	0.020	0.192	0.246	0.047	0.283	0.044

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.158	0.005	0.027	0.201	0.237	0.064	0.274	8.926

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.155	0.006	0.023	0.201	0.237	0.078	0.289	0.045

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.149	0.013	0.033	0.208	0.238	0.126	0.282	0.040

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.155	0.011	0.045	0.193	0.235	0.228	0.278	0.060

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.156	0.014	0.056	0.195	0.237	0.320	0.277	8.905

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.172	0.001	0.033	0.188	0.245	0.037	0.273	0.042

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	42	40	35	34	38	37	46	34
N.S.	1	1.05	1.00	0.88	0.85	0.95	0.92	1.15	0.85
time (sec)	N/A	0.190	0.009	0.074	0.187	0.247	0.060	0.271	0.037

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	44	48	49	45	47
N.S.	1	1.00	1.00	0.90	0.88	0.96	0.98	0.90	0.94
time (sec)	N/A	0.201	0.008	0.061	0.206	0.247	0.071	0.275	0.047

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	170	144	34	124	637	36	112	197
N.S.	1	0.92	0.78	0.18	0.67	3.44	0.19	0.61	1.06
time (sec)	N/A	0.448	0.171	0.064	0.302	0.900	0.812	0.291	9.639

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	232	363	325	297	690	50971	363
N.S.	1	1.00	8.00	12.52	11.21	10.24	23.79	1757.62	12.52
time (sec)	N/A	0.158	0.191	0.031	0.222	0.274	149.341	1.458	10.377

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.150	0.004	1.899	0.208	0.237	0.041	0.284	8.889

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.148	0.002	2.075	0.186	0.240	0.050	0.274	0.002

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.144	0.005	2.250	0.185	0.238	0.054	0.291	8.935

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.145	0.004	2.174	0.200	0.240	0.043	0.283	0.092

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.149	0.003	2.202	0.199	0.281	0.050	0.284	0.002

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.141	0.002	2.535	0.197	0.256	0.047	0.299	8.990

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	37	27	53	0	26
N.S.	1	1.00	1.00	1.17	1.61	1.17	2.30	0.00	1.13
time (sec)	N/A	0.166	0.022	1.804	0.191	0.258	0.328	0.000	9.039

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	26	25	29	27	28	49	0	31
N.S.	1	1.13	1.09	1.26	1.17	1.22	2.13	0.00	1.35
time (sec)	N/A	0.170	0.009	1.875	0.193	0.265	0.321	0.000	8.924

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	31	19	28	51	0	34
N.S.	1	1.00	1.00	2.07	1.27	1.87	3.40	0.00	2.27
time (sec)	N/A	0.160	0.005	1.777	0.200	0.259	0.353	0.000	8.953

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	25	24	25	16	26	31	0	26
N.S.	1	1.14	1.09	1.14	0.73	1.18	1.41	0.00	1.18
time (sec)	N/A	0.162	0.036	1.810	0.190	0.258	0.265	0.000	9.081

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	11	26	31	0	28
N.S.	1	1.00	1.00	1.80	0.73	1.73	2.07	0.00	1.87
time (sec)	N/A	0.155	0.024	1.769	0.199	0.258	0.293	0.000	9.130

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	8	8	8	9	8
N.S.	1	1.00	0.83	0.75	0.67	0.67	0.67	0.75	0.67
time (sec)	N/A	0.153	0.036	1.725	0.191	0.243	0.058	0.277	0.087

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	25	11	17	8	22	18	8
N.S.	1	1.00	1.79	0.79	1.21	0.57	1.57	1.29	0.57
time (sec)	N/A	0.150	0.020	1.744	0.187	0.256	0.106	0.293	9.221

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	32	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	0.67
time (sec)	N/A	0.150	0.002	1.713	0.290	0.256	0.074	0.280	0.091

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	33	24	28	31	24	31	0	26
N.S.	1	1.38	1.00	1.17	1.29	1.00	1.29	0.00	1.08
time (sec)	N/A	0.173	0.047	1.906	0.284	0.263	0.256	0.000	9.230

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	104	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	95	109	0	0	0	0	0	0
N.S.	1	0.96	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.344	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	103	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	1.626	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	69	99	0	0	0	0	0	0
N.S.	1	0.97	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	1.579	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	46	45	39	58	97	78	0	0
N.S.	1	0.90	0.88	0.76	1.14	1.90	1.53	0.00	0.00
time (sec)	N/A	0.178	0.011	2.068	0.308	0.270	0.913	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.259	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	77	0	0	0	0	0	97
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	1.59
time (sec)	N/A	0.213	0.122	0.000	0.000	0.000	0.000	0.000	9.142

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	131	131	0	0	0	0	0	0
N.S.	1	0.93	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.635	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	133	126	0	0	0	0	0	0
N.S.	1	1.04	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	1.905	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	101	117	0	0	0	0	0	0
N.S.	1	0.97	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.259	0.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	127	120	0	0	0	0	0	0
N.S.	1	1.04	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	1.617	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	64	58	51	73	120	88	0	0
N.S.	1	0.88	0.79	0.70	1.00	1.64	1.21	0.00	0.00
time (sec)	N/A	0.183	0.029	2.300	0.286	0.711	1.578	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	122	97	0	0	0	0	0	0
N.S.	1	1.04	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	95	94	0	0	0	0	0	0
N.S.	1	0.97	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	125	100	0	0	0	0	0	0
N.S.	1	1.02	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	97	96	0	0	0	0	0	0
N.S.	1	0.97	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	58	47	0	93	0	67	67
N.S.	1	1.00	1.14	0.92	0.00	1.82	0.00	1.31	1.31
time (sec)	N/A	0.240	0.029	0.102	0.000	0.263	0.000	0.284	9.154

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	62	61	53	108	0	69	55
N.S.	1	1.00	1.48	1.45	1.26	2.57	0.00	1.64	1.31
time (sec)	N/A	0.190	0.029	0.052	0.289	0.486	0.000	0.308	9.196

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	55	0	104	0	71	63
N.S.	1	1.00	1.29	1.08	0.00	2.04	0.00	1.39	1.24
time (sec)	N/A	0.208	0.038	0.055	0.000	0.532	0.000	0.284	9.310

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	69	74	0	112	0	0	0
N.S.	1	1.00	1.13	1.21	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.226	0.028	2.213	0.000	0.478	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	55	0	98	0	61	67
N.S.	1	1.00	1.25	1.04	0.00	1.85	0.00	1.15	1.26
time (sec)	N/A	0.232	0.033	0.126	0.000	0.305	0.000	0.285	9.211

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	68	81	34	118	0	63	54
N.S.	1	1.00	1.58	1.88	0.79	2.74	0.00	1.47	1.26
time (sec)	N/A	0.189	0.065	0.063	0.305	0.286	0.000	0.296	9.321

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	73	73	0	109	0	65	63
N.S.	1	1.00	1.38	1.38	0.00	2.06	0.00	1.23	1.19
time (sec)	N/A	0.211	0.073	0.085	0.000	0.276	0.000	0.286	9.233

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	105	0	118	0	0	0
N.S.	1	1.00	1.22	1.67	0.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.232	0.082	2.098	0.000	0.279	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	98	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.572	0.000	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	1.497	0.000	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.170	0.103	0.000	0.000	0.000	0.000	0.000	9.193

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	87	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	1.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	42	76	27	0	0
N.S.	1	1.00	1.00	0.84	1.35	2.45	0.87	0.00	0.00
time (sec)	N/A	0.163	0.006	2.068	0.303	0.518	0.631	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	66	0	0	0	0	0	0
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	0.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	98	117	0	0	0	0	0	0
N.S.	1	0.92	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	109	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	3.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	70	91	0	0	0	0	0	0
N.S.	1	0.97	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.219	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	1.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	49	52	42	61	148	185	0	0
N.S.	1	0.91	0.96	0.78	1.13	2.74	3.43	0.00	0.00
time (sec)	N/A	0.175	0.032	1.756	0.289	0.409	1.165	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	70	74	0	0	0	0	0	0
N.S.	1	0.97	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	70	74	0	0	0	0	0	0
N.S.	1	0.97	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.137	0.000	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	64	56	0	102	0	0	0
N.S.	1	1.00	2.00	1.75	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.177	0.025	0.294	0.000	0.343	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	60	49	0	80	0	40	0
N.S.	1	1.00	1.88	1.53	0.00	2.50	0.00	1.25	0.00
time (sec)	N/A	0.180	0.023	0.071	0.000	0.289	0.000	0.297	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	64	0	0	102	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.180	0.026	0.000	0.000	0.600	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	76	0	0	102	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.187	0.045	0.000	0.000	0.318	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	72	60	0	111	0	0	0
N.S.	1	1.00	2.18	1.82	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.173	0.236	2.440	0.000	0.573	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	68	51	0	88	0	47	0
N.S.	1	1.00	2.06	1.55	0.00	2.67	0.00	1.42	0.00
time (sec)	N/A	0.184	0.026	0.063	0.000	0.527	0.000	0.290	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	72	0	0	111	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.178	0.319	0.000	0.000	0.917	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	78	0	0	106	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.190	0.094	0.000	0.000	0.262	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.186	0.105	0.000	0.000	0.000	0.000	0.000	9.269

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	109	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	2.95	0.00	0.00	1.81
time (sec)	N/A	0.178	0.017	0.000	0.000	0.281	0.000	0.000	9.190

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.181	0.016	0.000	0.000	0.000	0.000	0.000	9.150

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.183	0.107	0.000	0.000	0.000	0.000	0.000	9.221

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	109	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	2.87	0.00	0.00	1.74
time (sec)	N/A	0.184	0.017	0.000	0.000	0.442	0.000	0.000	9.126

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.180	0.017	0.000	0.000	0.000	0.000	0.000	9.148

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	218	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.568	0.000	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	156	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.355	0.000	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	120	106	0	0	0	0	0	0
N.S.	1	1.18	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	123	116	0	0	0	0	0	0
N.S.	1	1.11	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	0.349	0.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	123	166	0	0	0	0	0	0
N.S.	1	1.11	1.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.625	0.000	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	177	0	0	0	0	0	82
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.252	0.234	0.000	0.000	0.000	0.000	0.000	9.170

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	101	134	0	0	0	0	0	82
N.S.	1	1.16	1.54	0.00	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.244	0.178	0.000	0.000	0.000	0.000	0.000	9.142

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	0	0	0	0	83
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.241	0.084	0.000	0.000	0.000	0.000	0.000	9.164

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0	83
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.246	0.124	0.000	0.000	0.000	0.000	0.000	9.377

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	185	0	0	0	0	0	83
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.249	0.283	0.000	0.000	0.000	0.000	0.000	9.510

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	16	0	16	0	50	15
N.S.	1	1.00	1.06	0.89	0.00	0.89	0.00	2.78	0.83
time (sec)	N/A	0.164	0.024	0.058	0.000	0.731	0.000	0.296	9.101

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	19	0	11	27
N.S.	1	1.00	1.00	0.90	0.00	0.95	0.00	0.55	1.35
time (sec)	N/A	0.143	0.043	2.030	0.000	0.281	0.000	0.284	9.155

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.146	0.002	1.794	0.286	0.246	0.085	0.311	9.125

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.88
time (sec)	N/A	0.162	0.014	1.893	0.211	0.248	0.000	0.294	9.120

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16
time (sec)	N/A	0.154	0.002	1.890	0.207	0.248	0.000	0.290	8.945

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	14	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.56	0.88
time (sec)	N/A	0.167	0.013	2.412	0.224	0.244	0.000	0.284	8.941

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	988	61	61	0	0	0	0	0	42
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	0.199	10.024	0.000	0.000	0.000	0.000	0.000	9.109

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	487	61	61	0	0	0	0	0	42
N.S.	1	0.13	0.13	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.199	10.021	0.000	0.000	0.000	0.000	0.000	9.100

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	92	92	0	0	0	0	0	0
N.S.	1	1.03	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	72	74	0	0	0	0	0	0
N.S.	1	1.09	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	95	83	0	0	0	0	0	0
N.S.	1	1.13	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	64	0	0	76
N.S.	1	1.00	0.98	0.00	0.00	1.45	0.00	0.00	1.73
time (sec)	N/A	0.174	0.076	0.000	0.000	0.468	0.000	0.000	9.155

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	61	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.190	0.002	0.000	0.000	0.266	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	76	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.212	0.088	0.000	0.000	0.255	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	79	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.177	0.004	0.000	0.000	0.268	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.070	1.808	0.245	0.359	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.00
time (sec)	N/A	0.191	0.069	1.802	0.239	0.423	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.067	1.829	0.234	0.690	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	47	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.206	0.052	0.000	0.000	0.253	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	45	0	0	54	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.202	0.031	0.000	0.000	0.259	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	40	0	0	76	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.181	0.144	0.000	0.000	0.267	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	38	86	0	64	0	0	0
N.S.	1	1.00	0.95	2.15	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.224	0.121	2.915	0.000	0.453	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [347] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	13	0.231
2	A	3	3	1.00	11	0.273
3	A	1	1	1.00	9	0.111
4	A	2	2	1.00	13	0.154
5	A	3	3	1.00	13	0.231
6	A	3	3	1.00	15	0.200
7	A	5	4	1.13	13	0.308
8	A	3	3	1.00	11	0.273
9	A	2	2	1.00	15	0.133
10	A	3	3	1.00	15	0.200
11	A	3	3	1.00	11	0.273
12	A	5	4	0.96	15	0.267
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	15	0.133
15	A	2	2	1.00	13	0.154
16	A	6	5	1.18	11	0.455
17	A	3	3	1.00	15	0.200
18	A	5	4	1.03	15	0.267
19	A	4	4	1.19	15	0.267
20	A	5	4	1.02	15	0.267
21	A	3	3	1.00	15	0.200
22	A	5	4	1.03	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	4	1.07	11	0.364
24	A	5	4	1.06	15	0.267
25	A	5	5	1.15	15	0.333
26	A	3	3	1.00	13	0.231
27	A	5	4	1.00	13	0.308
28	A	3	3	1.00	13	0.231
29	A	2	2	1.00	13	0.154
30	A	2	2	1.00	11	0.182
31	A	6	5	1.27	9	0.556
32	A	3	3	1.00	13	0.231
33	A	5	4	1.09	13	0.308
34	A	4	4	1.00	13	0.308
35	A	5	4	1.07	13	0.308
36	A	6	5	1.27	9	0.556
37	A	7	6	1.11	11	0.545
38	A	7	6	1.07	17	0.353
39	A	9	8	1.07	17	0.471
40	A	6	5	1.04	15	0.333
41	A	8	7	1.05	13	0.538
42	A	5	4	1.00	17	0.235
43	A	8	7	1.07	17	0.412
44	A	5	4	1.00	17	0.235
45	A	9	8	1.05	17	0.471
46	A	8	7	1.08	17	0.412
47	A	10	9	1.07	15	0.600
48	A	7	6	1.05	13	0.462
49	A	9	8	1.06	17	0.471
50	A	6	5	1.04	17	0.294
51	A	9	8	1.06	17	0.471
52	A	6	5	1.02	17	0.294
53	A	9	8	1.05	17	0.471
54	A	6	5	1.04	17	0.294
55	A	10	9	1.06	17	0.529
56	A	7	6	1.05	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	6	5	1.06	17	0.294
58	A	8	7	1.06	17	0.412
59	A	5	4	1.00	17	0.235
60	A	7	6	1.07	15	0.400
61	A	4	3	1.00	13	0.231
62	A	8	7	1.08	17	0.412
63	A	5	4	1.00	17	0.235
64	A	9	8	1.06	17	0.471
65	A	7	6	1.10	17	0.353
66	A	9	8	1.09	17	0.471
67	A	6	5	1.06	17	0.294
68	A	8	7	1.07	17	0.412
69	A	5	4	1.00	17	0.235
70	A	8	7	1.06	17	0.412
71	A	5	4	1.00	15	0.267
72	A	9	8	1.09	13	0.615
73	A	6	5	1.06	17	0.294
74	A	10	9	1.08	17	0.529
75	A	8	7	1.19	19	0.368
76	A	5	5	1.16	19	0.263
77	A	7	6	1.12	19	0.316
78	A	4	4	1.16	19	0.211
79	A	1	1	1.00	19	0.053
80	A	3	3	1.11	19	0.158
81	A	2	2	1.00	19	0.105
82	A	2	2	1.00	19	0.105
83	A	3	3	1.11	19	0.158
84	A	1	1	1.00	19	0.053
85	A	4	4	1.16	19	0.211
86	A	7	6	1.12	19	0.316
87	A	5	5	1.15	19	0.263
88	A	8	7	1.18	19	0.368
89	A	6	6	1.16	19	0.316
90	A	9	8	1.17	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	7	7	1.17	19	0.368
92	A	4	3	1.00	17	0.176
93	A	3	2	1.00	15	0.133
94	A	1	1	1.00	17	0.059
95	A	2	2	1.00	17	0.118
96	A	3	3	1.08	17	0.176
97	A	5	4	1.00	17	0.235
98	A	4	3	1.00	13	0.231
99	A	5	4	1.00	17	0.235
100	A	8	7	1.06	17	0.412
101	A	7	6	1.06	17	0.353
102	A	8	7	1.07	17	0.412
103	A	9	8	1.12	19	0.421
104	A	7	6	1.08	17	0.353
105	A	5	4	1.00	15	0.267
106	A	1	1	1.00	19	0.053
107	A	3	3	1.07	19	0.158
108	A	5	5	1.13	19	0.263
109	A	7	7	1.15	19	0.368
110	A	15	14	1.17	19	0.737
111	A	11	10	1.13	19	0.526
112	A	7	6	1.13	17	0.353
113	A	1	1	1.00	15	0.067
114	A	3	3	1.08	19	0.158
115	A	5	5	1.13	19	0.263
116	A	7	7	1.15	19	0.368
117	A	10	9	1.13	21	0.429
118	A	8	7	1.10	21	0.333
119	A	6	5	1.07	21	0.238
120	A	4	3	1.00	21	0.143
121	A	2	2	1.00	21	0.095
122	A	4	4	1.11	21	0.190
123	A	6	6	1.14	21	0.286
124	A	12	11	1.13	21	0.524

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	8	7	1.05	21	0.333
126	A	5	4	1.03	21	0.190
127	A	3	2	1.00	21	0.095
128	A	4	4	1.11	21	0.190
129	A	6	6	1.15	21	0.286
130	A	8	8	1.16	21	0.381
131	A	12	11	1.21	19	0.579
132	A	13	12	1.09	19	0.632
133	A	9	8	1.21	17	0.471
134	A	10	9	1.05	15	0.600
135	A	6	5	1.26	19	0.263
136	A	10	9	1.05	19	0.474
137	A	8	7	1.21	19	0.368
138	A	13	12	1.08	19	0.632
139	A	11	10	1.21	19	0.526
140	A	12	11	1.20	19	0.579
141	A	13	12	1.08	17	0.706
142	A	9	8	1.19	15	0.533
143	A	10	9	1.05	19	0.474
144	A	7	6	1.26	19	0.316
145	A	11	10	1.05	19	0.526
146	A	9	8	1.20	19	0.421
147	A	14	13	1.08	19	0.684
148	A	12	11	1.20	19	0.579
149	A	12	11	1.22	19	0.579
150	A	13	12	1.10	19	0.632
151	A	9	8	1.22	19	0.421
152	A	10	9	1.07	17	0.529
153	A	6	5	1.21	15	0.333
154	A	9	8	1.06	19	0.421
155	A	7	6	1.22	19	0.316
156	A	12	11	1.08	19	0.579
157	A	10	9	1.22	19	0.474
158	A	14	13	1.11	19	0.684

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	10	9	1.23	19	0.474
160	A	11	10	1.09	19	0.526
161	A	7	6	1.26	17	0.353
162	A	9	8	1.04	15	0.533
163	A	7	6	1.23	19	0.316
164	A	12	11	1.08	19	0.579
165	A	10	9	1.22	19	0.474
166	A	15	14	1.10	19	0.737
167	A	13	13	1.18	19	0.684
168	A	10	10	1.17	19	0.526
169	A	7	7	1.15	17	0.412
170	A	4	4	1.11	15	0.267
171	A	1	1	1.00	19	0.053
172	A	5	4	0.99	19	0.211
173	A	8	7	1.08	19	0.368
174	A	11	10	1.12	19	0.526
175	A	14	13	1.14	19	0.684
176	A	12	12	1.17	19	0.632
177	A	9	9	1.16	17	0.529
178	A	6	6	1.14	15	0.400
179	A	3	3	1.07	19	0.158
180	A	5	4	1.03	19	0.211
181	A	6	5	1.02	19	0.263
182	A	9	8	1.09	19	0.421
183	A	12	11	1.12	19	0.579
184	A	15	14	1.14	19	0.737
185	A	14	14	1.17	19	0.737
186	A	11	11	1.16	19	0.579
187	A	8	8	1.14	19	0.421
188	A	5	5	1.10	17	0.294
189	A	2	2	1.00	15	0.133
190	A	4	3	1.00	19	0.158
191	A	7	6	1.08	19	0.316
192	A	10	9	1.12	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	13	12	1.15	19	0.632
194	A	12	12	1.17	19	0.632
195	A	9	9	1.15	19	0.474
196	A	6	6	1.12	19	0.316
197	A	3	3	1.09	17	0.176
198	A	4	3	1.00	15	0.200
199	A	7	6	1.10	19	0.316
200	A	10	9	1.13	19	0.474
201	A	13	12	1.15	19	0.632
202	A	16	15	1.16	19	0.789
203	A	3	3	1.00	15	0.200
204	A	3	3	1.00	13	0.231
205	A	1	1	1.00	11	0.091
206	A	3	3	1.00	15	0.200
207	A	2	2	1.17	15	0.133
208	A	3	3	1.00	17	0.176
209	A	3	3	1.00	15	0.200
210	A	3	3	1.00	13	0.231
211	A	3	3	1.00	17	0.176
212	A	3	3	1.00	17	0.176
213	A	3	3	1.00	17	0.176
214	A	3	3	1.00	17	0.176
215	A	3	3	1.00	17	0.176
216	A	3	3	1.00	17	0.176
217	A	2	2	1.00	17	0.118
218	A	4	4	1.00	15	0.267
219	A	3	3	1.00	13	0.231
220	A	3	3	1.00	17	0.176
221	A	3	3	1.00	17	0.176
222	A	3	3	1.00	17	0.176
223	A	3	3	1.00	17	0.176
224	A	3	3	1.00	17	0.176
225	A	3	3	1.00	17	0.176
226	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	3	3	1.00	17	0.176
228	A	3	3	1.00	17	0.176
229	A	3	3	1.00	15	0.200
230	A	3	3	1.00	13	0.231
231	A	3	3	1.00	17	0.176
232	A	4	4	1.11	19	0.211
233	A	3	3	1.08	17	0.176
234	A	2	2	1.00	15	0.133
235	A	1	1	1.00	19	0.053
236	A	4	3	1.00	19	0.158
237	A	4	3	1.00	19	0.158
238	A	5	4	0.98	19	0.211
239	A	6	5	1.04	19	0.263
240	A	6	6	1.15	19	0.316
241	A	5	5	1.13	17	0.294
242	A	4	4	1.11	15	0.267
243	A	3	3	1.08	19	0.158
244	A	2	2	1.00	19	0.105
245	A	1	1	1.00	19	0.053
246	A	5	4	1.03	19	0.211
247	A	5	4	1.05	19	0.211
248	A	5	4	0.99	19	0.211
249	A	6	5	1.01	19	0.263
250	A	7	6	1.05	19	0.316
251	A	8	7	1.08	19	0.368
252	A	4	4	1.12	19	0.211
253	A	3	3	1.08	19	0.158
254	A	2	2	1.00	19	0.105
255	A	1	1	1.00	17	0.059
256	A	3	2	1.00	15	0.133
257	A	4	3	1.00	19	0.158
258	A	5	4	1.01	19	0.211
259	A	6	5	1.06	19	0.263
260	A	4	4	1.12	19	0.211

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	3	3	1.08	19	0.158
262	A	2	2	1.00	19	0.105
263	A	1	1	1.00	19	0.053
264	A	4	3	1.00	19	0.158
265	A	5	4	1.07	17	0.235
266	A	6	5	1.06	15	0.333
267	A	7	6	1.09	19	0.316
268	A	8	7	1.11	19	0.368
269	A	6	5	1.06	21	0.238
270	A	5	4	1.01	21	0.190
271	A	4	3	1.00	21	0.143
272	A	3	2	1.00	21	0.095
273	A	1	1	1.00	21	0.048
274	A	2	2	1.00	21	0.095
275	A	3	3	1.07	21	0.143
276	A	4	4	1.10	21	0.190
277	A	3	3	1.00	21	0.143
278	A	3	3	1.27	19	0.158
279	A	3	3	1.00	21	0.143
280	A	1	1	1.00	21	0.048
281	A	2	2	1.00	21	0.095
282	A	3	3	0.98	21	0.143
283	A	2	2	1.00	17	0.118
284	A	3	3	1.08	19	0.158
285	A	2	2	1.00	19	0.105
286	A	1	1	1.00	19	0.053
287	A	3	2	1.00	15	0.133
288	A	4	3	1.00	19	0.158
289	A	3	3	1.00	19	0.158
290	A	2	2	1.00	17	0.118
291	A	3	3	1.00	19	0.158
292	A	5	5	1.06	19	0.263
293	A	4	4	1.06	19	0.211
294	A	5	5	1.06	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
295	A	6	5	1.03	21	0.238
296	A	8	7	1.02	21	0.333
297	A	4	3	1.00	21	0.143
298	A	5	4	1.00	21	0.190
299	A	7	6	1.02	21	0.286
300	A	3	2	1.00	21	0.095
301	A	4	3	1.00	21	0.143
302	A	8	7	1.03	21	0.333
303	A	1	1	1.00	21	0.048
304	A	5	4	1.00	21	0.190
305	A	9	8	1.02	21	0.381
306	A	2	2	1.00	21	0.095
307	A	6	5	1.02	21	0.238
308	A	3	3	1.00	15	0.200
309	A	3	3	1.00	13	0.231
310	A	6	5	1.06	19	0.263
311	A	5	4	1.01	19	0.211
312	A	4	3	1.00	19	0.158
313	A	3	2	1.00	17	0.118
314	A	1	1	1.00	15	0.067
315	A	2	2	1.00	19	0.105
316	A	3	3	1.08	19	0.158
317	A	4	4	1.11	19	0.211
318	A	5	5	1.13	19	0.263
319	A	5	4	1.04	11	0.364
320	A	6	5	0.96	13	0.385
321	A	2	2	1.00	9	0.222
322	A	2	2	1.00	9	0.222
323	A	2	2	1.00	9	0.222
324	A	2	2	1.00	13	0.154
325	A	2	2	1.00	13	0.154
326	A	2	2	1.00	13	0.154
327	A	2	2	1.00	13	0.154
328	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	2	2	1.00	15	0.133
330	A	2	2	1.00	15	0.133
331	A	2	2	1.00	23	0.087
332	A	2	2	1.00	11	0.182
333	A	2	2	1.00	13	0.154
334	A	2	2	1.00	13	0.154
335	A	2	2	1.00	17	0.118
336	A	2	2	1.00	17	0.118
337	A	2	2	1.00	17	0.118
338	A	2	2	1.00	11	0.182
339	A	2	2	1.00	11	0.182
340	A	2	2	1.00	11	0.182
341	A	2	2	1.00	11	0.182
342	A	2	2	1.00	13	0.154
343	A	2	2	1.00	13	0.154
344	A	3	3	1.00	11	0.273
345	A	5	4	1.05	11	0.364
346	A	3	3	1.00	11	0.273
347	A	10	9	0.92	9	1.000
348	A	2	2	1.00	22	0.091
349	A	1	1	1.00	13	0.077
350	A	2	2	1.00	15	0.133
351	A	2	2	1.00	17	0.118
352	A	1	1	1.00	13	0.077
353	A	2	2	1.00	15	0.133
354	A	1	1	1.00	13	0.077
355	A	2	2	1.00	11	0.182
356	A	6	5	1.13	13	0.385
357	A	2	2	1.00	15	0.133
358	A	6	5	1.14	13	0.385
359	A	2	2	1.00	15	0.133
360	A	2	2	1.00	11	0.182
361	A	2	2	1.00	11	0.182
362	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	6	5	1.38	11	0.455
364	A	4	3	1.00	25	0.120
365	A	5	4	0.96	27	0.148
366	A	5	4	1.00	23	0.174
367	A	5	4	0.97	22	0.182
368	A	5	4	1.00	21	0.190
369	A	6	5	0.90	18	0.278
370	A	5	4	1.00	23	0.174
371	A	4	3	1.00	15	0.200
372	A	5	4	1.00	23	0.174
373	A	6	5	0.93	27	0.185
374	A	6	5	1.04	23	0.217
375	A	6	5	0.97	22	0.227
376	A	6	5	1.04	21	0.238
377	A	7	6	0.88	18	0.333
378	A	6	5	1.04	23	0.217
379	A	6	5	0.97	22	0.227
380	A	6	5	1.02	23	0.217
381	A	6	5	0.97	22	0.227
382	A	6	5	1.00	13	0.385
383	A	6	5	1.00	15	0.333
384	A	5	4	1.00	15	0.267
385	A	5	4	1.00	15	0.267
386	A	6	5	1.00	15	0.333
387	A	6	5	1.00	17	0.294
388	A	5	4	1.00	17	0.235
389	A	5	4	1.00	17	0.235
390	A	4	3	1.00	27	0.111
391	A	4	3	1.00	23	0.130
392	A	3	2	1.00	15	0.133
393	A	4	3	1.00	21	0.143
394	A	5	4	1.00	18	0.222
395	A	4	3	1.00	23	0.130
396	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	4	3	1.00	23	0.130
398	A	5	4	0.92	27	0.148
399	A	5	4	1.00	23	0.174
400	A	5	4	0.97	22	0.182
401	A	5	4	1.00	21	0.190
402	A	6	5	0.91	18	0.278
403	A	5	4	1.00	23	0.174
404	A	5	4	0.97	22	0.182
405	A	5	4	1.00	23	0.174
406	A	5	4	0.97	22	0.182
407	A	4	3	1.00	15	0.200
408	A	4	3	1.00	15	0.200
409	A	4	3	1.00	15	0.200
410	A	4	3	1.00	19	0.158
411	A	4	3	1.00	16	0.188
412	A	4	3	1.00	16	0.188
413	A	4	3	1.00	16	0.188
414	A	4	3	1.00	20	0.150
415	A	4	3	1.00	19	0.158
416	A	4	3	1.00	17	0.176
417	A	4	3	1.00	17	0.176
418	A	4	3	1.00	20	0.150
419	A	4	3	1.00	19	0.158
420	A	4	3	1.00	18	0.167
421	A	3	3	1.00	21	0.143
422	A	3	3	1.00	21	0.143
423	A	3	3	1.18	21	0.143
424	A	3	3	1.11	21	0.143
425	A	3	3	1.11	21	0.143
426	A	3	3	1.00	15	0.200
427	A	3	3	1.16	15	0.200
428	A	3	3	1.00	15	0.200
429	A	3	3	1.00	15	0.200
430	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	2	2	1.00	11	0.182
432	A	1	1	1.00	11	0.091
433	A	4	3	1.00	13	0.231
434	A	1	1	1.00	17	0.059
435	A	1	1	1.00	17	0.059
436	A	2	2	1.00	15	0.133
437	C	5	4	0.06	19	0.211
438	C	5	4	0.13	19	0.211
439	A	3	3	1.03	17	0.176
440	A	4	3	1.00	22	0.136
441	A	4	3	1.09	22	0.136
442	A	4	3	1.00	27	0.111
443	A	4	3	1.13	27	0.111
444	A	1	1	1.00	18	0.056
445	A	2	2	1.00	17	0.118
446	A	2	2	1.00	22	0.091
447	A	1	1	1.00	23	0.043
448	A	2	2	1.00	19	0.105
449	A	2	2	1.00	19	0.105
450	A	2	2	1.00	19	0.105
451	A	2	2	1.00	19	0.105
452	A	2	2	1.00	19	0.105
453	A	1	1	1.00	25	0.040
454	A	2	2	1.00	28	0.071

CHAPTER 3

Listing of integrals

3.1 $\int x^2(ax+bx^3)\,dx$

3.1.1 Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

output
```
1/4*a*x^4+1/6*b*x^6
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

input
```
Integrate[x^2*(a*x + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

3.1.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+bx^3\right)\,dx$$
$$\Big\downarrow \; 9$$
$$\int x^3\left(a+bx^2\right)dx$$
$$\Big\downarrow \; 244$$
$$\int \left(ax^3+bx^5\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

input
```
Int[x^2*(a*x + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

3.1.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
parallelrisch	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
gosper	$\frac{x^4(2b\,x^2+3a)}{12}$	16

input
```
int(x^2*(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/6*b*x^6
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3)\,dx=\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2(ax+bx^3)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

input
```
integrate(x**2*(b*x**3+a*x),x)
```

output
```
a*x**4/4 + b*x**6/6
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3)\,dx = \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3)\,dx = \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3)\,dx = \frac{b\,x^6}{6} + \frac{a\,x^4}{4}$$

input
```
int(x^2*(a*x + b*x^3),x)
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

3.2 $\int x(ax+bx^3)\,dx$

3.2.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x(ax+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

output
```
1/3*a*x^3+1/5*b*x^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(ax+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

input
```
Integrate[x*(a*x + b*x^3),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

3.2.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(ax+bx^3)\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^2(a+bx^2)\,dx$$

$$\Big\downarrow \text{244}$$

$$\int (ax^2+bx^4)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

input
```
Int[x*(a*x + b*x^3),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

3.2.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
parallelrisch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
gosper	$\frac{x^{3}(3b\,x^{2}+5a)}{15}$	16

input
```
int(x*(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/5*b*x^5
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3)\ dx=\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x*(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(ax+bx^3)\ dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

input
```
integrate(x*(b*x**3+a*x),x)
```

output
```
a*x**3/3 + b*x**5/5
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3)\,dx = \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

input
```
integrate(x*(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3)\,dx = \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

input
```
integrate(x*(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3)\,dx = \frac{b\,x^5}{5} + \frac{a\,x^3}{3}$$

input
```
int(x*(a*x + b*x^3),x)
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

3.3 $\int (ax + bx^3) \, dx$

3.3.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

output
```
1/2*a*x^2+1/4*b*x^4
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

input
```
Integrate[a*x + b*x^3,x]
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

3.3.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^3\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

input
```
Int[a*x + b*x^3,x]
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
risch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
parts	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
gosper	$\frac{x^{2}\left(b\,x^{2}+2a\right)}{4}$	15
default	$\frac{\left(b\,x^{2}+a\right)^{2}}{4b}$	15

input
```
int(b*x^3+a*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/4*b*x^4
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax + bx^3) \, dx = \frac{1}{4} bx^4 + \frac{1}{2} ax^2$$

input
```
integrate(b*x^3+a*x,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

input
```
integrate(b*x**3+a*x,x)
```

output
```
a*x**2/2 + b*x**4/4
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax + bx^3) \, dx = \frac{1}{4} bx^4 + \frac{1}{2} ax^2$$

input
```
integrate(b*x^3+a*x,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax + bx^3)\, dx = \frac{1}{4}\, bx^4 + \frac{1}{2}\, ax^2$$

input
```
integrate(b*x^3+a*x,x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax + bx^3)\, dx = \frac{b\,x^4}{4} + \frac{a\,x^2}{2}$$

input
```
int(a*x + b*x^3,x)
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

3.4 $\int \frac{ax+bx^3}{x} \, dx$

3.4.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{ax+bx^3}{x} \, dx = ax + \frac{bx^3}{3}$$

output
```
a*x+1/3*b*x^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x} \, dx = ax + \frac{bx^3}{3}$$

input
```
Integrate[(a*x + b*x^3)/x,x]
```

output
```
a*x + (b*x^3)/3
```

3.4.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {9, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3}{x} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \left(a+bx^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{bx^3}{3}$$

input
```
Int[(a*x + b*x^3)/x,x]
```

output
```
a*x + (b*x^3)/3
```

3.4.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{1}{3}b\,x^3$	11
norman	$ax+\frac{1}{3}b\,x^3$	11
risch	$ax+\frac{1}{3}b\,x^3$	11
parallelrisch	$ax+\frac{1}{3}b\,x^3$	11
parts	$ax+\frac{1}{3}b\,x^3$	11
gosper	$\frac{x(b\,x^2+3a)}{3}$	13

input
```
int((b*x^3+a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x^3
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax+bx^3}{x}\,dx = \frac{1}{3}\,bx^3 + ax$$

input
```
integrate((b*x^3+a*x)/x,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*x
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{ax+bx^3}{x}\,dx = ax + \frac{bx^3}{3}$$

input
```
integrate((b*x**3+a*x)/x,x)
```

output
```
a*x + b*x**3/3
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{1}{3} bx^3 + ax$$

input
```
integrate((b*x^3+a*x)/x,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + a*x
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{1}{3} bx^3 + ax$$

input
```
integrate((b*x^3+a*x)/x,x, algorithm="giac")
```

output
```
1/3*b*x^3 + a*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{b\,x^3}{3} + a\,x$$

input
```
int((a*x + b*x^3)/x,x)
```

output
```
a*x + (b*x^3)/3
```

3.5 $\int \frac{ax+bx^3}{x^2}\,dx$

3.5.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{ax+bx^3}{x^2}\,dx=\frac{bx^2}{2}+a\log(x)$$

output
```
1/2*b*x^2+a*ln(x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x^2}\,dx=\frac{bx^2}{2}+a\log(x)$$

input
```
Integrate[(a*x + b*x^3)/x^2,x]
```

output
```
(b*x^2)/2 + a*Log[x]
```

3.5.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3}{x^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{a+bx^2}{x}dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{a}{x}+bx\right)dx$$

$$\Big\downarrow \; 2009$$

$$a\log(x)+\frac{bx^2}{2}$$

input
```
Int[(a*x + b*x^3)/x^2,x]
```

output
```
(b*x^2)/2 + a*Log[x]
```

3.5.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
parallelrisch	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12

input
```
int((b*x^3+a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+a*ln(x)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx=\frac{1}{2}\,bx^2+a\log\left(x\right)$$

input
```
integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*log(x)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{ax+bx^3}{x^2}\,dx=a\log\left(x\right)+\frac{bx^2}{2}$$

input
```
integrate((b*x**3+a*x)/x**2,x)
```

output
```
a*log(x) + b*x**2/2
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2 + a\log\left(x\right)$$

input
```
integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*log(x)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

input
```
integrate((b*x^3+a*x)/x^2,x, algorithm="giac")
```

output
```
1/2*b*x^2 + 1/2*a*log(x^2)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{b\,x^2}{2} + a\,\ln\left(x\right)$$

input
```
int((a*x + b*x^3)/x^2,x)
```

output
```
(b*x^2)/2 + a*log(x)
```

3.6 $\int x^2(ax+bx^3)^2\,dx$

3.6.1 Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x^2(ax+bx^3)^2\,dx = \frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2(ax+bx^3)^2\,dx = \frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

input
```
Integrate[x^2*(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

3.6.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+bx^3\right)^2\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^4\left(a+bx^2\right)^2dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(a^2x^4+2abx^6+b^2x^8\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

input
```
Int[x^2*(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

3.6.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
parallelrisch	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
gosper	$\frac{x^5(35b^2x^4+90ab\,x^2+63a^2)}{315}$	27

input
```
int(x^2*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*a^2+2/7*a*b*x^7+1/9*b^2*x^9
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(ax+bx^3)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2(ax+bx^3)^2\,dx=\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{b^2x^9}{9}$$

input
```
integrate(x**2*(b*x**3+a*x)**2,x)
```

output
```
a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax + bx^3)^2 \, dx = \frac{1}{9} b^2 x^9 + \frac{2}{7} abx^7 + \frac{1}{5} a^2 x^5$$

input
```
integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax + bx^3)^2 \, dx = \frac{1}{9} b^2 x^9 + \frac{2}{7} abx^7 + \frac{1}{5} a^2 x^5$$

input
```
integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (ax + bx^3)^2 \, dx = \frac{a^2 x^5}{5} + \frac{2 a b x^7}{7} + \frac{b^2 x^9}{9}$$

input
```
int(x^2*(a*x + b*x^3)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

3.7 $\int x(ax+bx^3)^2\,dx$

3.7.1 Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x(ax+bx^3)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

3.7.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x(ax+bx^3)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

input
```
Integrate[x*(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

3.7.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^3\right)^2\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^3\left(a+bx^2\right)^2dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2\left(bx^2+a\right)^2dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^2x^6+2abx^4+a^2x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2x^4}{2}+\frac{2}{3}abx^6+\frac{b^2x^8}{4}\right)$$

input
```
Int[x*(a*x + b*x^3)^2,x]
```

output
```
((a^2*x^4)/2 + (2*a*b*x^6)/3 + (b^2*x^8)/4)/2
```

3.7.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
gosper	$\frac{x^4(3b^2x^4+8ab\,x^2+6a^2)}{24}$	27

input
```
int(x*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax+bx^3)^2\,dx = \frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax+bx^3)^2\,dx=\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

input
```
integrate(x*(b*x**3+a*x)**2,x)
```

output
```
a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax+bx^3)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax+bx^3)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax+bx^3)^2 \, dx = \frac{a^2 x^4}{4} + \frac{a b x^6}{3} + \frac{b^2 x^8}{8}$$

input
```
int(x*(a*x + b*x^3)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

3.8 $\int (ax + bx^3)^2 \, dx$

3.8.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int (ax + bx^3)^2 \, dx = \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (ax + bx^3)^2 \, dx = \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

input
```
Integrate[(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

3.8.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^3\right)^2 dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^2 \left(a+bx^2\right)^2 dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2x^2+2abx^4+b^2x^6\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

input
```
Int[(a*x + b*x^3)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

3.8.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.8.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
gosper	$\frac{x^3(15b^2x^4+42ab\,x^2+35a^2)}{105}$	27

input
```
int((b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax+bx^3)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (ax+bx^3)^2\,dx=\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

input
```
integrate((b*x**3+a*x)**2,x)
```

output
```
a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax + bx^3)^2 \, dx = \frac{1}{7} b^2 x^7 + \frac{2}{5} abx^5 + \frac{1}{3} a^2 x^3$$

input
```
integrate((b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax + bx^3)^2 \, dx = \frac{1}{7} b^2 x^7 + \frac{2}{5} abx^5 + \frac{1}{3} a^2 x^3$$

input
```
integrate((b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax + bx^3)^2 \, dx = \frac{a^2 x^3}{3} + \frac{2 a b x^5}{5} + \frac{b^2 x^7}{7}$$

input
```
int((a*x + b*x^3)^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

3.9 $\int \frac{(ax+bx^3)^2}{x} dx$

3.9.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{(ax+bx^3)^2}{x} dx = \frac{(a+bx^2)^3}{6b}$$

output
```
1/6*(b*x^2+a)^3/b
```

3.9.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x} dx = \frac{(a+bx^2)^3}{6b}$$

input
```
Integrate[(a*x + b*x^3)^2/x,x]
```

output
```
(a + b*x^2)^3/(6*b)
```

3.9.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x} \, dx$$

$$\Big\downarrow \; 9$$

$$\int x(a+bx^2)^2 \, dx$$

$$\Big\downarrow \; 241$$

$$\frac{(a+bx^2)^3}{6b}$$

input
```
Int[(a*x + b*x^3)^2/x,x]
```

output
```
(a + b*x^2)^3/(6*b)
```

3.9.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.9.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^2+a)^3}{6b}$	15
norman	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
parallelrisch	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
gosper	$\frac{x^2(b^2x^4+3ab\,x^2+3a^2)}{6}$	26
risch	$\frac{b^2x^6}{6}+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}+\frac{a^3}{6b}$	33

input
```
int((b*x^3+a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x^2+a)^3/b
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

input
```
integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

input
```
integrate((b*x**3+a*x)**2/x,x)
```

output
```
a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax + bx^3)^2}{x}\, dx = \frac{1}{6}\, b^2x^6 + \frac{1}{2}\, abx^4 + \frac{1}{2}\, a^2x^2$$

input
```
integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax + bx^3)^2}{x}\, dx = \frac{1}{6}\, b^2x^6 + \frac{1}{2}\, abx^4 + \frac{1}{2}\, a^2x^2$$

input
```
integrate((b*x^3+a*x)^2/x,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax + bx^3)^2}{x}\, dx = \frac{a^2\, x^2}{2} + \frac{a\, b\, x^4}{2} + \frac{b^2\, x^6}{6}$$

input
```
int((a*x + b*x^3)^2/x,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

3.10 $\int \frac{(ax+bx^3)^2}{x^2} \, dx$

3.10.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{(ax+bx^3)^2}{x^2} \, dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

3.10.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3)^2}{x^2} \, dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

input
```
Integrate[(a*x + b*x^3)^2/x^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

3.10.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int (a+bx^2)^2\,dx$$

$$\Big\downarrow \; 210$$

$$\int (a^2+2abx^2+b^2x^4)\,dx$$

$$\Big\downarrow \; 2009$$

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[(a*x + b*x^3)^2/x^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

3.10.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
parallelrisch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
gosper	$\frac{x\left(3b^2x^4+10ab\,x^2+15a^2\right)}{15}$	25
norman	$\frac{a^2x^2+\frac{1}{5}b^2x^6+\frac{2}{3}ab\,x^4}{x}$	28

input
```
int((b*x^3+a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx=a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

input
```
integrate((b*x**3+a*x)**2/x**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

input
```
int((a*x + b*x^3)^2/x^2,x)
```

output
```
a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.11 $\int (-4x+3x^3)^6 \, dx$

3.11.1 Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (-4x+3x^3)^6 \, dx = \frac{4096x^7}{7} - 2048x^9 + \frac{34560x^{11}}{11} - \frac{34560x^{13}}{13} + 1296x^{15} - \frac{5832x^{17}}{17} + \frac{729x^{19}}{19}$$

output
```
4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729
/19*x^19
```

3.11.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (-4x+3x^3)^6 \, dx = \frac{4096x^7}{7} - 2048x^9 + \frac{34560x^{11}}{11} - \frac{34560x^{13}}{13} + 1296x^{15} - \frac{5832x^{17}}{17} + \frac{729x^{19}}{19}$$

input
```
Integrate[(-4*x + 3*x^3)^6,x]
```

output
```
(4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 -
(5832*x^17)/17 + (729*x^19)/19
```

3.11.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x\right)^6 dx$$

$$\Big\downarrow \; 2027$$

$$\int x^6\left(3x^2-4\right)^6 dx$$

$$\Big\downarrow \; 244$$

$$\int \left(729x^{18}-5832x^{16}+19440x^{14}-34560x^{12}+34560x^{10}-18432x^8+4096x^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

input
```
Int[(-4*x + 3*x^3)^6,x]
```

output
```
(4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 -
(5832*x^17)/17 + (729*x^19)/19
```

3.11.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.11.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
norman	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
risch	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
parallelrisch	$\frac{4096}{7}x^{7}-2048x^{9}+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
gosper	$\frac{x^{7}\left(12405393x^{12}-110918808x^{10}+419026608x^{8}-859541760x^{6}+1015822080x^{4}-662165504x^{2}+189190144\right)}{323323}$	38

input
```
int((3*x^3-4*x)^6,x,method=_RETURNVERBOSE)
```

output
```
4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729
/19*x^19
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^{3}\right)^{6}\,dx=\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^{9}+\frac{4096}{7}\,x^{7}$$

input
```
integrate((3*x^3-4*x)^6,x, algorithm="fricas")
```

output
```
729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 2
048*x^9 + 4096/7*x^7
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(-4x+3x^3\right)^6\,dx = \frac{729x^{19}}{19} - \frac{5832x^{17}}{17} + 1296x^{15} - \frac{34560x^{13}}{13} + \frac{34560x^{11}}{11} - 2048x^9 + \frac{4096x^7}{7}$$

input
```
integrate((3*x**3-4*x)**6,x)
```

output
```
729*x**19/19 - 5832*x**17/17 + 1296*x**15 - 34560*x**13/13 + 34560*x**11/1
1 - 2048*x**9 + 4096*x**7/7
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx = \frac{729}{19}\,x^{19} - \frac{5832}{17}\,x^{17} + 1296\,x^{15} - \frac{34560}{13}\,x^{13} + \frac{34560}{11}\,x^{11} - 2048\,x^9 + \frac{4096}{7}\,x^7$$

input
```
integrate((3*x^3-4*x)^6,x, algorithm="maxima")
```

output
```
729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 2
048*x^9 + 4096/7*x^7
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx = \frac{729}{19}\,x^{19} - \frac{5832}{17}\,x^{17} + 1296\,x^{15} - \frac{34560}{13}\,x^{13} + \frac{34560}{11}\,x^{11} - 2048\,x^9 + \frac{4096}{7}\,x^7$$

input
```
integrate((3*x^3-4*x)^6,x, algorithm="giac")
```

output
```
729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 2
048*x^9 + 4096/7*x^7
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6 \, dx = \frac{729\,x^{19}}{19} - \frac{5832\,x^{17}}{17} + 1296\,x^{15} - \frac{34560\,x^{13}}{13} + \frac{34560\,x^{11}}{11} - 2048\,x^9 + \frac{4096\,x^7}{7}$$

input
```
int((4*x - 3*x^3)^6,x)
```

output
```
(4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 -
(5832*x^17)/17 + (729*x^19)/19
```

3.12 $\int \frac{x^4}{ax+bx^3} dx$

3.12.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{x^4}{ax+bx^3} dx = \frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}$$

output
```
1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{ax+bx^3} dx = \frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}$$

input
```
Integrate[x^4/(a*x + b*x^3),x]
```

output
```
x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

3.12.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax+bx^3}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x^3}{a+bx^2}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{x^2}{bx^2+a}dx^2$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(\frac{1}{b}-\frac{a}{b\,(bx^2+a)}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{x^2}{b}-\frac{a\log\left(a+bx^2\right)}{b^2}\right)$$

input
```
Int[x^4/(a*x + b*x^3),x]
```

output
```
(x^2/b - (a*Log[a + b*x^2])/b^2)/2
```

3.12.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-b\,x^{2}+a\ln\left(b\,x^{2}+a\right)}{2 b^{2}}$	23
default	$\frac{x^{2}}{2 b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2 b^{2}}$	24
norman	$\frac{x^{2}}{2 b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2 b^{2}}$	24
risch	$\frac{x^{2}}{2 b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2 b^{2}}$	24

input
```
int(x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^2+a*ln(b*x^2+a))/b^2
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{ax+bx^3}\,dx=\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{ax+bx^3}\,dx = -\frac{a\log(a+bx^2)}{2b^2} + \frac{x^2}{2b}$$

input
```
integrate(x**4/(b*x**3+a*x),x)
```

output
```
-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{ax+bx^3}\,dx = \frac{x^2}{2\,b} - \frac{a\log(bx^2+a)}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{ax+bx^3}\,dx = \frac{x^2}{2\,b} - \frac{a\log(|bx^2+a|)}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{ax+bx^3}\,dx = -\frac{a\,\ln\left(b\,x^2+a\right)-b\,x^2}{2\,b^2}$$

input
```
int(x^4/(a*x + b*x^3),x)
```

output
```
-(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

3.13 $\int \frac{x^3}{ax+bx^3} dx$

3.13.1 Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{ax+bx^3} dx = \frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
x/b-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax+bx^3} dx = \frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Integrate[x^3/(a*x + b*x^3),x]
```

output
```
x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

3.13.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax+bx^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2}{a+bx^2}dx$$

$$\Big\downarrow \; 262$$

$$\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Int[x^3/(a*x + b*x^3),x]
```

output
```
x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

3.13.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.13.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$\frac{x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^2}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2b^2}$	56

input
```
int(x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{x^3}{ax+bx^3}\,dx = \left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

input
```
integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
 -(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2} + \frac{x}{b}$$

input
```
integrate(x**3/(b*x**3+a*x),x)
```

output
```
sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b*
*3) + x)/2 + x/b
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{ax+bx^3}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

input
```
integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{ax+bx^3}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

input
```
integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

3.13.9 Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{ax + bx^3}\,dx = \frac{x}{b} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int(x^3/(a*x + b*x^3),x)
```

output
```
x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

3.14 $\int \frac{x^2}{ax+bx^3} \, dx$

3.14.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^2}{ax+bx^3} \, dx = \frac{\log(a+bx^2)}{2b}$$

output
```
1/2*ln(b*x^2+a)/b
```

3.14.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax+bx^3} \, dx = \frac{\log(a+bx^2)}{2b}$$

input
```
Integrate[x^2/(a*x + b*x^3),x]
```

output
```
Log[a + b*x^2]/(2*b)
```

3.14.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax+bx^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x}{a+bx^2}dx$$
$$\Big\downarrow \; 240$$
$$\frac{\log\left(a+bx^2\right)}{2b}$$

input
```
Int[x^2/(a*x + b*x^3),x]
```

output
```
Log[a + b*x^2]/(2*b)
```

3.14.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

3.14.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(b\,x^2+a)}{2b}$	14
norman	$\frac{\ln(b\,x^2+a)}{2b}$	14
risch	$\frac{\ln(b\,x^2+a)}{2b}$	14
parallelrisch	$\frac{\ln(b\,x^2+a)}{2b}$	14

input
```
int(x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(b*x^2+a)/b
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 + a)/b
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log\left(a+bx^2\right)}{2b}$$

input
```
integrate(x**2/(b*x**3+a*x),x)
```

output
```
log(a + b*x**2)/(2*b)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(bx^2+a)}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 + a)/b
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(|bx^2+a|)}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b
```

3.14.9 Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\ln(b\,x^2+a)}{2\,b}$$

input
```
int(x^2/(a*x + b*x^3),x)
```

output
```
log(a + b*x^2)/(2*b)
```

3.15 $\int \frac{x}{ax+bx^3}\,dx$

3.15.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{x}{ax+bx^3}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax+bx^3}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[x/(a*x + b*x^3),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.15.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {9, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax+bx^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{a+bx^2}dx$$
$$\Big\downarrow \; 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[x/(a*x + b*x^3),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.15.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.15.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

input
```
int(x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{x}{ax+bx^3}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(x/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{x}{ax+bx^3}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(x/(b*x**3+a*x),x)
```

output
```
-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-
1/(a*b)) + x)/2
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x}{ax+bx^3}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(x/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x}{ax+bx^3}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(x/(b*x^3+a*x),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{x}{ax+bx^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(x/(a*x + b*x^3),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.16 $\int \frac{1}{ax+bx^3}\,dx$

3.16.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{ax+bx^3}\,dx=\frac{\log(x)}{a}-\frac{\log(a+bx^2)}{2a}$$

output
```
ln(x)/a-1/2*ln(b*x^2+a)/a
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^3}\,dx=\frac{\log(x)}{a}-\frac{\log(a+bx^2)}{2a}$$

input
```
Integrate[(a*x + b*x^3)^(-1),x]
```

output
```
Log[x]/a - Log[a + b*x^2]/(2*a)
```

3.16.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2026, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x(a+bx^2)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2}dx^2}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\log(a+bx^2)}{a}\right)$$

input
```
Int[(a*x + b*x^3)^(-1),x]
```

output
```
(Log[x^2]/a - Log[a + b*x^2]/a)/2
```

3.16.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.16.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
parallelrisch	$\frac{2\ln(x)-\ln(b\,x^2+a)}{2a}$	21

input
```
int(1/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/2*ln(b*x^2+a)/a
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{ax + bx^3} \, dx = -\frac{\log (bx^2 + a) - 2 \log (x)}{2 a}$$

input
```
integrate(1/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
-1/2*(log(b*x^2 + a) - 2*log(x))/a
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{ax + bx^3} \, dx = \frac{\log (x)}{a} - \frac{\log \left(\frac{a}{b} + x^2\right)}{2a}$$

input
```
integrate(1/(b*x**3+a*x),x)
```

output
```
log(x)/a - log(a/b + x**2)/(2*a)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{ax + bx^3} \, dx = -\frac{\log (bx^2 + a)}{2 a} + \frac{\log (x)}{a}$$

input
```
integrate(1/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + a)/a + log(x)/a
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{ax+bx^3}\,dx = \frac{\log(x^2)}{2\,a} - \frac{\log(|bx^2+a|)}{2\,a}$$

input
```
integrate(1/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{ax+bx^3}\,dx = -\frac{\ln(b\,x^2+a) - 2\,\ln(x)}{2\,a}$$

input
```
int(1/(a*x + b*x^3),x)
```

output
```
-(log(a + b*x^2) - 2*log(x))/(2*a)
```

3.17 $\int \frac{1}{x(ax+bx^3)}\,dx$

3.17.1 Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{x(ax+bx^3)}\,dx = -\frac{1}{ax} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-1/a/x-arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(ax+bx^3)}\,dx = -\frac{1}{ax} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a*x + b*x^3)),x]
```

output
```
-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

3.17.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(ax+bx^3)}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^2(a+bx^2)}dx$$
$$\Big\downarrow \text{ 264}$$
$$-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}$$
$$\Big\downarrow \text{ 218}$$
$$-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

input
```
Int[1/(x*(a*x + b*x^3)),x]
```

output
```
-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

3.17.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.17.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}$	30
risch	$-\frac{1}{ax}+\frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a^2}-\frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a^2}$	58

input
```
int(1/x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax},-\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

input
```
integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a
*x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{x(ax+bx^3)}\,dx = \frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2} - \frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2} - \frac{1}{ax}$$

input
```
integrate(1/x/(b*x**3+a*x),x)
```

output
```
sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sq
rt(-b/a**3)/b + x)/2 - 1/(a*x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(ax+bx^3)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{1}{ax}$$

input
```
integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(ax+bx^3)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{1}{ax}$$

input
```
integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(ax+bx^3)}\,dx = -\frac{1}{a\,x} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(1/(x*(a*x + b*x^3)),x)
```

output
```
- 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.18 $\int \frac{1}{x^2(ax+bx^3)}\,dx$

3.18.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}$$

output
```
-1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

3.18.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

3.18.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^3)} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^3 (a+bx^2)}dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+a)}dx^2$$
$$\Big\downarrow \; 54$$
$$\frac{1}{2}\int \left(\frac{b^2}{a^2 (bx^2+a)}-\frac{b}{a^2x^2}+\frac{1}{ax^4}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-\frac{b\log(x^2)}{a^2}+\frac{b\log(a+bx^2)}{a^2}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^2*(a*x + b*x^3)),x]
```

output
```
(-(1/(a*x^2)) - (b*Log[x^2])/a^2 + (b*Log[a + b*x^2])/a^2)/2
```

3.18.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.18.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln(b\,x^2+a)}{2a^2}$	32
norman	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln(b\,x^2+a)}{2a^2}$	32
parallelrisch	$-\frac{2b\ln(x)x^2-b\ln(b\,x^2+a)x^2+a}{2x^2a^2}$	33
risch	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln(-b\,x^2-a)}{2a^2}$	35

input
```
int(1/x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(ax+bx^3)}\,dx=\frac{bx^2\log(bx^2+a)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (ax + bx^3)} dx = -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

input
```
integrate(1/x**2/(b*x**3+a*x),x)
```

output
```
-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (ax + bx^3)} dx = \frac{b \log(bx^2 + a)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2 (ax + bx^3)} dx = -\frac{b \log(x^2)}{2a^2} + \frac{b \log(|bx^2 + a|)}{2a^2} + \frac{bx^2 - a}{2a^2 x^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2
*x^2)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\,(ax+bx^3)}\,dx = \frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}-\frac{b\,\ln\left(x\right)}{a^2}$$

input
```
int(1/(x^2*(a*x + b*x^3)),x)
```

output
```
(b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

3.19 $\int \frac{1}{x^3(ax+bx^3)} \, dx$

3.19.1 Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{x^3(ax+bx^3)} \, dx = -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(ax+bx^3)} \, dx = -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^3*(a*x + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

3.19.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(ax+bx^3)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^4(a+bx^2)}dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{b\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^3*(a*x + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^
(3/2)))/a
```

3.19.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.19.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b}{x\,a^{2}}+\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$	39
risch	$\frac{\frac{b\,x^{2}}{a^{2}}-\frac{1}{3a}}{x^{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{2}+b^{3}\right)}\_R\ln\left(\left(3a^{5}\_R^{2}+2b^{3}\right)x-a^{3}b\_R\right)\right)}{2}$	64

input
```
int(1/x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+b/x/a^2+b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{x^3 (ax + bx^3)} dx = \left[\frac{3 bx^3 \sqrt{-\frac{b}{a}} \log \left(\frac{bx^2 + 2 ax \sqrt{-\frac{b}{a}} - a}{bx^2 + a} \right) + 6 bx^2 - 2 a}{6 a^2 x^3}, \frac{3 bx^3 \sqrt{\frac{b}{a}} \arctan \left(x \sqrt{\frac{b}{a}} \right) + 3 bx^2 - a}{3 a^2 x^3} \right]$$

input
```
integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
 6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*
b*x^2 - a)/(a^2*x^3)]
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(37) = 74.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^3 (ax + bx^3)} dx = -\frac{\sqrt{-\frac{b^3}{a^5}} \log \left(-\frac{a^3 \sqrt{-\frac{b^3}{a^5}}}{b^2} + x \right)}{2} + \frac{\sqrt{-\frac{b^3}{a^5}} \log \left(\frac{a^3 \sqrt{-\frac{b^3}{a^5}}}{b^2} + x \right)}{2} + \frac{-a + 3bx^2}{3a^2 x^3}$$

input
```
integrate(1/x**3/(b*x**3+a*x),x)
```

output
```
-sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5
)*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3 (ax + bx^3)} \, dx = \frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

input
```
integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3 (ax + bx^3)} \, dx = \frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

input
```
integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (ax + bx^3)} \, dx = \frac{b^{3/2} \operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{3\,a} - \frac{b\,x^2}{a^2}}{x^3}$$

input
```
int(1/(x^3*(a*x + b*x^3)),x)
```

output
```
(b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

3.20 $\int \frac{1}{x^4(ax+bx^3)}\,dx$

3.20.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^4(ax+bx^3)}\,dx = -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}$$

output
```
-1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

3.20.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(ax+bx^3)}\,dx = -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}$$

input
```
Integrate[1/(x^4*(a*x + b*x^3)),x]
```

output
```
-1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(
2*a^3)
```

3.20.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(ax+bx^3\right)} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^5 \left(a+bx^2\right)}dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{1}{x^6 \left(bx^2+a\right)}dx^2$$
$$\Big\downarrow \; 54$$
$$\frac{1}{2}\int \left(-\frac{b^3}{a^3 \left(bx^2+a\right)}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^4}+\frac{1}{ax^6}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(\frac{b^2 \log \left(x^2\right)}{a^3}-\frac{b^2 \log \left(a+bx^2\right)}{a^3}+\frac{b}{a^2x^2}-\frac{1}{2ax^4}\right)$$

input
```
Int[1/(x^4*(a*x + b*x^3)),x]
```

output
```
(-1/2*1/(a*x^4) + b/(a^2*x^2) + (b^2*Log[x^2])/a^3 - (b^2*Log[a + b*x^2])/
a^3)/2
```

3.20.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{4a\,x^4}+\frac{b}{2a^2x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}$	44
norman	$\frac{-\frac{1}{4a}+\frac{b\,x^2}{2a^2}}{x^4}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}$	46
risch	$\frac{-\frac{1}{4a}+\frac{b\,x^2}{2a^2}}{x^4}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}$	46
parallelrisch	$\frac{4b^2\ln(x)x^4-2b^2\ln\left(b\,x^2+a\right)x^4+2ab\,x^2-a^2}{4a^3x^4}$	48

input
```
int(1/x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (ax + bx^3)}\, dx = -\frac{2\, b^2 x^4 \log \left(b x^2 + a\right) - 4\, b^2 x^4 \log \left(x\right) - 2\, a b x^2 + a^2}{4\, a^3 x^4}$$

input
```
integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*
x^4)
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4 (ax + bx^3)}\, dx = \frac{-a + 2bx^2}{4a^2x^4} + \frac{b^2 \log \left(x\right)}{a^3} - \frac{b^2 \log \left(\frac{a}{b} + x^2\right)}{2a^3}$$

input
```
integrate(1/x**4/(b*x**3+a*x),x)
```

output
```
(-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2
*a**3)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (ax + bx^3)}\, dx = -\frac{b^2 \log \left(b x^2 + a\right)}{2\, a^3} + \frac{b^2 \log \left(x\right)}{a^3} + \frac{2\, b x^2 - a}{4\, a^2 x^4}$$

input
```
integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
-1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{b^2 \log (x^2)}{2 a^3} - \frac{b^2 \log (|bx^2 + a|)}{2 a^3} - \frac{3 b^2 x^4 - 2 abx^2 + a^2}{4 a^3 x^4}$$

input
```
integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 -
2*a*b*x^2 + a^2)/(a^3*x^4)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{b^2 \ln (x)}{a^3} - \frac{b^2 \ln (b x^2 + a)}{2 a^3} - \frac{\frac{1}{4 a} - \frac{b x^2}{2 a^2}}{x^4}$$

input
```
int(1/(x^4*(a*x + b*x^3)),x)
```

output
```
(b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^
2))/x^4
```

3.21 $\int \frac{x^2}{(ax+bx^3)^2}\,dx$

3.21.1 Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx=\frac{x}{2a(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx=\frac{x}{2a(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[x^2/(a*x + b*x^3)^2,x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

3.21.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^3)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{(a+bx^2)^2}dx$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\,(a+bx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\,(a+bx^2)}$$

input
```
Int[x^2/(a*x + b*x^3)^2,x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

3.21.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.21.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	62

input
```
int(x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx$$
$$=\left[\frac{2\,abx-(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^2b^2x^2+a^3b)},\frac{abx+(bx^2+a)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(a^2b^2x^2+a^3b)}\right]$$

input
```
integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(x**2/(b*x**3+a*x)**2,x)
```

output
```
x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) +
x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2\,(abx^2+a^2)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2\,(bx^2+a)a}$$

input
```
integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2\,a\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(x^2/(a*x + b*x^3)^2,x)
```

output
```
x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

3.22 $\int \frac{x}{(ax+bx^3)^2} dx$

3.22.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x}{(ax+bx^3)^2} dx = \frac{1}{2a(a+bx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx^2)}{2a^2}$$

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x}{(ax+bx^3)^2} dx = \frac{\frac{a}{a+bx^2} + 2\log(x) - \log(a+bx^2)}{2a^2}$$

input
```
Integrate[x/(a*x + b*x^3)^2,x]
```

output
```
(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^3)^2}dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{1}{x(a+bx^2)^2}dx$$
$$\Big\downarrow \ 243$$
$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)^2}dx^2$$
$$\Big\downarrow \ 54$$
$$\frac{1}{2}\int \left(-\frac{b}{a^2(bx^2+a)}-\frac{b}{a(bx^2+a)^2}+\frac{1}{a^2x^2}\right)dx^2$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\left(-\frac{\log(a+bx^2)}{a^2}+\frac{\log(x^2)}{a^2}+\frac{1}{a(a+bx^2)}\right)$$

input
```
Int[x/(a*x + b*x^3)^2,x]
```

output
```
(1/(a*(a + b*x^2)) + Log[x^2]/a^2 - Log[a + b*x^2]/a^2)/2
```

3.22.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$\frac{\ln(x)}{a^2}-\frac{b\left(\frac{\ln\left(b\,x^2+a\right)}{b}-\frac{a}{b\left(b\,x^2+a\right)}\right)}{2a^2}$	42
parallelrisch	$\frac{2b\ln(x)x^2-b\ln(b\,x^2+a)x^2-b\,x^2+2a\ln(x)-a\ln(b\,x^2+a)}{2a^2(b\,x^2+a)}$	60

input
```
int(x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x}{(ax+bx^3)^2}\,dx = -\frac{(bx^2+a)\log(bx^2+a)-2\,(bx^2+a)\log(x)-a}{2\,(a^2bx^2+a^3)}$$

input
```
integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 +
a^3)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2a^2+2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate(x/(b*x**3+a*x)**2,x)
```

output
```
1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2(abx^2+a^2)} - \frac{\log(bx^2+a)}{2a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{\log(x^2)}{2a^2} - \frac{\log(|bx^2+a|)}{2a^2} + \frac{bx^2+2a}{2(bx^2+a)a^2}$$

input
```
integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
 + a)*a^2)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{1}{2\,a\,(b\,x^2+a)} - \frac{\ln(b\,x^2+a)}{2\,a^2}$$

input
```
int(x/(a*x + b*x^3)^2,x)
```

output
```
log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

3.23 $\int \frac{1}{(ax+bx^3)^2} dx$

3.23.1 Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{1}{(ax+bx^3)^2} dx = -\frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{(ax+bx^3)^2} dx = -\frac{1}{a^2x} - \frac{bx}{2a^2(a+bx^2)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[(a*x + b*x^3)^(-2),x]
```

output
```
-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2))
```

3.23.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2026, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^3)^2}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^2(a+bx^2)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3\int \frac{1}{x^2(bx^2+a)}dx}{2a}+\frac{1}{2ax(a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{3\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}$$

input
```
Int[(a*x + b*x^3)^(-2),x]
```

output
```
1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/a^(3/2)))/(2*a)
```

3.23.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.23.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b \left(\frac{x}{2 b \,x^{2}+2 a}+\frac{3 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{2}}$	45
risch	$\frac{-\frac{3 b \,x^{2}}{2 a^{2}}-\frac{1}{a}}{x \left(b \,x^{2}+a \right)}+\frac{3 \left(\sum_{\_R =\operatorname{RootOf}\left(a^{5} \_Z^{2}+b \right)}\_R \ln \left(\left(3 a^{5} \_R^{2}+2 b \right) x +a^{3} \_R \right)\right)}{4}$	68

input
```
int(1/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{1}{(ax+bx^3)^2}\,dx = \left[-\frac{6\,bx^2 - 3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2+a}\right) + 4\,a}{4\,(a^2bx^3 + a^3x)},\right.$$
$$\left. -\frac{3\,bx^2 + 3\,(bx^3+ax)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 2\,a}{2\,(a^2bx^3 + a^3x)}\right]$$

input
```
integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a
*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int \frac{1}{(ax+bx^3)^2}\,dx = \frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} + \frac{-2a - 3bx^2}{2a^3x + 2a^2bx^3}$$

input
```
integrate(1/(b*x**3+a*x)**2,x)
```

output
```
3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**
3*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{3\,bx^2+2\,a}{2\,(a^2bx^3+a^3x)} - \frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

input
```
integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a^2)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

input
```
integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
 + a*x)*a^2)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int(1/(a*x + b*x^3)^2,x)
```

output
```
- (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^
(1/2)))/(2*a^(5/2))
```

3.24 $\int \frac{1}{x(ax+bx^3)^2} dx$

3.24.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x(ax+bx^3)^2} dx = -\frac{1}{2a^2x^2} - \frac{b}{2a^2(a+bx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx^2)}{a^3}$$

output
```
-1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

3.24.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x(ax+bx^3)^2} dx = -\frac{a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log(a+bx^2)}{2a^3}$$

input
```
Integrate[1/(x*(a*x + b*x^3)^2),x]
```

output
```
-1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

3.24.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (ax+bx^3)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^3 (a+bx^2)^2}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2}\int \left(\frac{2b^2}{a^3 (bx^2+a)}+\frac{b^2}{a^2 (bx^2+a)^2}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^4}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{2b\log \left(x^2\right)}{a^3}+\frac{2b\log \left(a+bx^2\right)}{a^3}-\frac{b}{a^2 (a+bx^2)}-\frac{1}{a^2x^2}\right)$$

input
```
Int[1/(x*(a*x + b*x^3)^2),x]
```

output
```
(-(1/(a^2*x^2)) - b/(a^2*(a + b*x^2)) - (2*b*Log[x^2])/a^3 + (2*b*Log[a +
b*x^2])/a^3)/2
```

3.24.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
norman	$\frac{\frac{b^{2} x^{4}}{a^{3}}-\frac{1}{2 a}}{x^{2} \left(b \,x^{2}+a \right)}+\frac{b \ln \left(b \,x^{2}+a \right)}{a^{3}}-\frac{2 b \ln \left(x \right)}{a^{3}}$	52
risch	$\frac{-\frac{b \,x^{2}}{a^{2}}-\frac{1}{2 a}}{x^{2} \left(b \,x^{2}+a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{b \ln \left(-b \,x^{2}-a \right)}{a^{3}}$	54
default	$-\frac{1}{2 a^{2} x^{2}}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{b^{2} \left(\frac{2 \ln \left(b \,x^{2}+a \right)}{b}-\frac{a}{b \left(b \,x^{2}+a \right)}\right)}{2 a^{3}}$	55
parallelrisch	$-\frac{4 b^{2} \ln \left(x \right) x^{4}-2 b^{2} \ln \left(b \,x^{2}+a \right) x^{4}-2 b^{2} x^{4}+4 \ln \left(x \right) x^{2} a b -2 \ln \left(b \,x^{2}+a \right) x^{2} a b +a^{2}}{2 a^{3} x^{2} \left(b \,x^{2}+a \right)}$	80

input
```
int(1/x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2/a^3*x^4-1/2/a)/x^2/(b*x^2+a)+b*ln(b*x^2+a)/a^3-2*b*ln(x)/a^3
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

input
```
integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4
+ a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = \frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

input
```
integrate(1/x/(b*x**3+a*x)**2,x)
```

output
```
(-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/
b + x**2)/a**3
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

input
```
integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(
x)/a^3
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\,(ax+bx^3)^2}\,dx = -\frac{b\log(x^2)}{a^3} + \frac{b\log(|bx^2+a|)}{a^3} - \frac{2\,bx^2+a}{2\,(bx^4+ax^2)a^2}$$

input
```
integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 +
a*x^2)*a^2)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\,(ax+bx^3)^2}\,dx = \frac{b\,\ln(b\,x^2+a)}{a^3} - \frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2} - \frac{2\,b\,\ln(x)}{a^3}$$

input
```
int(1/(x*(a*x + b*x^3)^2),x)
```

output
```
(b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*lo
g(x))/a^3
```

3.25 $\int \frac{1}{x^2(ax+bx^3)^2}\,dx$

3.25.1 Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx=-\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/
a^(1/2))/a^(7/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx=-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3)^2),x]
```

output
```
-1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

3.25.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {9, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^3)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^4 (a+bx^2)^2}dx$$

$$\Big\downarrow \; 253$$

$$\frac{5\int \frac{1}{x^4(bx^2+a)}dx}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{5\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

input
```
Int[1/(x^2*(a*x + b*x^3)^2),x]
```

output
```
1/(2*a*x^3*(a + b*x^2)) + (5*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/(2*a)
```

3.25.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.25.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{1}{3a^{2}x^{3}}+\frac{2b}{a^{3}x}+\frac{b^{2}\left(\frac{x}{2b\,x^{2}+2a}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}$	55
risch	$\frac{\frac{5b^{2}x^{4}}{2a^{3}}+\frac{5b\,x^{2}}{3a^{2}}-\frac{1}{3a}}{x^{3}\left(b\,x^{2}+a\right)}+\frac{5\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{4a^{4}}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^{4}}$	91

input
```
int(1/x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3+2*b/a^3/x+b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2)))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.53

$$\int \frac{1}{x^2 (ax+bx^3)^2} dx = \left[\frac{30 b^2x^4 + 20 abx^2 + 15 (b^2x^5 + abx^3)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2+2 ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 4 a^2}{12 (a^3bx^5 + a^4x^3)}, \frac{15 b^2x^4 + 10 abx^2 + 15 (b^2x^5 + a\right.$$

input
```
integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1
/6*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sq
rt(b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \frac{1}{x^2 (ax+bx^3)^2} dx = -\frac{5\sqrt{-\frac{b^3}{a^7}} \log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}} \log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2x^4}{6a^4x^3 + 6a^3bx^5}$$

input
```
integrate(1/x**2/(b*x**3+a*x)**2,x)
```

output
```
-5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/
a**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*
b**2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (ax+bx^3)^2} dx = \frac{15 b^2 x^4 + 10 abx^2 - 2 a^2}{6 (a^3 bx^5 + a^4 x^3)} + \frac{5 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} a^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 (ax+bx^3)^2} dx = \frac{5 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} a^3} + \frac{b^2 x}{2 (bx^2 + a) a^3} + \frac{6 bx^2 - a}{3 a^3 x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3
) + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (ax+bx^3)^2} dx = \frac{\frac{5 b x^2}{3 a^2} - \frac{1}{3 a} + \frac{5 b^2 x^4}{2 a^3}}{b x^5 + a x^3} + \frac{5 b^{3/2} \operatorname{atan}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{2 a^{7/2}}$$

input
```
int(1/(x^2*(a*x + b*x^3)^2),x)
```

output
```
((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b
^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

3.26 $\int \frac{x^5}{x-x^3}\,dx$

3.26.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^5}{x-x^3}\,dx = -x-\frac{x^3}{3}+\operatorname{arctanh}(x)$$

output
```
-x-1/3*x^3+arctanh(x)
```

3.26.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{x^5}{x-x^3}\,dx = -x-\frac{x^3}{3}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

input
```
Integrate[x^5/(x - x^3),x]
```

output
```
-x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2
```

3.26.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x-x^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^4}{1-x^2}dx$$
$$\Big\downarrow \; 254$$
$$\int \left(-x^2+\frac{1}{1-x^2}-1\right)dx$$
$$\Big\downarrow \; 2009$$
$$\text{arctanh}(x)-\frac{x^3}{3}-x$$

input
```
Int[x^5/(x - x^3),x]
```

output
```
-x - x^3/3 + ArcTanh[x]
```

3.26.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

method	result	size
meijerg	$-\frac{i\left(-\frac{2ix\left(5x^2+15\right)}{15}+2i\operatorname{arctanh}(x)\right)}{2}$	21
default	$-\frac{x^3}{3}-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	22
norman	$-\frac{x^3}{3}-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	22
risch	$-\frac{x^3}{3}-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	22
parallelrisch	$-\frac{x^3}{3}-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	22

input
```
int(x^5/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-2/15*I*x*(5*x^2+15)+2*I*arctanh(x))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{1}{3}\,x^3 - x + \frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

input
```
integrate(x^5/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{x^3}{3} - x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
integrate(x**5/(-x**3+x),x)
```

output
```
-x**3/3 - x - log(x - 1)/2 + log(x + 1)/2
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{1}{3}\,x^3 - x + \frac{1}{2}\,\log(x+1) - \frac{1}{2}\,\log(x-1)$$

input
```
integrate(x^5/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{1}{3}\,x^3 - x + \frac{1}{2}\,\log(|x+1|) - \frac{1}{2}\,\log(|x-1|)$$

input
```
integrate(x^5/(-x^3+x),x, algorithm="giac")
```

output
```
-1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.26.9 Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{x-x^3}\,dx = \operatorname{atanh}(x) - x - \frac{x^3}{3}$$

input
```
int(x^5/(x - x^3),x)
```

output
```
atanh(x) - x - x^3/3
```

3.27 $\int \frac{x^4}{x-x^3} \, dx$

3.27.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{x^4}{x-x^3} \, dx = -\frac{x^2}{2} - \frac{1}{2} \log \left(1-x^2\right)$$

output
```
-1/2*x^2-1/2*ln(-x^2+1)
```

3.27.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{x-x^3} \, dx = -\frac{x^2}{2} - \frac{1}{2} \log \left(-1+x^2\right)$$

input
```
Integrate[x^4/(x - x^3),x]
```

output
```
-1/2*x^2 - Log[-1 + x^2]/2
```

3.27.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x-x^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^3}{1-x^2}dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{x^2}{1-x^2}dx^2$$
$$\Big\downarrow \; 49$$
$$\frac{1}{2}\int \left(\frac{1}{1-x^2}-1\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-x^2-\log\left(1-x^2\right)\right)$$

input
```
Int[x^4/(x - x^3),x]
```

output
```
(-x^2 - Log[1 - x^2])/2
```

3.27.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{x^2}{2}-\frac{\ln(x^2-1)}{2}$	15
meijerg	$-\frac{x^2}{2}-\frac{\ln(-x^2+1)}{2}$	17
default	$-\frac{x^2}{2}-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	19
norman	$-\frac{x^2}{2}-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	19
parallelrisch	$-\frac{x^2}{2}-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	19

input
```
int(x^4/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-1/2*ln(x^2-1)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx=-\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-1\right)$$

input
```
integrate(x^4/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*x^2 - 1/2*log(x^2 - 1)
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{x^2}{2} - \frac{\log(x^2-1)}{2}$$

input
```
integrate(x**4/(-x**3+x),x)
```

output
```
-x**2/2 - log(x**2 - 1)/2
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log(x+1) - \frac{1}{2}\,\log(x-1)$$

input
```
integrate(x^4/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2*x^2 - 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(|x^2-1|\right)$$

input
```
integrate(x^4/(-x^3+x),x, algorithm="giac")
```

output
```
-1/2*x^2 - 1/2*log(abs(x^2 - 1))
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{\ln\left(x^2-1\right)}{2}-\frac{x^2}{2}$$

input
```
int(x^4/(x - x^3),x)
```

output
```
- log(x^2 - 1)/2 - x^2/2
```

3.28 $\int \frac{x^3}{x-x^3}\,dx$

3.28.1 Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{x^3}{x-x^3}\,dx=-x+\operatorname{arctanh}(x)$$

output
```
-x+arctanh(x)
```

3.28.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(6) = 12.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{x-x^3}\,dx=-x-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

input
```
Integrate[x^3/(x - x^3),x]
```

output
```
-x - Log[1 - x]/2 + Log[1 + x]/2
```

3.28.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x-x^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2}{1-x^2}dx$$

$$\Big\downarrow \; 262$$

$$\int \frac{1}{1-x^2}dx-x$$

$$\Big\downarrow \; 219$$

$$\operatorname{arctanh}(x)-x$$

input
```
Int[x^3/(x - x^3),x]
```

output
```
-x + ArcTanh[x]
```

3.28.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.28.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

method	result	size
meijerg	$\frac{i(2ix-2i\operatorname{arctanh}(x))}{2}$	14
default	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17
norman	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17
risch	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17
parallelrisch	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17

input
```
int(x^3/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(2*I*x-2*I*arctanh(x))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{x^3}{x-x^3}\,dx = -x+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

input
```
integrate(x^3/(-x^3+x),x, algorithm="fricas")
```

output
```
-x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.28.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{x^3}{x-x^3}\,dx = -x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
integrate(x**3/(-x**3+x),x)
```

output
```
-x - log(x - 1)/2 + log(x + 1)/2
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{x^3}{x-x^3}\,dx = -x + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate(x^3/(-x^3+x),x, algorithm="maxima")
```

output
```
-x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.00

$$\int \frac{x^3}{x-x^3}\,dx = -x + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

input
```
integrate(x^3/(-x^3+x),x, algorithm="giac")
```

output
```
-x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{x - x^3} \, dx = \operatorname{atanh}(x) - x$$

input
```
int(x^3/(x - x^3),x)
```

output
```
atanh(x) - x
```

3.29 $\int \frac{x^2}{x-x^3} \, dx$

3.29.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^2}{x-x^3} \, dx = -\frac{1}{2} \log \left(1-x^2\right)$$

output
```
-1/2*ln(-x^2+1)
```

3.29.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{x-x^3} \, dx = -\frac{1}{2} \log \left(1-x^2\right)$$

input
```
Integrate[x^2/(x - x^3),x]
```

output
```
-1/2*Log[1 - x^2]
```

3.29.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {9, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x-x^3}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{x}{1-x^2}dx$$
$$\Big\downarrow \ 240$$
$$-\frac{1}{2}\log\left(1-x^2\right)$$

input
```
Int[x^2/(x - x^3),x]
```

output
```
-1/2*Log[1 - x^2]
```

3.29.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

3.29.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\ln(x^2-1)}{2}$	9
meijerg	$-\frac{\ln(-x^2+1)}{2}$	11
default	$-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	14
norman	$-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	14

input
```
int(x^2/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2-1)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{1}{2}\log\left(x^2-1\right)$$

input
```
integrate(x^2/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*log(x^2 - 1)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate(x**2/(-x**3+x),x)
```

output
```
-log(x**2 - 1)/2
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate(x^2/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2*log(x + 1) - 1/2*log(x - 1)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

input
```
integrate(x^2/(-x^3+x),x, algorithm="giac")
```

output
```
-1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{\ln(x^2-1)}{2}$$

input
```
int(x^2/(x - x^3),x)
```

output
```
-log(x^2 - 1)/2
```

3.30 $\int \frac{x}{x-x^3} \, dx$

3.30.1 Optimal result

Integrand size = 11, antiderivative size = 2

$$\int \frac{x}{x-x^3} \, dx = \operatorname{arctanh}(x)$$

output
```
arctanh(x)
```

3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(2) = 4.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{x}{x-x^3} \, dx = -\frac{1}{2} \log(1-x) + \frac{1}{2} \log(1+x)$$

input
```
Integrate[x/(x - x^3),x]
```

output
```
-1/2*Log[1 - x] + Log[1 + x]/2
```

3.30.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x-x^3} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{1-x^2}dx$$
$$\Big\downarrow \text{ 219}$$
$$\text{arctanh}(x)$$

input
```
Int[x/(x - x^3),x]
```

output
```
ArcTanh[x]
```

3.30.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.30.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
risch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

input
```
int(x/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
arctanh(x)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{x}{x-x^3}\,dx = \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate(x/(-x^3+x),x, algorithm="fricas")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{x}{x-x^3}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
integrate(x/(-x**3+x),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{x}{x-x^3} \, dx = \frac{1}{2} \log \left(x+1\right) - \frac{1}{2} \log \left(x-1\right)$$

input
```
integrate(x/(-x^3+x),x, algorithm="maxima")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{x}{x-x^3} \, dx = \frac{1}{2} \log \left(|x+1|\right) - \frac{1}{2} \log \left(|x-1|\right)$$

input
```
integrate(x/(-x^3+x),x, algorithm="giac")
```

output
```
1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{x}{x-x^3}\,dx = \operatorname{atanh}(x)$$

input
```
int(x/(x - x^3),x)
```

output
```
atanh(x)
```

3.31 $\int \frac{1}{x-x^3} dx$

3.31.1 Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{1}{x-x^3} dx = \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

output
```
ln(x)-1/2*ln(-x^2+1)
```

3.31.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x-x^3} dx = \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[(x - x^3)^(-1),x]
```

output
```
Log[x] - Log[1 - x^2]/2
```

3.31.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2026, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-x^3} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{1}{x(1-x^2)}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2(1-x^2)}dx^2$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2+\int \frac{1}{1-x^2}dx^2\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\left(\int \frac{1}{1-x^2}dx^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\log\left(x^2\right)-\log\left(1-x^2\right)\right)$$

input
```
Int[(x - x^3)^(-1),x]
```

output
```
(Log[x^2] - Log[1 - x^2])/2
```

3.31.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.31.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^2-1)}{2}$	12
default	$\ln(x)-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	16
norman	$\ln(x)-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	16
parallelrisch	$\ln(x)-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	16
meijerg	$\ln(x)+\frac{i\pi}{2}-\frac{\ln(-x^2+1)}{2}$	18

input
```
int(1/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/2*ln(x^2-1)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{x - x^3} \, dx = -\frac{1}{2} \log \left(x^2 - 1\right) + \log \left(x\right)$$

input
```
integrate(1/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*log(x^2 - 1) + log(x)
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{x - x^3} \, dx = \log \left(x\right) - \frac{\log \left(x^2 - 1\right)}{2}$$

input
```
integrate(1/(-x**3+x),x)
```

output
```
log(x) - log(x**2 - 1)/2
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x - x^3} \, dx = -\frac{1}{2} \log \left(x + 1\right) - \frac{1}{2} \log \left(x - 1\right) + \log \left(x\right)$$

input
```
integrate(1/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{x-x^3}\,dx = \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/(-x^3+x),x, algorithm="giac")
```

output
```
1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{x-x^3}\,dx = \ln(x) - \frac{\ln(x^2-1)}{2}$$

input
```
int(1/(x - x^3),x)
```

output
```
log(x) - log(x^2 - 1)/2
```

3.32 $\int \frac{1}{x(x-x^3)}\,dx$

3.32.1 Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{x\,(x-x^3)}\,dx = -\frac{1}{x}+\operatorname{arctanh}(x)$$

output
```
-1/x+arctanh(x)
```

3.32.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int \frac{1}{x\,(x-x^3)}\,dx = -\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

input
```
Integrate[1/(x*(x - x^3)),x]
```

output
```
-x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

3.32.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x-x^3)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^2(1-x^2)}dx$$

$$\Big\downarrow \text{ 264}$$

$$\int \frac{1}{1-x^2}dx-\frac{1}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}(x)-\frac{1}{x}$$

input
```
Int[1/(x*(x - x^3)),x]
```

output
```
-x^(-1) + ArcTanh[x]
```

3.32.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.32.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

method	result	size
meijerg	$\frac{i\left(\frac{2i}{x}-2i\,\mathrm{arctanh}\left(x\right)\right)}{2}$	16
default	$-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}-\frac{1}{x}$	19
norman	$-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}-\frac{1}{x}$	19
risch	$-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}-\frac{1}{x}$	19
parallelrisch	$\frac{\ln\left(1+x\right)x-\ln\left(-1+x\right)x-2}{2x}$	21

input
```
int(1/x/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(2*I/x-2*I*arctanh(x))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(x-x^3\right)}\,dx=\frac{x\log\left(x+1\right)-x\log\left(x-1\right)-2}{2\,x}$$

input
```
integrate(1/x/(-x^3+x),x, algorithm="fricas")
```

output
```
1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{x\left(x-x^{3}\right)}\,dx = -\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}-\frac{1}{x}$$

input
```
integrate(1/x/(-x**3+x),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 - 1/x
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\left(x-x^{3}\right)}\,dx = -\frac{1}{x}+\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

input
```
integrate(1/x/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(x-x^{3}\right)}\,dx = -\frac{1}{x}+\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

input
```
integrate(1/x/(-x^3+x),x, algorithm="giac")
```

output
```
-1/x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(x-x^3)}\,dx = \operatorname{atanh}(x) - \frac{1}{x}$$

input
```
int(1/(x*(x - x^3)),x)
```

output
```
atanh(x) - 1/x
```

3.33 $\int \frac{1}{x^2(x-x^3)}\,dx$

3.33.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x^2(x-x^3)}\,dx=-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

output
```
-1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

3.33.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(x-x^3)}\,dx=-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[1/(x^2*(x - x^3)),x]
```

output
```
-1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

3.33.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x-x^3)} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^3 (1-x^2)}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2} \int \frac{1}{x^4 (1-x^2)}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$\frac{1}{2} \int \left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{1-x^2}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2} \left(-\frac{1}{x^2}+\log \left(x^2\right)-\log \left(1-x^2\right)\right)$$

input
```
Int[1/(x^2*(x - x^3)),x]
```

output
```
(-x^(-2) + Log[x^2] - Log[1 - x^2])/2
```

3.33.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.33.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln\left(x^2-1\right)}{2}$	17
default	$-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	21
norman	$-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	21
meijerg	$-\frac{1}{2x^2}+\ln\left(x\right)+\frac{i\pi}{2}-\frac{\ln\left(-x^2+1\right)}{2}$	23
parallelrisch	$\frac{2\ln(x)x^2-\ln(1+x)x^2-\ln(-1+x)x^2-1}{2x^2}$	33

input
```
int(1/x^2/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+ln(x)-1/2*ln(x^2-1)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(x-x^3\right)}\,dx=-\frac{x^2\log\left(x^2-1\right)-2\,x^2\log\left(x\right)+1}{2\,x^2}$$

input
```
integrate(1/x^2/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 (x-x^3)} dx = \log(x) - \frac{\log(x^2-1)}{2} - \frac{1}{2x^2}$$

input
```
integrate(1/x**2/(-x**3+x),x)
```

output
```
log(x) - log(x**2 - 1)/2 - 1/(2*x**2)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (x-x^3)} dx = -\frac{1}{2x^2} - \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1) + \log(x)$$

input
```
integrate(1/x^2/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2 (x-x^3)} dx = -\frac{x^2+1}{2x^2} + \frac{1}{2}\log(x^2) - \frac{1}{2}\log(|x^2-1|)$$

input
```
integrate(1/x^2/(-x^3+x),x, algorithm="giac")
```

output
```
-1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2\,(x-x^3)}\,dx = \ln(x) - \frac{\ln(x^2-1)}{2} - \frac{1}{2\,x^2}$$

input
```
int(1/(x^2*(x - x^3)),x)
```

output
```
log(x) - log(x^2 - 1)/2 - 1/(2*x^2)
```

3.34 $\int \frac{1}{x^3(x-x^3)}\,dx$

3.34.1 Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{x^3(x-x^3)}\,dx = -\frac{1}{3x^3}-\frac{1}{x}+\operatorname{arctanh}(x)$$

output
```
-1/3/x^3-1/x+arctanh(x)
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1}{x^3(x-x^3)}\,dx = -\frac{1}{3x^3}-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

input
```
Integrate[1/(x^3*(x - x^3)),x]
```

output
```
-1/3*1/x^3 - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

3.34.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x-x^3)}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^4(1-x^2)}dx$$
$$\Big\downarrow \text{ 264}$$
$$\int \frac{1}{x^2(1-x^2)}dx-\frac{1}{3x^3}$$
$$\Big\downarrow \text{ 264}$$
$$\int \frac{1}{1-x^2}dx-\frac{1}{3x^3}-\frac{1}{x}$$
$$\Big\downarrow \text{ 219}$$
$$\text{arctanh}(x)-\frac{1}{3x^3}-\frac{1}{x}$$

input
```
Int[1/(x^3*(x - x^3)),x]
```

output
```
-1/3*1/x^3 - x^(-1) + ArcTanh[x]
```

3.34.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.34.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
meijerg	$-\frac{i\left(-\frac{2i}{x}-\frac{2i}{3x^3}+2i\,\mathrm{arctanh}(x)\right)}{2}$	22
default	$-\frac{1}{3x^3}-\frac{1}{x}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	24
norman	$\frac{-\frac{1}{3}-x^2}{x^3}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	25
risch	$\frac{-\frac{1}{3}-x^2}{x^3}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	25
parallelrisch	$\frac{3\ln(1+x)x^3-3\ln(-1+x)x^3-2-6x^2}{6x^3}$	31

input
```
int(1/x^3/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-2*I/x-2/3*I/x^3+2*I*arctanh(x))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3\,(x-x^3)}\,dx = \frac{3\,x^3\log\left(x+1\right)-3\,x^3\log\left(x-1\right)-6\,x^2-2}{6\,x^3}$$

input
```
integrate(1/x^3/(-x^3+x),x, algorithm="fricas")
```

output
```
1/6*(3*x^3*log(x + 1) - 3*x^3*log(x - 1) - 6*x^2 - 2)/x^3
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^3 (x - x^3)} \, dx = -\frac{\log (x - 1)}{2} + \frac{\log (x + 1)}{2} - \frac{3x^2 + 1}{3x^3}$$

input
```
integrate(1/x**3/(-x**3+x),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^3 (x - x^3)} \, dx = -\frac{3\,x^2 + 1}{3\,x^3} + \frac{1}{2}\,\log (x + 1) - \frac{1}{2}\,\log (x - 1)$$

input
```
integrate(1/x^3/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/3*(3*x^2 + 1)/x^3 + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^3 (x - x^3)} \, dx = -\frac{3\,x^2 + 1}{3\,x^3} + \frac{1}{2}\,\log (|x + 1|) - \frac{1}{2}\,\log (|x - 1|)$$

input
```
integrate(1/x^3/(-x^3+x),x, algorithm="giac")
```

output
```
-1/3*(3*x^2 + 1)/x^3 + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.34.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (x - x^3)} \, dx = \operatorname{atanh}(x) - \frac{x^2 + \frac{1}{3}}{x^3}$$

input
```
int(1/(x^3*(x - x^3)),x)
```

output
```
atanh(x) - (x^2 + 1/3)/x^3
```

3.35 $\int \frac{1}{x^4(x-x^3)}\,dx$

3.35.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{x^4(x-x^3)}\,dx = -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

output
```
-1/4/x^4-1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

3.35.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(x-x^3)}\,dx = -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[1/(x^4*(x - x^3)),x]
```

output
```
-1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

3.35.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x - x^3)} dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^5 (1 - x^2)} dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2} \int \frac{1}{x^6 (1 - x^2)} dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2} \int \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \frac{1}{1 - x^2} \right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(-\frac{1}{2x^4} - \frac{1}{x^2} + \log \left(x^2\right) - \log \left(1 - x^2\right) \right)$$

input
```
Int[1/(x^4*(x - x^3)),x]
```

output
```
(-1/2*1/x^4 - x^(-2) + Log[x^2] - Log[1 - x^2])/2
```

3.35.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4}+\ln\left(x\right)-\frac{\ln\left(x^2-1\right)}{2}$	23
default	$-\frac{1}{4x^4}-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	26
norman	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4}+\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	27
meijerg	$-\frac{1}{4x^4}-\frac{1}{2x^2}+\ln\left(x\right)+\frac{i\pi}{2}-\frac{\ln\left(-x^2+1\right)}{2}$	28
parallelrisch	$\frac{4\ln\left(x\right)x^4-2\ln\left(1+x\right)x^4-2\ln\left(-1+x\right)x^4-1-2x^2}{4x^4}$	38

input
```
int(1/x^4/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
(-1/4-1/2*x^2)/x^4+ln(x)-1/2*ln(x^2-1)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4\left(x-x^3\right)}\,dx=-\frac{2\,x^4\log\left(x^2-1\right)-4\,x^4\log\left(x\right)+2\,x^2+1}{4\,x^4}$$

input
```
integrate(1/x^4/(-x^3+x),x, algorithm="fricas")
```

output
```
-1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (x - x^3)} \, dx = \log(x) - \frac{\log(x^2 - 1)}{2} - \frac{2x^2 + 1}{4x^4}$$

input
```
integrate(1/x**4/(-x**3+x),x)
```

output
```
log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (x - x^3)} \, dx = -\frac{2\,x^2 + 1}{4\,x^4} - \frac{1}{2}\log(x + 1) - \frac{1}{2}\log(x - 1) + \log(x)$$

input
```
integrate(1/x^4/(-x^3+x),x, algorithm="maxima")
```

output
```
-1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (x - x^3)} \, dx = -\frac{3\,x^4 + 2\,x^2 + 1}{4\,x^4} + \frac{1}{2}\log(x^2) - \frac{1}{2}\log(|x^2 - 1|)$$

input
```
integrate(1/x^4/(-x^3+x),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4 (x - x^3)} \, dx = \ln(x) - \frac{\ln(x^2 - 1)}{2} - \frac{\frac{x^2}{2} + \frac{1}{4}}{x^4}$$

input
```
int(1/(x^4*(x - x^3)),x)
```

output
```
log(x) - log(x^2 - 1)/2 - (x^2/2 + 1/4)/x^4
```

3.36 $\int \frac{1}{x+bx^3}\,dx$

3.36.1 Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{1}{x+bx^3}\,dx = \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

output
```
ln(x)-1/2*ln(b*x^2+1)
```

3.36.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+bx^3}\,dx = \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

input
```
Integrate[(x + b*x^3)^(-1),x]
```

output
```
Log[x] - Log[1 + b*x^2]/2
```

3.36.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2026, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^3+x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x(bx^2+1)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+1)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-b\int \frac{1}{bx^2+1}dx^2\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\log\left(x^2\right)-b\int \frac{1}{bx^2+1}dx^2\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(x^2\right)-\log\left(bx^2+1\right)\right)$$

input
```
Int[(x + b*x^3)^(-1),x]
```

output
```
(Log[x^2] - Log[1 + b*x^2])/2
```

3.36.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.36.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)-\frac{\ln(b\,x^2+1)}{2}$	14
norman	$\ln(x)-\frac{\ln(b\,x^2+1)}{2}$	14
risch	$\ln(x)-\frac{\ln(b\,x^2+1)}{2}$	14
parallelrisch	$\ln(x)-\frac{\ln(b\,x^2+1)}{2}$	14
meijerg	$\ln(x)+\frac{\ln(b)}{2}-\frac{\ln(b\,x^2+1)}{2}$	18

input
```
int(1/(b*x^3+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/2*ln(b*x^2+1)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+bx^3}\,dx = -\frac{1}{2}\log\left(bx^2+1\right)+\log\left(x\right)$$

input
```
integrate(1/(b*x^3+x),x, algorithm="fricas")
```

output
```
-1/2*log(b*x^2 + 1) + log(x)
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{x+bx^3}\,dx = \log\left(x\right)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}$$

input
```
integrate(1/(b*x**3+x),x)
```

output
```
log(x) - log(x**2 + 1/b)/2
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+bx^3}\,dx = -\frac{1}{2}\log\left(bx^2+1\right)+\log\left(x\right)$$

input
```
integrate(1/(b*x^3+x),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + 1) + log(x)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+bx^3}\,dx = \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

input
```
integrate(1/(b*x^3+x),x, algorithm="giac")
```

output
```
1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

3.36.9 Mupad [B] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x+bx^3}\,dx = \ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

input
```
int(1/(x + b*x^3),x)
```

output
```
log(x) - log((3*b*x^2)/2 + 3/2)/2
```

3.37 $\int \frac{1}{-x+bx^3} dx$

3.37.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{-x+bx^3} dx = -\log(x) + \frac{1}{2}\log\left(1-bx^2\right)$$

output
```
-ln(x)+1/2*ln(-b*x^2+1)
```

3.37.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+bx^3} dx = -\log(x) + \frac{1}{2}\log\left(1-bx^2\right)$$

input
```
Integrate[(-x + b*x^3)^(-1),x]
```

output
```
-Log[x] + Log[1 - b*x^2]/2
```

3.37.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {2026, 243, 25, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^3-x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x(bx^2-1)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int -\frac{1}{x^2(1-bx^2)}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{x^2(1-bx^2)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(-b\int \frac{1}{1-bx^2}dx^2-\int \frac{1}{x^2}dx^2\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(-b\int \frac{1}{1-bx^2}dx^2-\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(1-bx^2\right)-\log\left(x^2\right)\right)$$

input
```
Int[(-x + b*x^3)^(-1),x]
```

output
```
(-Log[x^2] + Log[1 - b*x^2])/2
```

3.37.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.37.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
default	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
norman	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
parallelrisch	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
risch	$-\ln\left(x\right)+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	17
meijerg	$-\ln\left(x\right)-\frac{\ln\left(-b\right)}{2}+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	23

input
```
int(1/(b*x^3-x),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+1/2*ln(b*x^2-1)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{-x+bx^3}\,dx = \frac{1}{2}\log\left(bx^2-1\right)-\log\left(x\right)$$

input
```
integrate(1/(b*x^3-x),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 - 1) - log(x)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{-x+bx^3}\,dx = -\log\left(x\right)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

input
```
integrate(1/(b*x**3-x),x)
```

output
```
-log(x) + log(x**2 - 1/b)/2
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{-x+bx^3}\,dx = \frac{1}{2}\log\left(bx^2-1\right)-\log\left(x\right)$$

input
```
integrate(1/(b*x^3-x),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 - 1) - log(x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+bx^3}\,dx = -\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

input
```
integrate(1/(b*x^3-x),x, algorithm="giac")
```

output
```
-1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{-x+bx^3}\,dx = \frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2}-\ln\left(x\right)$$

input
```
int(-1/(x - b*x^3),x)
```

output
```
log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

3.38 $\int x^3\sqrt{ax+bx^3}\,dx$

3.38.1 Optimal result

Integrand size = 17, antiderivative size = 163

$$\int x^3\sqrt{ax+bx^3}\,dx = -\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}$$

output
```
-20/231*a^2*(b*x^3+a*x)^(1/2)/b^2+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b+2/11*x^4*
(b*x^3+a*x)^(1/2)+10/231*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)
/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)^(1/2)
```

3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int x^3\sqrt{ax+bx^3}\,dx = \frac{2\sqrt{x\left(a+bx^2\right)}\left(\sqrt{1+\frac{bx^2}{a}}\left(-5a^2+2abx^2+7b^2x^4\right)+5a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^3*Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 + 7*b^2*x^
4) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b^2*Sqrt[
1 + (b*x^2)/a])
```

3.38.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1927, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{11}a\int \frac{x^4}{\sqrt{bx^3+ax}}dx+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

$\Big\downarrow$ 761

$$\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}$$

input
```
Int[x^3*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^4*Sqrt[a*x + b*x^3])/11 + (2*a*((2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*
a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])))/(7*b)))/11
```

3.38.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.38.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{2\left(-21b^{2}x^{4}-6ab\,x^{2}+10a^{2}\right)x\left(b\,x^{2}+a\right)}{231b^{2}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{10a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{231b^{3}\sqrt{b\,x^{3}+ax}}$
default	$\frac{2x^{4}\sqrt{b\,x^{3}+ax}}{11}+\frac{4a\,x^{2}\sqrt{b\,x^{3}+ax}}{77b}-\frac{20a^{2}\sqrt{b\,x^{3}+ax}}{231b^{2}}+\frac{10a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{231b^{3}\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{2x^{4}\sqrt{b\,x^{3}+ax}}{11}+\frac{4a\,x^{2}\sqrt{b\,x^{3}+ax}}{77b}-\frac{20a^{2}\sqrt{b\,x^{3}+ax}}{231b^{2}}+\frac{10a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{231b^{3}\sqrt{b\,x^{3}+ax}}$

input
```
int(x^3*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(-21*b^2*x^4-6*a*b*x^2+10*a^2)/b^2*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+
10/231*a^3/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*
(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+
a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)
)
```

3.38.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int x^3\sqrt{ax+bx^3}\,dx = \frac{2\left(10\,a^3\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(21\,b^3x^4+6\,ab^2x^2-10\,a^2b\right)\sqrt{bx^3+ax}\right)}{231\,b^3}$$

input
```
integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(10*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (21*b^3*x^4 + 6*
a*b^2*x^2 - 10*a^2*b)*sqrt(b*x^3 + a*x))/b^3
```

3.38.6 Sympy [F]

$$\int x^3\sqrt{ax+bx^3}\,dx = \int x^3\sqrt{x\left(a+bx^2\right)}\,dx$$

input
```
integrate(x**3*(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x*(a + b*x**2)), x)
```

3.38.7 Maxima [F]

$$\int x^3\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

3.38.8 Giac [F]

$$\int x^3\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{ax+bx^3}\,dx = \int x^3\,\sqrt{b\,x^3+a\,x}\,dx$$

input
```
int(x^3*(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^3*(a*x + b*x^3)^(1/2), x)
```

3.39 $\int x^2\sqrt{ax+bx^3}\,dx$

3.39.1 Optimal result

Integrand size = 17, antiderivative size = 281

$$\int x^2\sqrt{ax+bx^3}\,dx=-\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}$$

output
```
-4/15*a^2*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+4/45*a
*x*(b*x^3+a*x)^(1/2)/b+2/9*x^3*(b*x^3+a*x)^(1/2)+4/15*a^(9/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
)*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x
*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a
*x)^(1/2)-2/15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int x^2\sqrt{ax+bx^3}\,dx = \frac{2x\sqrt{x(a+bx^2)}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^2*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometr
ic2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

3.39.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1927, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{9}a\int \frac{x^3}{\sqrt{bx^3+ax}}dx+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int \frac{x}{\sqrt{bx^3+ax}}dx}{5b}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+$$

$$\frac{2}{9}x^3\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1510}$$

3.39. $\int x^2\sqrt{ax+bx^3}\,dx$

$$\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\right.}{5b\sqrt{ax+bx^3}}\right.$$

$$\frac{2}{9}x^3\sqrt{ax+bx^3}$$

input
```
Int[x^2*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^3*Sqrt[a*x + b*x^3])/9 + (2*a*((2*x*Sqrt[a*x + b*x^3])/(5*b) - (6*a*S
qrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*
x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a
+ b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*Sqrt[a*x + b*x^3])))/9
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.39. $\int x^2\sqrt{ax+bx^3}\,dx$

3.39.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.70

method	result
default	$\frac{2x^3\sqrt{b x^3+ax}}{9}+\frac{4ax\sqrt{b x^3+ax}}{45b}-\frac{2a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+ \cdots\right)}{15b^2\sqrt{b x^3+ax}}$
elliptic	$\frac{2x^3\sqrt{b x^3+ax}}{9}+\frac{4ax\sqrt{b x^3+ax}}{45b}-\frac{2a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+ \cdots\right)}{15b^2\sqrt{b x^3+ax}}$
risch	$\frac{2x^2\left(5b\,x^2+2a\right)\left(b\,x^2+a\right)}{45b\sqrt{x\left(b\,x^2+a\right)}}-\frac{2a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\cdots\right)}{\cdots}\right)}{15b^2\sqrt{b x^3+ax}}$

input
```
int(x^2*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^3*(b*x^3+a*x)^(1/2)+4/45*a*x*(b*x^3+a*x)^(1/2)/b-2/15*a^2/b^2*(-a*b)
^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)
^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+
(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(
1/2)))
```

3.39.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int x^2\sqrt{ax+bx^3}\,dx$$
$$=\frac{2\left(6\,a^2\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(5\,b^2x^3+2\,abx\right)\sqrt{bx^3+ax}\right)}{45\,b^2}$$

input
```
integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(6*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) + (5*b^2*x^3 + 2*a*b*x)*sqrt(b*x^3 + a*x))/b^2
```

3.39.6 Sympy [F]

$$\int x^2\sqrt{ax+bx^3}\,dx = \int x^2\sqrt{x\,(a+bx^2)}\,dx$$

input
```
integrate(x**2*(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x*(a + b*x**2)), x)
```

3.39.7 Maxima [F]

$$\int x^2\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x^2\,dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

3.39.8 Giac [F]

$$\int x^2\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x^2\,dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{ax+bx^3}\,dx = \int x^2\,\sqrt{b\,x^3+a\,x}\,dx$$

input
```
int(x^2*(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^2*(a*x + b*x^3)^(1/2), x)
```

3.40 $\int x\sqrt{ax+bx^3}\,dx$

3.40.1 Optimal result

Integrand size = 15, antiderivative size = 137

$$\int x\sqrt{ax+bx^3}\,dx=\frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}$$

output
```
4/21*a*(b*x^3+a*x)^(1/2)/b+2/7*x^2*(b*x^3+a*x)^(1/2)-2/21*a^(7/4)*(cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x
^3+a*x)^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

$$\int x\sqrt{ax+bx^3}\,dx=\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x*Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometric
2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

3.40.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1927, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{7}a\int \frac{x^2}{\sqrt{bx^3+ax}}dx+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

input
```
Int[x*Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a*x + b*x^3])/7 + (2*a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x +
b*x^3])))/7
```

3.40.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.40.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{2x^2\sqrt{b\,x^3+ax}}{7}+\frac{4a\sqrt{b\,x^3+ax}}{21b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^2\sqrt{b\,x^3+ax}}$	146
elliptic	$\frac{2x^2\sqrt{b\,x^3+ax}}{7}+\frac{4a\sqrt{b\,x^3+ax}}{21b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^2\sqrt{b\,x^3+ax}}$	146
risch	$\frac{2(3b\,x^2+2a)x(b\,x^2+a)}{21b\sqrt{x(b\,x^2+a)}}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^2\sqrt{b\,x^3+ax}}$	147

input
```
int(x*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2*(b*x^3+a*x)^(1/2)+4/21*a*(b*x^3+a*x)^(1/2)/b-2/21*a^2/b^2*(-a*b)^(
1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((
x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.40.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.36

$$\int x\sqrt{ax+bx^3}\,dx=-\frac{2\left(2\,a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(3\,b^2x^2+2\,ab)\sqrt{bx^3+ax}\right)}{21\,b^2}$$

input
```
integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/21*(2*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (3*b^2*x^2 + 2*a*
b)*sqrt(b*x^3 + a*x))/b^2
```

3.40.6 Sympy [F]

$$\int x\sqrt{ax+bx^3}\,dx=\int x\sqrt{x\left(a+bx^2\right)}\,dx$$

input
```
integrate(x*(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(x*(a + b*x**2)), x)
```

3.40.7 Maxima [F]

$$\int x\sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x, x)
```

3.40.8 Giac [F]

$$\int x\sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)*x, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{ax+bx^3}\,dx=\int x\,\sqrt{b\,x^3+a\,x}\,dx$$

input
```
int(x*(a*x + b*x^3)^(1/2),x)
```

output
```
int(x*(a*x + b*x^3)^(1/2), x)
```

3.41 $\int \sqrt{ax+bx^3}\,dx$

3.41.1 Optimal result

Integrand size = 13, antiderivative size = 255

$$\int \sqrt{ax+bx^3}\,dx = \frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3} - \frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}} + \frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}$$

output
```
4/5*a*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*x
^3+a*x)^(1/2)-4/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)+2/5*a^(5/4)*(cos(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x
)^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.20

$$\int \sqrt{ax+bx^3}\,dx=\frac{2x\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)])/
(3*Sqrt[1 + (b*x^2)/a])
```

3.41.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^3}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{5}a\int \frac{x}{\sqrt{bx^3+ax}}dx+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{834}$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \; 761$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

$$\Big\downarrow \; 1510$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}$$

input
```
Int[Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[a*x + b*x^3])/5 + (4*a*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sq
rt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x]
)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a]
+ Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[
(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*Sqrt[a*
x + b*x^3])
```

3.41. $\int \sqrt{ax+bx^3}\,dx$

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.41.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x\sqrt{b\,x^{3}+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{5b\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{2x\sqrt{b\,x^{3}+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{5b\sqrt{b\,x^{3}+ax}}$
risch	$\frac{2x^{2}\left(b\,x^{2}+a\right)}{5\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{5b\sqrt{b\,x^{3}+ax}}$

input
```
int((b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(b*x^3+a*x)^(1/2)+2/5*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)
*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/
b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.41.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \sqrt{ax+bx^3}\,dx = \frac{2\left(\sqrt{bx^3+ax}bx-2\,a\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b}$$

input
```
integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a*x)*b*x - 2*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weiers
trassPInverse(-4*a/b, 0, x)))/b
```

3.41.6 Sympy [F]

$$\int \sqrt{ax+bx^3}\,dx = \int \sqrt{ax+bx^3}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2),x)
```

output
```
Integral(sqrt(a*x + b*x**3), x)
```

3.41.7 Maxima [F]

$$\int \sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x), x)
```

3.41.8 Giac [F]

$$\int \sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x), x)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \sqrt{ax+bx^3}\,dx = \frac{2\,x\,\sqrt{b\,x^3+a\,x}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{b\,x^2}{a}\right)}{3\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int((a*x + b*x^3)^(1/2),x)
```

output
```
(2*x*(a*x + b*x^3)^(1/2)*hypergeom([-1/2, 3/4], 7/4, -(b*x^2)/a))/(3*((b*x
^2)/a + 1)^(1/2))
```

3.42 $\int \frac{\sqrt{ax+bx^3}}{x} \, dx$

3.42.1 Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{\sqrt{ax+bx^3}}{x} \, dx = \frac{2}{3}\sqrt{ax+bx^3} + \frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
2/3*(b*x^3+a*x)^(1/2)+2/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)
/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

3.42.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{ax+bx^3}}{x} \, dx = \frac{2\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x,x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sq
rt[1 + (b*x^2)/a]
```

3.42.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x}dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{3}a\int \frac{1}{\sqrt{bx^3+ax}}dx+\frac{2}{3}\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

input
```
Int[Sqrt[a*x + b*x^3]/x,x]
```

output
```
(2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])
```

3.42.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.42.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{2\sqrt{b\,x^{3}+ax}}{3}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b\sqrt{b\,x^{3}+ax}}$	124
elliptic	$\frac{2\sqrt{b\,x^{3}+ax}}{3}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b\sqrt{b\,x^{3}+ax}}$	124
risch	$\frac{2x\left(b\,x^{2}+a\right)}{3\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b\sqrt{b\,x^{3}+ax}}$	132

input
```
int((b*x^3+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a*x)^(1/2)+2/3*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b
)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1
/2),1/2*2^(1/2))
```

3.42.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2\left(2\,a\sqrt{b}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}b\right)}{3\,b}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
2/3*(2*a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*b)/
b
```

3.42.6 Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \int \frac{\sqrt{x\,(a+bx^2)}}{x}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x, x)
```

3.42.7 Maxima [F]

$$\int \frac{\sqrt{ax + bx^3}}{x} \, dx = \int \frac{\sqrt{bx^3 + ax}}{x} \, dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x, x)
```

3.42.8 Giac [F]

$$\int \frac{\sqrt{ax + bx^3}}{x} \, dx = \int \frac{\sqrt{bx^3 + ax}}{x} \, dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x, x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax + bx^3}}{x} \, dx = \int \frac{\sqrt{b\,x^3 + a\,x}}{x} \, dx$$

input
```
int((a*x + b*x^3)^(1/2)/x,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x, x)
```

3.43 $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$

3.43.1 Optimal result

Integrand size = 17, antiderivative size = 248

$$\begin{aligned}\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx &= \frac{4\sqrt{b}x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\\ &\quad-\frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}\\ &\quad+\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt{ax+bx^3}}\end{aligned}$$

output
```
4*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)^
(1/2)/x-4*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+2*a^(1/4)*b^(1/4)*(cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*El
lipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(
1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = -\frac{2\sqrt{x\,(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x^2,x]
```

output
```
(-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/
(x*Sqrt[1 + (b*x^2)/a])
```

3.43.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1926, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$$

$$\Big\downarrow \text{ 1926}$$

$$2b\int \frac{x}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \ 761$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}$$

$$\Big\downarrow \ 1510$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{\sqrt{ax+bx^3}} \qquad \frac{2\sqrt{ax+bx^3}}{x}$$

input
```
Int[Sqrt[a*x + b*x^3]/x^2,x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/x + (4*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqr
t[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] +
 Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(
b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/Sqrt[a*x +
b*x^3]
```

3.43. $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.43. $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$

3.43.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.71

method	result
default	$-\frac{2(bx^2+a)}{\sqrt{x(bx^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{bx^3+ax}}$
risch	$-\frac{2(bx^2+a)}{\sqrt{x(bx^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{bx^3+ax}}$
elliptic	$-\frac{2(bx^2+a)}{\sqrt{x(bx^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1
/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.43.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$$
$$=-\frac{2\left(2\sqrt{b}x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{x}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2*(2*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0,
x)) + sqrt(b*x^3 + a*x))/x
```

3.43.6 Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{x\left(a+bx^2\right)}}{x^2}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x**2, x)
```

3.43.7 Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

3.43.8 Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx=\int \frac{\sqrt{b\,x^3+a\,x}}{x^2}\,dx$$

input
```
int((a*x + b*x^3)^(1/2)/x^2,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x^2, x)
```

3.44 $\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx$

3.44.1 Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=-\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}$$

output
```
-2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x
^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^3+a*x)^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=-\frac{2\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x^3,x]
```

output
```
(-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/
(3*x^2*Sqrt[1 + (b*x^2)/a])
```

3.44.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1926, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x^3}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{2}{3}b\int \frac{1}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

input
```
Int[Sqrt[a*x + b*x^3]/x^3,x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(3*x^2) + (2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a*x + b*x^3])
```

3.44.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.44.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\sqrt{b\,x^{3}+a x}}{3x^{2}}+\frac{2\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+a x}}$	123
elliptic	$-\frac{2\sqrt{b\,x^{3}+a x}}{3x^{2}}+\frac{2\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+a x}}$	123
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3x\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+a x}}$	130

input
```
int(1/x^3*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*
b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(
1/2),1/2*2^(1/2))
```

3.44.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\frac{2\left(2\sqrt{b}x^2\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}\right)}{3x^2}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x))/
x^2
```

3.44.6 Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\int \frac{\sqrt{x\,(a+bx^2)}}{x^3}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x**3, x)
```

3.44.7 Maxima [F]

$$\int \frac{\sqrt{ax + bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3 + ax}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

3.44.8 Giac [F]

$$\int \frac{\sqrt{ax + bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3 + ax}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax + bx^3}}{x^3}\,dx = \int \frac{\sqrt{b\,x^3 + a\,x}}{x^3}\,dx$$

input
```
int((a*x + b*x^3)^(1/2)/x^3,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x^3, x)
```

3.45 $\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$

3.45.1 Optimal result

Integrand size = 17, antiderivative size = 283

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} - \frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}} + \frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}$$

output
```
4/5*b^(3/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*x^3
+a*x)^(1/2)/x^3-4/5*b*(b*x^3+a*x)^(1/2)/a/x-4/5*b^(5/4)*(cos(2*arctan(b^(1
/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elli
pticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1
/2)+2/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2
))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

3.45.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a*x + b*x^3]/x^4,x]
```

output
```
(-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])
/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

3.45.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{2}{5}b\int \frac{1}{x\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{2}{5}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{5}b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$\Big\downarrow$ 834

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$\Big\downarrow$ 27

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$\Big\downarrow$ 761

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

$\Big\downarrow$ 1510

$$\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{\ldots}}{\sqrt[4]{a}\ldots}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\ldots\right.}{a\sqrt{ax+bx^3}}\ldots\right.$$

$$\frac{2\sqrt{ax+bx^3}}{5x^3}$$

3.45. $\displaystyle\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$

input
```
Int[Sqrt[a*x + b*x^3]/x^4,x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(5*x^3) + (2*b*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b
*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b
]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[
a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2]
)/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x + b*x^3])))/5
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.45.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(bx^2+a)(2bx^2+a)}{5x^2\sqrt{x(bx^2+a)}\,a}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\cdots}\right)}{\cdots}\right)}{5a\sqrt{bx^3+ax}}$
default	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right)}{5a\sqrt{bx^3+ax}}$
elliptic	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right)}{5a\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(2*b*x^2+a)/x^2/(x*(b*x^2+a))^(1/2)/a+2/5/a*b*(-a*b)^(1/2)*
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/
b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^
(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.45.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\left(2\,b^{\frac{3}{2}}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\,(2\,bx^2+a)\right)}{5\,ax^3}$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/5*(2*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) + sqrt(b*x^3 + a*x)*(2*b*x^2 + a))/(a*x^3)
```

3.45.6 Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{x\,(a+bx^2)}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x*(a + b*x**2))/x**4, x)
```

3.45.7 Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

3.45.8 Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \int \frac{\sqrt{b\,x^3+a\,x}}{x^4}\,dx$$

input
```
int((a*x + b*x^3)^(1/2)/x^4,x)
```

output
```
int((a*x + b*x^3)^(1/2)/x^4, x)
```

3.46 $\int x^2(ax+bx^3)^{3/2}\,dx$

3.46.1 Optimal result

Integrand size = 17, antiderivative size = 186

$$\int x^2\left(ax+bx^3\right)^{3/2}\,dx=-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}$$

output
```
2/15*x^3*(b*x^3+a*x)^(3/2)-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+8/385*a^2*x^2*(
b*x^3+a*x)^(1/2)/b+4/55*a*x^4*(b*x^3+a*x)^(1/2)+4/231*a^(15/4)*(cos(2*arct
an(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)
))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+
a*x)^(1/2)
```

3.46.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int x^2(ax+bx^3)^{3/2}\,dx = \frac{2\sqrt{x(a+bx^2)}\left(-\left((5a-11bx^2)(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}\right)+5a^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\right.\right.}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^2*(a*x + b*x^3)^(3/2),x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/
a]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqr
t[1 + (b*x^2)/a])
```

3.46.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1927, 1927, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(ax+bx^3)^{3/2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{5}a\int x^3\sqrt{bx^3+ax}dx+\frac{2}{15}x^3(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\int\frac{x^4}{\sqrt{bx^3+ax}}dx+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int\frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3(ax+bx^3)^{3/2}$$

$$\Big\downarrow \; 1930$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int\frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \; 1917$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \; 266$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{ax+bx^3}\right)+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \; 761$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}\right)+\frac{2}{11}x^4\sqrt{\right.$$

$$\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}$$

input
```
Int[x^2*(a*x + b*x^3)^(3/2),x]
```

3.46. $\int x^2(ax+bx^3)^{3/2}\,dx$

output
```
(2*x^3*(a*x + b*x^3)^(3/2))/15 + (2*a*((2*x^4*Sqrt[a*x + b*x^3])/11 + (2*a
*((2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a
^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[
a*x + b*x^3])))/(7*b)))/11))/5
```

3.46.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.46.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{2(-77b^3x^6-119a\,b^2x^4-12a^2b\,x^2+20a^3)x(b\,x^2+a)}{1155b^2\sqrt{x(b\,x^2+a)}}+\frac{4a^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{231b^3\sqrt{b\,x^3+ax}}$
default	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-}}{231b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-}}{231b^3\sqrt{b\,x^3+ax}}$

input
```
int(x^2*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(-77*b^3*x^6-119*a*b^2*x^4-12*a^2*b*x^2+20*a^3)/b^2*x*(b*x^2+a)/(x
*(b*x^2+a))^(1/2)+4/231*a^4/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)
*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^
(1/2),1/2*2^(1/2))
```

3.46.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int x^2(ax+bx^3)^{3/2}\,dx=\frac{2\left(20\,a^4\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(77\,b^4x^6+119\,ab^3x^4+12\,a^2b^2x^2-20\,a^3b)\sqrt{bx^3}\right.}{1155\,b^3}$$

input
```
integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(20*a^4*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (77*b^4*x^6 + 1
19*a*b^3*x^4 + 12*a^2*b^2*x^2 - 20*a^3*b)*sqrt(b*x^3 + a*x))/b^3
```

3.46.6 Sympy [F]

$$\int x^2 \left(ax + bx^3\right)^{3/2} \, dx = \int x^2 \left(x \left(a + bx^2\right)\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**2*(x*(a + b*x**2))**(3/2), x)
```

3.46.7 Maxima [F]

$$\int x^2 \left(ax + bx^3\right)^{3/2} \, dx = \int \left(bx^3 + ax\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

3.46.8 Giac [F]

$$\int x^2 \left(ax + bx^3\right)^{3/2} \, dx = \int \left(bx^3 + ax\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (ax + bx^3)^{3/2} \, dx = \int x^2 \left(b\,x^3 + a\,x\right)^{3/2} dx$$

input
```
int(x^2*(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^2*(a*x + b*x^3)^(3/2), x)
```

3.47 $\int x(ax+bx^3)^{3/2}\,dx$

3.47.1 Optimal result

Integrand size = 15, antiderivative size = 304

$$\int x\left(ax+bx^3\right)^{3/2}\,dx=-\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}$$

output
```
2/13*x^2*(b*x^3+a*x)^(3/2)-8/65*a^3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2)
)/(b*x^3+a*x)^(1/2)+8/195*a^2*x*(b*x^3+a*x)^(1/2)/b+4/39*a*x^3*(b*x^3+a*x)
^(1/2)+8/65*a^(13/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b
^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-4/65*a^(13/4)*(cos(2*arctan(b^(
1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1
/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(
1/2)
```

3.47.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.28

$$\int x\left(ax+bx^3\right)^{3/2}\,dx=\frac{2x\sqrt{x(a+bx^2)}\left(\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{13b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x*(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeo
metric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

3.47.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1927, 1927, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^3\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{13}a\int x^2\sqrt{bx^3+ax}dx+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\int\frac{x^3}{\sqrt{bx^3+ax}}dx+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int\frac{x}{\sqrt{bx^3+ax}}dx}{5b}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)+\frac{2}{9}x^3\sqrt{ax+bx^3}\right)+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{6}{13}a\Bigg(\frac{2}{9}a\Bigg(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

$$\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{ 1510}$$

3.47. $\int x\left(ax+bx^3\right)^{3/2}\,dx$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\cdots}}{\cdots}\right.}{5b\sqrt{ax+bx^3}}\right.\right.$$

$$\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

input
```
Int[x*(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x^2*(a*x + b*x^3)^(3/2))/13 + (6*a*((2*x^3*Sqrt[a*x + b*x^3])/9 + (2*a*
((2*x*Sqrt[a*x + b*x^3])/(5*b) - (6*a*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[
x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sq
rt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*
Sqrt[a*x + b*x^3])))/9))/13
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.47. $\int x(ax+bx^3)^{3/2}\,dx$

3.47.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.69

method	result
risch	$\frac{2x^2(15b^2x^4+25abx^2+4a^2)(bx^2+a)}{195b\sqrt{x(bx^2+a)}} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right.}{65b^2\sqrt{bx^3+ax}}$
default	$\frac{2bx^5\sqrt{bx^3+ax}}{13} + \frac{10ax^3\sqrt{bx^3+ax}}{39} + \frac{8a^2x\sqrt{bx^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\right.}{}\right.}{65b^2\sqrt{bx^3+ax}}$
elliptic	$\frac{2bx^5\sqrt{bx^3+ax}}{13} + \frac{10ax^3\sqrt{bx^3+ax}}{39} + \frac{8a^2x\sqrt{bx^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\right.}{}\right.}{65b^2\sqrt{bx^3+ax}}$

input
```
int(x*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/195*x^2*(15*b^2*x^4+25*a*b*x^2+4*a^2)/b*(b*x^2+a)/(x*(b*x^2+a))^(1/2)-4/
65/b^2*a^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)
^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1
/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2)))
```

3.47.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int x(ax+bx^3)^{3/2}\,dx = \frac{2\left(12\,a^3\sqrt{b}\,\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(15\,b^3x^5+25\,ab^2x^3+\right.\right.}{195\,b^2}$$

input
```
integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/195*(12*a^3*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) + (15*b^3*x^5 + 25*a*b^2*x^3 + 4*a^2*b*x)*sqrt(b*x^3 + a*x))/b^2
```

3.47.6 Sympy [F]

$$\int x(ax+bx^3)^{3/2}\,dx = \int x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x*(x*(a + b*x**2))**(3/2), x)
```

3.47.7 Maxima [F]

$$\int x(ax+bx^3)^{3/2}\,dx = \int (bx^3+ax)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x, x)
```

3.47.8 Giac [F]

$$\int x(ax+bx^3)^{3/2}\,dx = \int (bx^3+ax)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)*x, x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int x(ax + bx^3)^{3/2}\, dx = \int x\,\left(b\,x^3 + a\,x\right)^{3/2}\, dx$$

input
```
int(x*(a*x + b*x^3)^(3/2),x)
```

output
```
int(x*(a*x + b*x^3)^(3/2), x)
```

3.48 $\int (ax+bx^3)^{3/2}\,dx$

3.48.1 Optimal result

Integrand size = 13, antiderivative size = 158

$$\int (ax+bx^3)^{3/2}\,dx = \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x(ax+bx^3)^{3/2} - \frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}$$

output
```
2/11*x*(b*x^3+a*x)^(3/2)+8/77*a^2*(b*x^3+a*x)^(1/2)/b+12/77*a*x^2*(b*x^3+a
*x)^(1/2)-4/77*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+
x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

3.48.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int (ax+bx^3)^{3/2}\,dx = \frac{2\sqrt{x(a+bx^2)}\left((a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{11b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2),x]
```

output
```
(2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeome
tric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

3.48.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1910, 1927, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{6}{11}a\int x\sqrt{bx^3+ax}dx+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\int \frac{x^2}{\sqrt{bx^3+ax}}dx+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+bx^3}\right)+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)+\frac{2}{7}x^2\sqrt{ax+b\right.$$
$$\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

input
```
Int[(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x*(a*x + b*x^3)^(3/2))/11 + (6*a*((2*x^2*Sqrt[a*x + b*x^3])/7 + (2*a*((
2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a
^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])))/7))/11
```

3.48.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.48.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

method	result
risch	$\frac{2(7b^2x^4+13abx^2+4a^2)x(bx^2+a)}{77b\sqrt{x(bx^2+a)}}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77b^2\sqrt{bx^3+ax}}$
default	$\frac{2bx^4\sqrt{bx^3+ax}}{11}+\frac{26ax^2\sqrt{bx^3+ax}}{77}+\frac{8a^2\sqrt{bx^3+ax}}{77b}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{77b^2\sqrt{bx^3+ax}}$
elliptic	$\frac{2bx^4\sqrt{bx^3+ax}}{11}+\frac{26ax^2\sqrt{bx^3+ax}}{77}+\frac{8a^2\sqrt{bx^3+ax}}{77b}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{77b^2\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/77*(7*b^2*x^4+13*a*b*x^2+4*a^2)/b*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)-4/77/b
^2*a^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/
2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.48.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \left(ax+bx^3\right)^{3/2}\,dx = -\frac{2\left(4\,a^3\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\left(7\,b^3x^4+13\,ab^2x^2+4\,a^2b\right)\sqrt{bx^3+ax}\right)}{77\,b^2}$$

input
```
integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/77*(4*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (7*b^3*x^4 + 13*a
*b^2*x^2 + 4*a^2*b)*sqrt(b*x^3 + a*x))/b^2
```

3.48.6 Sympy [F]

$$\int \left(ax+bx^3\right)^{3/2}\,dx = \int \left(ax+bx^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**3)**(3/2), x)
```

3.48.7 Maxima [F]

$$\int \left(ax+bx^3\right)^{3/2}\,dx = \int \left(bx^3+ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2), x)
```

3.48.8 Giac [F]

$$\int \left(ax+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2), x)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.25

$$\int \left(ax+bx^3\right)^{3/2}\,dx=\frac{2\,x\,\left(b\,x^3+a\,x\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{5}{4};\ \frac{9}{4};\ -\frac{b\,x^2}{a}\right)}{5\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a*x + b*x^3)^(3/2),x)
```

output
```
(2*x*(a*x + b*x^3)^(3/2)*hypergeom([-3/2, 5/4], 9/4, -(b*x^2)/a))/(5*((b*x
^2)/a + 1)^(3/2))
```

3.49 $\int \frac{(ax+bx^3)^{3/2}}{x} \, dx$

3.49.1 Optimal result

Integrand size = 17, antiderivative size = 275

$$\int \frac{(ax+bx^3)^{3/2}}{x} \, dx = \frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}(ax+bx^3)^{3/2} - \frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}} + \frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}$$

output
```
2/9*(b*x^3+a*x)^(3/2)+8/15*a^2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)+4/15*a*x*(b*x^3+a*x)^(1/2)-8/15*a^(9/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellip
ticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2
))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/
2)+4/15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2
))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

3.49.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx=\frac{2ax\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x,x]
```

output
```
(2*a*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]
)/(3*Sqrt[1 + (b*x^2)/a])
```

3.49.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1927, 1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{2}{3}a\int \sqrt{bx^3+ax}dx+\frac{2}{9}(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\int \frac{x}{\sqrt{bx^3+ax}}dx+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{3}a\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}(ax+bx^3)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)+$$

$$\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{3}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{a}\ldots}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5\sqrt{ax+bx^3}}\right.$$

$$\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

3.49. $\int\frac{\left(ax+bx^3\right)^{3/2}}{x}dx$

input
```
Int[(a*x + b*x^3)^(3/2)/x,x]
```

output
```
(2*(a*x + b*x^3)^(3/2))/9 + (2*a*((2*x*Sqrt[a*x + b*x^3])/5 + (4*a*Sqrt[x]
*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) +
(a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^
2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(
3/4)*Sqrt[a + b*x^2])))/(5*Sqrt[a*x + b*x^3])))/3
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.49.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.71

method	result
default	$\frac{2b\,x^3\sqrt{b\,x^3+ax}}{9}+\frac{22ax\sqrt{b\,x^3+ax}}{45}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2b\,x^3\sqrt{b\,x^3+ax}}{9}+\frac{22ax\sqrt{b\,x^3+ax}}{45}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(5b\,x^2+11a\right)\left(b\,x^2+a\right)}{45\sqrt{x\left(b\,x^2+a\right)}}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\right.}{}\right.}{15b\sqrt{b\,x^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/9*b*x^3*(b*x^3+a*x)^(1/2)+22/45*a*x*(b*x^3+a*x)^(1/2)+4/15*a^2*(-a*b)^(1
/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^
(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(
-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1
/2)))
```

3.49.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = -\frac{2\left(12\,a^2\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-(5\,b^2x^3+11\,abx)\sqrt{bx^3+ax}\right)}{45\,b}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
-2/45*(12*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) - (5*b^2*x^3 + 11*a*b*x)*sqrt(b*x^3 + a*x))/b
```

3.49.6 Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x, x)
```

3.49.7 Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x, x)
```

3.49.8 Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x, x)
```

3.50 $\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx$

3.50.1 Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x} + \frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
2/7*(b*x^3+a*x)^(3/2)/x+4/7*a*(b*x^3+a*x)^(1/2)+4/7*a^(7/4)*(cos(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x
)^(1/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^2,x]
```

output
```
(2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/
Sqrt[1 + (b*x^2)/a]
```

3.50.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1927, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{7}a\int \frac{\sqrt{bx^3+ax}}{x}dx+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{1927}$$

$$\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{bx^3+ax}}dx+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{7}a\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{7}a\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2(ax+bx^3)^{3/2}}{7x}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}a\left(\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^2,x]
```

output
```
(2*(a*x + b*x^3)^(3/2))/(7*x) + (6*a*((2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x +
b*x^3])))/7
```

3.50.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.50.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{2(b x^2+3a)x(b x^2+a)}{7\sqrt{x(b x^2+a)}}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b x^3+ax}}$	143
default	$\frac{2b x^2\sqrt{b x^3+ax}}{7}+\frac{6a\sqrt{b x^3+ax}}{7}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b x^3+ax}}$	144
elliptic	$\frac{2b x^2\sqrt{b x^3+ax}}{7}+\frac{6a\sqrt{b x^3+ax}}{7}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b x^3+ax}}$	144

input
```
int((b*x^3+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*(b*x^2+3*a)*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+4/7*a^2*(-a*b)^(1/2)/b*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.50.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \frac{2\left(4\,a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(b^2x^2+3\,ab)\sqrt{bx^3+ax}\right)}{7\,b}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2/7*(4*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (b^2*x^2 + 3*a*b)*s
qrt(b*x^3 + a*x))/b
```

3.50.6 Sympy [F]

$$\int \frac{(ax + bx^3)^{3/2}}{x^2} \, dx = \int \frac{(x(a + bx^2))^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**2,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**2, x)
```

3.50.7 Maxima [F]

$$\int \frac{(ax + bx^3)^{3/2}}{x^2} \, dx = \int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

3.50.8 Giac [F]

$$\int \frac{(ax + bx^3)^{3/2}}{x^2} \, dx = \int \frac{(bx^3 + ax)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^2}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^2,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^2, x)
```

3.51 $\int \frac{(ax+bx^3)^{3/2}}{x^3} dx$

3.51.1 Optimal result

Integrand size = 17, antiderivative size = 274

$$\int \frac{(ax+bx^3)^{3/2}}{x^3} dx = \frac{24a\sqrt{b}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} - \frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} + \frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$

output
```
-2*(b*x^3+a*x)^(3/2)/x^2+24/5*a*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b
*x^3+a*x)^(1/2)+12/5*b*x*(b*x^3+a*x)^(1/2)-24/5*a^(5/4)*b^(1/4)*(cos(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1
/2)+12/5*a^(5/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1
/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = -\frac{2a\sqrt{x\,(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^3,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)]
)/(x*Sqrt[1 + (b*x^2)/a])
```

3.51.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1926, 1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{1926}$$

$$6b\int \sqrt{bx^3+ax}dx-\frac{2(ax+bx^3)^{3/2}}{x^2}$$

$$\Big\downarrow \text{1910}$$

$$6b\left(\frac{2}{5}a\int \frac{x}{\sqrt{bx^3+ax}}dx+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2(ax+bx^3)^{3/2}}{x^2}$$

$$\Big\downarrow \text{1938}$$

$$6b\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2(ax+bx^3)^{3/2}}{x^2}$$

$$\Big\downarrow \text{266}$$

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\big\downarrow$ 834

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\big\downarrow$ 27

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\big\downarrow$ 761

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

$\big\downarrow$ 1510

3.51. $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^3}\,dx$

$$6b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5\sqrt{ax+bx^3}}\right.$$

$$\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^3,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/x^2 + 6*b*((2*x*Sqrt[a*x + b*x^3])/5 + (4*a*Sqrt[
x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x))
+ (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*
x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b
^(3/4)*Sqrt[a + b*x^2])))/(5*Sqrt[a*x + b*x^3]))
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.51. $\int \frac{(ax+bx^3)^{3/2}}{x^3} \, dx$

3.51.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(b\,x^2+a)(-b\,x^2+5a)}{5\sqrt{x(b\,x^2+a)}}+\frac{12a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\ldots}\right.}{\ldots}\right.}{5\sqrt{b\,x^3+ax}}$
default	$-\frac{2(b\,x^2+a)a}{\sqrt{x(b\,x^2+a)}}+\frac{2bx\sqrt{b\,x^3+ax}}{5}+\frac{12a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\ldots\right.}{5\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{2(b\,x^2+a)a}{\sqrt{x(b\,x^2+a)}}+\frac{2bx\sqrt{b\,x^3+ax}}{5}+\frac{12a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\ldots\right.}{5\sqrt{b\,x^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(-b*x^2+5*a)/(x*(b*x^2+a))^(1/2)+12/5*a*(-a*b)^(1/2)*((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)
^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*Elli
pticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/
b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.51.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx =$$
$$-\frac{2\left(12\,a\sqrt{b}x\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\sqrt{bx^3+ax}(bx^2-5\,a)\right)}{5\,x}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-2/5*(12*a*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) - sqrt(b*x^3 + a*x)*(b*x^2 - 5*a))/x
```

3.51.6 Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**3,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**3, x)
```

3.51.7 Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

3.51.8 Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^3}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^3,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^3, x)
```

3.52 $\int \frac{(ax+bx^3)^{3/2}}{x^4} dx$

3.52.1 Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \frac{(ax+bx^3)^{3/2}}{x^4} dx = \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}$$

output

```
-2/3*(b*x^3+a*x)^(3/2)/x^3+4/3*b*(b*x^3+a*x)^(1/2)+4/3*a^(3/4)*b^(3/4)*(co
s(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)
/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(
a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+
a*x)^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.40

$$\int \frac{(ax+bx^3)^{3/2}}{x^4} dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^4,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)]
)/(3*x^2*Sqrt[1 + (b*x^2)/a])
```

3.52.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1926, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \; 1926$$

$$2b\int \frac{\sqrt{bx^3+ax}}{x}dx-\frac{2(ax+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 1927$$

$$2b\left(\frac{2}{3}a\int \frac{1}{\sqrt{bx^3+ax}}dx+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2(ax+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 1917$$

$$2b\left(\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2(ax+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 266$$

$$2b\left(\frac{4a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2(ax+bx^3)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 761$$

$$2b\left(\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^4,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(3*x^3) + 2*b*((2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/
4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x
+ b*x^3]))
```

3.52.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.52. $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^4}\,dx$

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.52.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{2a\sqrt{b\,x^{3}+ax}}{3x^{2}}+\frac{2b\sqrt{b\,x^{3}+ax}}{3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	139
risch	$-\frac{2\left(b\,x^{2}+a\right)\left(-b\,x^{2}+a\right)}{3x\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	139
elliptic	$-\frac{2a\sqrt{b\,x^{3}+ax}}{3x^{2}}+\frac{2b\sqrt{b\,x^{3}+ax}}{3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	139

input
```
int((b*x^3+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*a*(b*x^3+a*x)^(1/2)/x^2+2/3*b*(b*x^3+a*x)^(1/2)+4/3*a*(-a*b)^(1/2)*((
x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.52.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.34

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx=\frac{2\left(4\,a\sqrt{b}x^{2}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^{3}+ax}(bx^{2}-a)\right)}{3\,x^{2}}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
2/3*(4*a*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)
*(b*x^2 - a))/x^2
```

3.52.6 Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**4,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**4, x)
```

3.52.7 Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

3.52.8 Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^4}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^4,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^4, x)
```

3.53 $\int \frac{(ax+bx^3)^{3/2}}{x^5} \, dx$

3.53.1 Optimal result

Integrand size = 17, antiderivative size = 277

$$\int \frac{(ax+bx^3)^{3/2}}{x^5} \, dx = \frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} - \frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} + \frac{12\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$

output
```
-2/5*(b*x^3+a*x)^(3/2)/x^4+24/5*b^(3/2)*x*(b*x^2+a)/(a^(1/2)+x*b^(1/2))/(b
*x^3+a*x)^(1/2)-12/5*b*(b*x^3+a*x)^(1/2)/x-24/5*a^(1/4)*b^(5/4)*(cos(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1
/2)+12/5*a^(1/4)*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1
/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^5,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)
])/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

3.53.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1926, 1926, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{5}b\int \frac{\sqrt{bx^3+ax}}{x^2}dx-\frac{2(ax+bx^3)^{3/2}}{5x^4}$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{5}b\left(2b\int \frac{x}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2(ax+bx^3)^{3/2}}{5x^4}$$

$$\Big\downarrow \text{1938}$$

$$\frac{6}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2(ax+bx^3)^{3/2}}{5x^4}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$$\Big\downarrow \; 834$$

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$$\Big\downarrow \; 27$$

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$$\Big\downarrow \; 761$$

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

$$\Big\downarrow \; 1510$$

3.53. $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^5}\,dx$

$$\frac{6}{5}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{\sqrt{ax+bx^3}}\right.$$

$$\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^5,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(5*x^4) + (6*b*((-2*Sqrt[a*x + b*x^3])/x + (4*b*S
qrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*
x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a
+ b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(2*b^(3/4)*Sqrt[a + b*x^2])))/Sqrt[a*x + b*x^3]))/5
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.53.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2(bx^2+a)(7bx^2+a)}{5x^2\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\cdots}\right.}{\cdots}\right.}{5\sqrt{bx^3+ax}}$
default	$-\frac{2a\sqrt{bx^3+ax}}{5x^3}-\frac{14(bx^2+a)b}{5\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right.}{5\sqrt{bx^3+ax}}$
elliptic	$-\frac{2a\sqrt{bx^3+ax}}{5x^3}-\frac{14(bx^2+a)b}{5\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right.}{5\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(7*b*x^2+a)/x^2/(x*(b*x^2+a))^(1/2)+12/5*b*(-a*b)^(1/2)*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*E
llipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/
2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.53.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = -\frac{2\left(12\,b^{\frac{3}{2}}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(7\,bx^2+a)\right)}{5\,x^3}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-2/5*(12*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) + sqrt(b*x^3 + a*x)*(7*b*x^2 + a))/x^3
```

3.53.6 Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx=\int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**5,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**5, x)
```

3.53.7 Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

3.53.8 Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^5}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^5,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^5, x)
```

3.54 $\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx$

3.54.1 Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = -\frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}$$

output
```
-2/7*(b*x^3+a*x)^(3/2)/x^5-4/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b^(7/4)*(cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*
x^3+a*x)^(1/2)
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7x^4\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^6,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)
])/(7*x^4*Sqrt[1 + (b*x^2)/a])
```

3.54.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1926, 1926, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{7}b\int \frac{\sqrt{bx^3+ax}}{x^3}dx-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{1926}$$

$$\frac{6}{7}b\left(\frac{2}{3}b\int \frac{1}{\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{7}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{7}b\left(\frac{4b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}b\left(\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^6,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(7*x^5) + (6*b*((-2*Sqrt[a*x + b*x^3])/(3*x^2) +
(2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*a^(1/4)*S
qrt[a*x + b*x^3])))/7
```

3.54.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.54.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

method	result	size
risch	$-\frac{2(b\,x^2+a)(3b\,x^2+a)}{7x^3\sqrt{x(b\,x^2+a)}}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^3+ax}}$	139
default	$-\frac{2a\sqrt{b\,x^3+ax}}{7x^4}-\frac{6b\sqrt{b\,x^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^3+ax}}$	142
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{7x^4}-\frac{6b\sqrt{b\,x^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^3+ax}}$	142

input
```
int((b*x^3+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^2+a)*(3*b*x^2+a)/x^3/(x*(b*x^2+a))^(1/2)+4/7*b*(-a*b)^(1/2)*((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.54.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx=\frac{2\left(4\,b^{\frac{3}{2}}x^4\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^3+ax}(3\,bx^2+a)\right)}{7\,x^4}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
2/7*(4*b^(3/2)*x^4*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*(
3*b*x^2 + a))/x^4
```

3.54.6 Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**6,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**6, x)
```

3.54.7 Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

3.54.8 Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^6}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^6,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^6, x)
```

3.55 $\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx$

3.55.1 Optimal result

Integrand size = 17, antiderivative size = 306

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} - \frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}} + \frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$

output
```
-2/9*(b*x^3+a*x)^(3/2)/x^6+8/15*b^(5/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/
(b*x^3+a*x)^(1/2)-4/15*b*(b*x^3+a*x)^(1/2)/x^3-8/15*b^2*(b*x^3+a*x)^(1/2)/
a/x-8/15*b^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+4/15*b^(9/4)*(cos(2*arctan(b^(1/4)*
x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*
x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

3.55.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{bx^2}{a}\right)}{9x^5\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^7,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)
])/(9*x^5*Sqrt[1 + (b*x^2)/a])
```

3.55.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1926, 1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{2}{3}b\int \frac{\sqrt{bx^3+ax}}{x^4}dx-\frac{2(ax+bx^3)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{1926}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\int \frac{1}{x\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2(ax+bx^3)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{1931}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2(ax+bx^3)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int\frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)-\frac{2\sqrt{ax+bx^3}}{5x^3}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

$$\Big\downarrow \text{1510}$$

3.55. $\int\frac{\left(ax+bx^3\right)^{3/2}}{x^7}dx$

$$\frac{2}{3}b\left(\frac{2}{5}b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}\right.\right.$$

$$\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^7,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(9*x^6) + (2*b*((-2*Sqrt[a*x + b*x^3])/(5*x^3) +
(2*b*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((S
qrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[
b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4
)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*
(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[
2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(
a*Sqrt[a*x + b*x^3])))/5))/3
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.55. $\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx$

3.55.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(b x^2+a)(12b^2x^4+11ab x^2+5a^2)}{45x^4\sqrt{x(b x^2+a)}\,a}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right.}{15a\sqrt{b x^3+ax}}$
default	$-\frac{2a\sqrt{b x^3+ax}}{9x^5}-\frac{22b\sqrt{b x^3+ax}}{45x^3}-\frac{8(b x^2+a)b^2}{15a\sqrt{x(b x^2+a)}}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\right.\right.}{}\right.}{15a\sqrt{b x^3+ax}}$
elliptic	$-\frac{2a\sqrt{b x^3+ax}}{9x^5}-\frac{22b\sqrt{b x^3+ax}}{45x^3}-\frac{8(b x^2+a)b^2}{15a\sqrt{x(b x^2+a)}}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\right.\right.}{}\right.}{15a\sqrt{b x^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/45*(b*x^2+a)*(12*b^2*x^4+11*a*b*x^2+5*a^2)/x^4/(x*(b*x^2+a))^(1/2)/a+4/
15*b^2/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(
1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2
),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),1/2*2^(1/2)))
```

3.55.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx =$$

$$-\frac{2\left(12\,b^{\frac{5}{2}}x^5\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(12\,b^2x^4+11\,abx^2+5\,a^2)\sqrt{bx^3+a}\right.}{45\,ax^5}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-2/45*(12*b^(5/2)*x^5*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/
b, 0, x)) + (12*b^2*x^4 + 11*a*b*x^2 + 5*a^2)*sqrt(b*x^3 + a*x))/(a*x^5)
```

3.55.6 Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**7,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**7, x)
```

3.55.7 Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

3.55.8 Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax + bx^3)^{3/2}}{x^7} \, dx = \int \frac{(b\,x^3 + a\,x)^{3/2}}{x^7} \, dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^7,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^7, x)
```

3.56 $\int \frac{(ax+bx^3)^{3/2}}{x^8} dx$

3.56.1 Optimal result

Integrand size = 17, antiderivative size = 163

$$\int \frac{(ax+bx^3)^{3/2}}{x^8} dx = -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} - \frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}$$

output
```
-2/11*(b*x^3+a*x)^(3/2)/x^7-12/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3+
a*x)^(1/2)/a/x^2-4/77*b^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a
^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \frac{(ax+bx^3)^{3/2}}{x^8} dx = -\frac{2a\sqrt{x\,(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^2}{a}\right)}{11x^6\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a*x + b*x^3)^(3/2)/x^8,x]
```

output
```
(-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a
)])/(11*x^6*Sqrt[1 + (b*x^2)/a])
```

3.56.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1926, 1926, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{6}{11}b\int \frac{\sqrt{bx^3+ax}}{x^5}dx-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\int \frac{1}{x^2\sqrt{bx^3+ax}}dx-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)-\frac{2\sqrt{ax+bx^3}}{7x^4}\right)-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

input
```
Int[(a*x + b*x^3)^(3/2)/x^8,x]
```

output
```
(-2*(a*x + b*x^3)^(3/2))/(11*x^7) + (6*b*((-2*Sqrt[a*x + b*x^3])/(7*x^4) +
 (2*b*((-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[x])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])))/7))/11
```

3.56.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.56.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{2(bx^2+a)(4b^2x^4+13abx^2+7a^2)}{77x^5\sqrt{x(bx^2+a)}\,a}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77a\sqrt{bx^3+ax}}$
default	$-\frac{2a\sqrt{bx^3+ax}}{11x^6}-\frac{26b\sqrt{bx^3+ax}}{77x^4}-\frac{8b^2\sqrt{bx^3+ax}}{77a\,x^2}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{77a\sqrt{bx^3+ax}}$
elliptic	$-\frac{2a\sqrt{bx^3+ax}}{11x^6}-\frac{26b\sqrt{bx^3+ax}}{77x^4}-\frac{8b^2\sqrt{bx^3+ax}}{77a\,x^2}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{77a\sqrt{bx^3+ax}}$

input
```
int((b*x^3+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/77*(b*x^2+a)*(4*b^2*x^4+13*a*b*x^2+7*a^2)/x^5/(x*(b*x^2+a))^(1/2)/a-4/7
7*b^2/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1
/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.56.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx =$$
$$-\frac{2\left(4\,b^{\frac{5}{2}}x^6\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(4\,b^2x^4+13\,abx^2+7\,a^2)\sqrt{bx^3+ax}\right)}{77\,ax^6}$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-2/77*(4*b^(5/2)*x^6*weierstrassPInverse(-4*a/b, 0, x) + (4*b^2*x^4 + 13*a
*b*x^2 + 7*a^2)*sqrt(b*x^3 + a*x))/(a*x^6)
```

3.56.6 Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x**3+a*x)**(3/2)/x**8,x)
```

output
```
Integral((x*(a + b*x**2))**(3/2)/x**8, x)
```

3.56.7 Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

3.56.8 Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^8}\,dx$$

input
```
int((a*x + b*x^3)^(3/2)/x^8,x)
```

output
```
int((a*x + b*x^3)^(3/2)/x^8, x)
```

3.57 $\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$

3.57.1 Optimal result

Integrand size = 17, antiderivative size = 140

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = -\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}$$

output
```
-10/21*a*(b*x^3+a*x)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x)^(1/2)/b+5/21*a^(7/4)*(c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4
)/(b*x^3+a*x)^(1/2)
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{2x\left(-5a^2-2abx^2+3b^2x^4+5a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{21b^2\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[x^4/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[x*(a + b*x^2)])
```

3.57.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}$$

$$\Big\downarrow \text{761}$$

$$\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}$$

input
```
Int[x^4/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*a*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(
3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*
x + b*x^3])))/(7*b)
```

3.57.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.57.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{2(-3b\,x^2+5a)x(b\,x^2+a)}{21b^2\sqrt{x(b\,x^2+a)}}+\frac{5a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^3\sqrt{b\,x^3+ax}}$	147
default	$\frac{2x^2\sqrt{b\,x^3+ax}}{7b}-\frac{10a\sqrt{b\,x^3+ax}}{21b^2}+\frac{5a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^3\sqrt{b\,x^3+ax}}$	149
elliptic	$\frac{2x^2\sqrt{b\,x^3+ax}}{7b}-\frac{10a\sqrt{b\,x^3+ax}}{21b^2}+\frac{5a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^3\sqrt{b\,x^3+ax}}$	149

input
```
int(x^4/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-3*b*x^2+5*a)/b^2*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+5/21*a^2/b^3*(-a*
b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/
(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*Elliptic
F(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.57.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx=\frac{2\left(5\,a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(3\,b^2x^2-5\,ab\right)\sqrt{bx^3+ax}\right)}{21\,b^3}$$

input
```
integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(5*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (3*b^2*x^2 - 5*a*b
)*sqrt(b*x^3 + a*x))/b^3
```

3.57.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(x**4/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x*(a + b*x**2)), x)
```

3.57.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^3 + a*x), x)
```

3.57.8 Giac [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(b*x^3 + a*x), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx=\int \frac{x^4}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x^4/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^4/(a*x + b*x^3)^(1/2), x)
```

3.58 $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$

3.58.1 Optimal result

Integrand size = 17, antiderivative size = 258

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = -\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b} + \frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}} - \frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}$$

output
```
-6/5*a*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*
x^3+a*x)^(1/2)/b+6/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*
x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(
1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-3/5*a^(5/4)*(cos(2*arct
an(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)
))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+
a*x)^(1/2)
```

3.58.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx=\frac{2x^2\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[x^3/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^2*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -((b*x^2)/a)]))/(5*b*Sqrt[x*(a + b*x^2)])
```

3.58.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int \frac{x}{\sqrt{bx^3+ax}}dx}{5b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 761$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 1510$$

$$\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}$$

input
```
Int[x^3/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[a*x + b*x^3])/(5*b) - (6*a*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x
]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqr
t[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*S
qrt[a*x + b*x^3])
```

3.58. $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.58.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}} \cdots\right.}{b}\right.}{5b^2\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}} \cdots\right.}{b}\right.}{5b^2\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5b\sqrt{x\left(b\,x^2+a\right)}}-\frac{3a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}} \cdots\right.}{b}\right.}{5b^2\sqrt{b\,x^3+ax}}$

input
```
int(x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(b*x^3+a*x)^(1/2)/b-3/5*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(
1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.58.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+ax}bx+3\,a\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b^2}$$

input
```
integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a*x)*b*x + 3*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weiers
trassPInverse(-4*a/b, 0, x)))/b^2
```

3.58.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x*(a + b*x**2)), x)
```

3.58.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(b*x^3 + a*x), x)
```

3.58.8 Giac [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(b*x^3 + a*x), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx=\int \frac{x^3}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x^3/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^3)^(1/2), x)
```

3.59 $\int \frac{x^2}{\sqrt{ax+bx^3}} \, dx$

3.59.1 Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{x^2}{\sqrt{ax+bx^3}} \, dx = \frac{2\sqrt{ax+bx^3}}{3b} - \frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

output
```
2/3*(b*x^3+a*x)^(1/2)/b-1/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b
^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+
a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

3.59.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{ax+bx^3}} \, dx = \frac{2x\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^2/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((b*x^2)/a)]))/(3*b*Sqrt[x*(a + b*x^2)])
```

3.59.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

input
```
Int[x^2/Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/
a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])
```

3.59.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.59.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{2\sqrt{b\,x^{3}+a x}}{3b}-\frac{a\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3b^{2}\sqrt{b\,x^{3}+a x}}$	127
elliptic	$\frac{2\sqrt{b\,x^{3}+a x}}{3b}-\frac{a\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3b^{2}\sqrt{b\,x^{3}+a x}}$	127
risch	$\frac{2x\left(b\,x^{2}+a\right)}{3b\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{a\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3b^{2}\sqrt{b\,x^{3}+a x}}$	135

input
```
int(x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a*x)^(1/2)/b-1/3*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/
2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),1/2*2^(1/2))
```

3.59.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = -\frac{2\left(a\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}b\right)}{3\,b^2}$$

input
```
integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*b)/b
^2
```

3.59.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x*(a + b*x**2)), x)
```

3.59.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^3 + a*x), x)
```

3.59.8 Giac [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^3 + a*x), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x^2/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^3)^(1/2), x)
```

3.60 $\int \frac{x}{\sqrt{ax+bx^3}} dx$

3.60.1 Optimal result

Integrand size = 15, antiderivative size = 229

$$\int \frac{x}{\sqrt{ax+bx^3}} dx = \frac{2x(a+bx^2)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}$$

output
```
2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*a^(1/4)*(cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/
(b*x^3+a*x)^(1/2)+a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.23

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx=\frac{2x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])
/(3*Sqrt[x*(a + b*x^2)])
```

3.60.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{\sqrt{ax+bx^3}}$$

input
```
Int[x/Sqrt[a*x + b*x^3],x]
```

output
```
(2*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt
[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqr
t[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/Sqrt[a*x + b*x^3]
```

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.60.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{b\sqrt{b\,x^{3}+ax}}$	158
elliptic	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{b\sqrt{b\,x^{3}+ax}}$	158

input
```
int(x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-
2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2
^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)
,1/2*2^(1/2)))
```

3.60.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.10

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = -\frac{2\,\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{\sqrt{b}}$$

input
```
integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))/sqrt(b)
```

3.60.6 Sympy [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(x/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(x*(a + b*x**2)), x)
```

3.60.7 Maxima [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^3 + a*x), x)
```

3.60.8 Giac [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*x^3 + a*x), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(x/(a*x + b*x^3)^(1/2),x)
```

output
```
int(x/(a*x + b*x^3)^(1/2), x)
```

3.61 $\int \frac{1}{\sqrt{ax+bx^3}}\,dx$

3.61.1 Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx=\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1
/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1
/4)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx=\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{x\,(a+bx^2)}}$$

input
```
Integrate[1/Sqrt[a*x + b*x^3],x]
```

output
```
(2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/S
qrt[x*(a + b*x^2)]
```

3.61.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$
$$\Big\downarrow \text{1917}$$
$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{\sqrt{ax+bx^3}}$$
$$\Big\downarrow \text{266}$$
$$\frac{2\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{ax+bx^3}}$$
$$\Big\downarrow \text{761}$$
$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

input
```
Int[1/Sqrt[a*x + b*x^3],x]
```

output
```
(Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[
a*x + b*x^3])
```

3.61.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.61.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}$	108
elliptic	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}$	108

input
```
int(1/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*El
lipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.61.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.15

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{2\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(-4*a/b, 0, x)/sqrt(b)
```

3.61.6 Sympy [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**3), x)
```

3.61.7 Maxima [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^3 + a*x), x)
```

3.61.8 Giac [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^3 + a*x), x)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{2\,x\,\sqrt{\frac{b\,x^2}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;-\frac{b\,x^2}{a}\right)}{\sqrt{b\,x^3+a\,x}}$$

input
```
int(1/(a*x + b*x^3)^(1/2),x)
```

output
```
(2*x*((b*x^2)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^2)/a))/(a*x +
b*x^3)^(1/2)
```

3.62 $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$

3.62.1 Optimal result

Integrand size = 17, antiderivative size = 253

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \frac{2\sqrt{b}x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax} - \frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}$$

output
```
2*x*(b*x^2+a)*b^(1/2)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x
)^(1/2)/a/x-2*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*
b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+b^(1/4)*(cos(2*arctan(b^(1/4)*
x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*
x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/S
qrt[x*(a + b*x^2)]
```

3.62.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$$

$$\Big\downarrow \text{1931}$$

$$\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{1938}$$

$$\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \text{834}$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \; 761$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}$$

$$\Big\downarrow \; 1510$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}$$

$$\frac{2\sqrt{ax+bx^3}}{ax}$$

input
```
Int[1/(x*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]
*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt
[a*x + b*x^3])
```

3.62. $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.62.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2(b x^2+a)}{a\sqrt{x(b x^2+a)}}+\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) \cdots}{\sqrt{-ab}}} \cdots\right)}{b} \cdots\right)}{a\sqrt{b x^3+a x}}$
risch	$-\frac{2(b x^2+a)}{a\sqrt{x(b x^2+a)}}+\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) \cdots}{\sqrt{-ab}}} \cdots\right)}{b} \cdots\right)}{a\sqrt{b x^3+a x}}$
elliptic	$-\frac{2(b x^2+a)}{a\sqrt{x(b x^2+a)}}+\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) \cdots}{\sqrt{-ab}}} \cdots\right)}{b} \cdots\right)}{a\sqrt{b x^3+a x}}$

input
```
int(1/x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^2+a)/a/(x*(b*x^2+a))^(1/2)+1/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b
)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.62.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = -\frac{2\left(\sqrt{b}x\,\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{ax}$$

input
```
integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)
) + sqrt(b*x^3 + a*x))/(a*x)
```

3.62.6 Sympy [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x\sqrt{x\left(a+bx^2\right)}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x*(a + b*x**2))), x)
```

3.62.7 Maxima [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}\,x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

3.62.8 Giac [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}\,x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x*(a*x + b*x^3)^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^3)^(1/2)), x)
```

3.63 $\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx$

3.63.1 Optimal result

Integrand size = 17, antiderivative size = 119

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}$$

output
```
-2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b
*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x\sqrt{x\left(a+bx^2\right)}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/(
3*x*Sqrt[x*(a + b*x^2)])
```

3.63.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}dx$$
$$\Big\downarrow \text{1931}$$
$$-\frac{b\int \frac{1}{\sqrt{bx^3+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$
$$\Big\downarrow \text{1917}$$
$$-\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$
$$\Big\downarrow \text{266}$$
$$-\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$
$$\Big\downarrow \text{761}$$
$$-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

input
```
Int[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])
```

3.63.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.63.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\sqrt{b\,x^{3}+a x}}{3 a\,x^{2}}-\frac{\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3 a\sqrt{b\,x^{3}+a x}}$	129
elliptic	$-\frac{2\sqrt{b\,x^{3}+a x}}{3 a\,x^{2}}-\frac{\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3 a\sqrt{b\,x^{3}+a x}}$	129
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3 a x\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{3 a\sqrt{b\,x^{3}+a x}}$	136

input
```
int(1/x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1
/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2),1/2*2^(1/2))
```

3.63.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\left(\sqrt{b}x^2\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}\right)}{3\,ax^2}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x))/(
a*x^2)
```

3.63.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x**2))), x)
```

3.63.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

3.63.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^3)^(1/2)), x)
```

3.64 $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$

3.64.1 Optimal result

Integrand size = 17, antiderivative size = 286

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx
&= -\frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} \\
&\quad + \frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}} \\
&\quad - \frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

output
```
-6/5*b^(3/2)*x*(b*x^2+a)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*
x^3+a*x)^(1/2)/a/x^3+6/5*b*(b*x^3+a*x)^(1/2)/a^2/x+6/5*b^(5/4)*(cos(2*arct
an(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)
))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+
a*x)^(1/2)-3/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^2\sqrt{x\,(a+bx^2)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])/
(5*x^2*Sqrt[x*(a + b*x^2)])
```

3.64.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{1938}$$

$$-\frac{3b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{266}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{834}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

$$\Big\downarrow \text{1510}$$

3.64. $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$

$$-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\cdots}{\sqrt{b}}\cdots\right.}{a\sqrt{ax+bx^3}}\cdots\right.}{5a}\cdots$$

$$\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

input
```
Int[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x + b*x^3])/(5*a*x^3) - (3*b*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2
*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt
[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[
b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqr
t[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x + b*x^3])))/(5*a)
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.64.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2(bx^2+a)(-3bx^2+a)}{5a^2x^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\cdots}\right)}{\cdots}\right)}{5a^2\sqrt{bx^3+ax}}\cdots$
default	$-\frac{2\sqrt{bx^3+ax}}{5ax^3}+\frac{6(bx^2+a)b}{5a^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right)}{5a^2\sqrt{bx^3+ax}}\cdots$
elliptic	$-\frac{2\sqrt{bx^3+ax}}{5ax^3}+\frac{6(bx^2+a)b}{5a^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right)}{5a^2\sqrt{bx^3+ax}}\cdots$

input
```
int(1/x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)*(-3*b*x^2+a)/a^2/x^2/(x*(b*x^2+a))^(1/2)-3/5*b/a^2*(-a*b)^(
1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(
1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-
a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/
2)))
```

3.64.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \frac{2\left(3\,b^{\frac{3}{2}}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(3\,bx^2-a)\right)}{5\,a^2x^3}$$

input
```
integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(3*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
0, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 - a))/(a^2*x^3)
```

3.64.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x^3\sqrt{x\,(a+bx^2)}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x*(a + b*x**2))), x)
```

3.64.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

3.64.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx=\int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^3)^(1/2)), x)
```

3.65 $\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx$

3.65.1 Optimal result

Integrand size = 17, antiderivative size = 161

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}$$

output
```
-x^5/b/(b*x^3+a*x)^(1/2)-15/7*a*(b*x^3+a*x)^(1/2)/b^3+9/7*x^2*(b*x^3+a*x)^
(1/2)/b^2+15/14*a^(7/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+
x*b^(1/2))^2)^(1/2)/b^(13/4)/(b*x^3+a*x)^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \frac{x\left(-15a^2-6abx^2+2b^2x^4+15a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b^3\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^7/(a*x + b*x^3)^(3/2),x]
```

output
```
(x*(-15*a^2 - 6*a*b*x^2 + 2*b^2*x^4 + 15*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(7*b^3*Sqrt[x*(a + b*x^2)])
```

3.65.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1928, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\int \frac{x^4}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{9\left(\frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{5a\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{7b}\right)}{2b}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^7/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^5/(b*Sqrt[a*x + b*x^3])) + (9*((2*x^2*Sqrt[a*x + b*x^3])/(7*b) - (5*a*
((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])))/(7*b)))/(2*b)
```

3.65.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.65.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

method	result
default	$-\frac{x a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b x^3+ax}}{7b^2}-\frac{8a\sqrt{b x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{14b^4\sqrt{b x^3+ax}}$
elliptic	$-\frac{x a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b x^3+ax}}{7b^2}-\frac{8a\sqrt{b x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{14b^4\sqrt{b x^3+ax}}$
risch	$-\frac{2\left(-b x^2+4a\right)\left(b x^2+a\right)x}{7b^3\sqrt{x\left(b x^2+a\right)}}+\frac{a^2\left(\frac{11\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b x^3+ax}}-7a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\cdots\right.\right.}{7b^3}$

input
```
int(x^7/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*x*a^2/((x^2+a/b)*b*x)^(1/2)+2/7*x^2*(b*x^3+a*x)^(1/2)/b^2-8/7*a*(b*
x^3+a*x)^(1/2)/b^3+15/14*a^2/b^4*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2
)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)
^(1/2),1/2*2^(1/2))
```

3.65.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \frac{15\,(a^2bx^2+a^3)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(2\,b^3x^4-6\,ab^2x^2-15\,a^2b)\sqrt{bx^3+}}{7\,(b^5x^2+ab^4)}$$

input
```
integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/7*(15*(a^2*b*x^2 + a^3)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (2*b
^3*x^4 - 6*a*b^2*x^2 - 15*a^2*b)*sqrt(b*x^3 + a*x))/(b^5*x^2 + a*b^4)
```

3.65.6 Sympy [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**7/(x*(a + b*x**2))**(3/2), x)
```

3.65.7 Maxima [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

3.65.8 Giac [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^7/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^7/(a*x + b*x^3)^(3/2), x)
```

3.66 $\int \frac{x^6}{(ax+bx^3)^{3/2}} dx$

3.66.1 Optimal result

Integrand size = 17, antiderivative size = 279

$$\int \frac{x^6}{(ax+bx^3)^{3/2}} dx = -\frac{x^4}{b\sqrt{ax+bx^3}} - \frac{21ax(a+bx^2)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} + \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}} - \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}$$

output
```
-x^4/b/(b*x^3+a*x)^(1/2)-21/5*a*x*(b*x^2+a)/b^(5/2)/(a^(1/2)+x*b^(1/2))/(b
*x^3+a*x)^(1/2)+7/5*x*(b*x^3+a*x)^(1/2)/b^2+21/5*a^(5/4)*(cos(2*arctan(b^(
1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ell
ipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1
/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^3+a*x)^
(1/2)-21/10*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(11/4)/(b*x^3+a*x)^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.24

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \frac{2x^2\left(-7a+bx^2+7a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^6/(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x^2*(-7*a + b*x^2 + 7*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2,
 7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[x*(a + b*x^2)])
```

3.66.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1928, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{7\int \frac{x^3}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\int \frac{x}{\sqrt{bx^3+ax}}dx}{5b}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{3a\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 266$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 834$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 761$$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{5b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 1510$$

3.66. $\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx$

$$\frac{7\left(\frac{2x\sqrt{ax+bx^3}}{5b}-\frac{6a\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\cdots\right.}{5b\sqrt{ax+bx^3}}\cdots\right.}{2b}$$

$$\frac{x^4}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^6/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^4/(b*Sqrt[a*x + b*x^3])) + (7*((2*x*Sqrt[a*x + b*x^3])/(5*b) - (6*a*Sq
rt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x
)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a +
 b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(
2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b*Sqrt[a*x + b*x^3])))/(2*b)
```

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.66. $\int \frac{x^6}{(ax+bx^3)^{3/2}}\, dx$

3.66.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72

method	result
default	$\frac{x^2 a}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x\sqrt{b\,x^3+ax}}{5b^2}-\frac{21a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{10b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{x^2 a}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x\sqrt{b\,x^3+ax}}{5b^2}-\frac{21a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{10b^3\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5b^2\sqrt{x\left(b\,x^2+a\right)}}-\left(a\,\frac{8\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{x+\frac{\sqrt{-ab}}{b}}{\sqrt{-ab}}}\right.}{b}\right.}{b\sqrt{b\,x^3+ax}}\right.$

input
```
int(x^6/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*x^2*a/((x^2+a/b)*b*x)^(1/2)+2/5*x*(b*x^3+a*x)^(1/2)/b^2-21/10*a/b^3*
(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)
/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*
(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(
1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1
/2*2^(1/2)))
```

3.66.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.27

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \frac{21\,(abx^2+a^2)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(2\,b^2x^3 -}{5\,(b^4x^2+ab^3)}$$

input
```
integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(a*b*x^2 + a^2)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInv
erse(-4*a/b, 0, x)) + (2*b^2*x^3 + 7*a*b*x)*sqrt(b*x^3 + a*x))/(b^4*x^2 +
a*b^3)
```

3.66.6 Sympy [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**6/(x*(a + b*x**2))**(3/2), x)
```

3.66.7 Maxima [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

3.66.8 Giac [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^6/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^6/(a*x + b*x^3)^(3/2), x)
```

3.67 $\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx$

3.67.1 Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx=-\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}$$

output
```
-x^3/b/(b*x^3+a*x)^(1/2)+5/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a^(3/4)*(cos(2*arct
an(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)
))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+
a*x)^(1/2)
```

3.67.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx=\frac{x\left(5a+2bx^2-5a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b^2\sqrt{x\,(a+bx^2)}}$$

input
```
Integrate[x^5/(a*x + b*x^3)^(3/2),x]
```

output
```
(x*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((b*x^2)/a)]))/(3*b^2*Sqrt[x*(a + b*x^2)])
```

3.67.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1928, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{5\int \frac{x^2}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^3+ax}}dx}{3b}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{2a\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3b\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\right)}{2b}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^5/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^3/(b*Sqrt[a*x + b*x^3])) + (5*((2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*
Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b
*x^3])))/(2*b)
```

3.67.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.67.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

method	result
default	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6b^3\sqrt{b\,x^3+ax}}$
risch	$\frac{2\left(b\,x^2+a\right)x}{3b^2\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\left(\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^3+ax}}-3a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{(x+\cdots}}{\cdots}\right.\right.}{3b^2}$

input
```
int(x^5/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*x*a/((x^2+a/b)*b*x)^(1/2)+2/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a/b^3*(-a*b)
^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.67.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = -\frac{5\,(abx^2+a^2)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(2\,b^2x^2+5\,ab)\sqrt{bx^3+ax}}{3\,(b^4x^2+ab^3)}$$

input
```
integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(a*b*x^2 + a^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (2*b^2
*x^2 + 5*a*b)*sqrt(b*x^3 + a*x))/(b^4*x^2 + a*b^3)
```

3.67.6 Sympy [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**5/(x*(a + b*x**2))**(3/2), x)
```

3.67.7 Maxima [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

3.67.8 Giac [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^5/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^5/(a*x + b*x^3)^(3/2), x)
```

3.68 $\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$

3.68.1 Optimal result

Integrand size = 17, antiderivative size = 253

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{ax+bx^3}} + \frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}$$

output
```
-x^2/b/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+
a*x)^(1/2)-3*a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b
^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)+3/2*a^(1/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellip
ticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2
))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/
2)
```

3.68.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{2x^2\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^4/(a*x + b*x^3)^(3/2),x]
```

output
```
(-2*x^2*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^
2)/a)]))/(b*Sqrt[x*(a + b*x^2)])
```

3.68.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1928, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{3\int \frac{x}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{2b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{834}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{b\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{3\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{b\sqrt{ax+bx^3}}$$

$$\frac{x^2}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^4/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x^2/(b*Sqrt[a*x + b*x^3])) + (3*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*S
qrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a]
 + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan
[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(b*Sqrt[a
*x + b*x^3])
```

3.68. $\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$

3.68.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.68. $\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$

3.68.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.72

method	result
default	$-\frac{x^{2}}{b\sqrt{\left(x^{2}+\frac{a}{b}\right) b x}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\,\sqrt{-\frac{x b}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\right.}{b}\right.}{2b^{2}\sqrt{b\,x^{3}+a x}}$
elliptic	$-\frac{x^{2}}{b\sqrt{\left(x^{2}+\frac{a}{b}\right) b x}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\,\sqrt{-\frac{x b}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\right.}{b}\right.}{2b^{2}\sqrt{b\,x^{3}+a x}}$

input
```
int(x^4/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*x^2/((x^2+a/b)*b*x)^(1/2)+3/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b
)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.68.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{\sqrt{bx^3+ax}bx+3\left(bx^2+a\right)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b^3x^2+ab^2}$$

input
```
integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*x^3 + a*x)*b*x + 3*(b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0,
 weierstrassPInverse(-4*a/b, 0, x)))/(b^3*x^2 + a*b^2)
```

3.68.6 Sympy [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**4/(x*(a + b*x**2))**(3/2), x)
```

3.68.7 Maxima [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

3.68.8 Giac [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{x^4}{\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

input
```
int(x^4/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^4/(a*x + b*x^3)^(3/2), x)
```

3.69 $\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx$

3.69.1 Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = -\frac{x}{b\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}$$

output
```
-x/b/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/
2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \frac{x\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^3/(a*x + b*x^3)^(3/2),x]
```

output
```
(x*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)
]))/(b*Sqrt[x*(a + b*x^2)])
```

3.69.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1928, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{\int \frac{1}{\sqrt{bx^3+ax}}dx}{2b}-\frac{x}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{2b\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{b\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

input
```
Int[x^3/(a*x + b*x^3)^(3/2),x]
```

output
```
-(x/(b*Sqrt[a*x + b*x^3])) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)],
 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a*x + b*x^3])
```

3.69.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

3.69.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{x}{b\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^2\sqrt{b\,x^3+ax}}$	130
elliptic	$-\frac{x}{b\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^2\sqrt{b\,x^3+ax}}$	130

input
```
int(x^3/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*x/((x^2+a/b)*b*x)^(1/2)+1/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^
(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2),1/2*2^(1/2))
```

3.69.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \frac{(bx^2+a)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^3+ax}b}{b^3x^2+ab^2}$$

input
```
integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)
*b)/(b^3*x^2 + a*b^2)
```

3.69.6 Sympy [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**3/(x*(a + b*x**2))**(3/2), x)
```

3.69.7 Maxima [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

3.69.8 Giac [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^3)^(3/2), x)
```

3.70 $\int \frac{x^2}{(ax+bx^3)^{3/2}} dx$

3.70.1 Optimal result

Integrand size = 17, antiderivative size = 254

$$\int \frac{x^2}{(ax+bx^3)^{3/2}} dx = \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{x(a+bx^2)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}} - \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}$$

output
```
x^2/a/(b*x^3+a*x)^(1/2)-x*(b*x^2+a)/a/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a
*x)^(1/2)+(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1
/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^
(1/2)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)-1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx=\frac{2x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)}{3a\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x^2/(a*x + b*x^3)^(3/2),x]
```

output
```
(2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)])
/(3*a*Sqrt[x*(a + b*x^2)])
```

3.70.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1929, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 1929$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+ax}}dx}{2a}$$

$$\Big\downarrow \; 1938$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{2a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 266$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 834$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \textbf{1510}$$

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}$$

input
```
Int[x^2/(a*x + b*x^3)^(3/2),x]
```

output
```
x^2/(a*Sqrt[a*x + b*x^3]) - (Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a
 + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sq
rt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(
1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x +
b*x^3])
```

3.70. $\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx$

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.70. $\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx$

3.70.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.72

method	result
default	$\frac{x^2}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{2ab\sqrt{b\,x^3+ax}}$
elliptic	$\frac{x^2}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{2ab\sqrt{b\,x^3+ax}}$

input
```
int(x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^2/a/((x^2+a/b)*b*x)^(1/2)-1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(
1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.70.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx=\frac{\sqrt{bx^3+ax}bx+(bx^2+a)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{ab^2x^2+a^2b}$$

input
```
integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(b*x^3 + a*x)*b*x + (b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0, we
ierstrassPInverse(-4*a/b, 0, x)))/(a*b^2*x^2 + a^2*b)
```

3.70.6 Sympy [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(x(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**2/(x*(a + b*x**2))**(3/2), x)
```

3.70.7 Maxima [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

3.70.8 Giac [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^3)^(3/2), x)
```

3.71 $\int \frac{x}{(ax+bx^3)^{3/2}}\,dx$

3.71.1 Optimal result

Integrand size = 15, antiderivative size = 114

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{x}{a\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

output
```
x/a/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/a^(5/4)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{x + x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{a\sqrt{x(a+bx^2)}}$$

input
```
Integrate[x/(a*x + b*x^3)^(3/2),x]
```

output
```
(x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])
/(a*Sqrt[x*(a + b*x^2)])
```

3.71.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1929, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 1929$$

$$\frac{\int \frac{1}{\sqrt{bx^3+ax}}dx}{2a}+\frac{x}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 1917$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{2a\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 266$$

$$\frac{\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \; 761$$

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

input
```
Int[x/(a*x + b*x^3)^(3/2),x]
```

output
```
x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])
```

3.71.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

3.71.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2ab\sqrt{b\,x^{3}+ax}}$	132
elliptic	$\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2ab\sqrt{b\,x^{3}+ax}}$	132

input
```
int(x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/a/((x^2+a/b)*b*x)^(1/2)+1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/
2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),1/2*2^(1/2))
```

3.71.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.45

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{(bx^2+a)\sqrt{b}\,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}\,b}{ab^2x^2+a^2b}$$

input
```
integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)
*b)/(a*b^2*x^2 + a^2*b)
```

3.71.6 Sympy [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(x(a+bx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x/(x*(a + b*x**2))**(3/2), x)
```

3.71.7 Maxima [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^3 + a*x)^(3/2), x)
```

3.71.8 Giac [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b*x^3 + a*x)^(3/2), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^3)^(3/2),x)
```

output
```
int(x/(a*x + b*x^3)^(3/2), x)
```

3.72 $\int \frac{1}{(ax+bx^3)^{3/2}} dx$

3.72.1 Optimal result

Integrand size = 13, antiderivative size = 273

$$\int \frac{1}{(ax+bx^3)^{3/2}} dx = \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\sqrt{b}x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} - \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{7/4}\sqrt{ax+bx^3}} + \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}$$

output
```
1/a/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)*b^(1/2)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3
+a*x)^(1/2)-3*(b*x^3+a*x)^(1/2)/a^2/x-3*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1
/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(si
n(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)+3/2*
b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/a^(7/4)/(b*x^3+a*x)^(1/2)
```

3.72.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.19

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{a\sqrt{x(a+bx^2)}}$$

input
```
Integrate[(a*x + b*x^3)^(-3/2),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/(
a*Sqrt[x*(a + b*x^2)])
```

3.72.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {1912, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{3\int \frac{1}{x\sqrt{bx^3+ax}}dx}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{3\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 834

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 761

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{2a}+\frac{1}{a\sqrt{ax+bx^3}}$$

$\Big\downarrow$ 1510

3.72. $\int \frac{1}{(ax+bx^3)^{3/2}}\,dx$

$$\frac{3\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\frac{\sqrt{x}\sqrt{\cdots}}{\sqrt{a}\cdots}}{\sqrt{b}}\cdots\right)}{a\sqrt{ax+bx^3}}\cdots\right)}{2a}$$

$$\frac{1}{a\sqrt{ax+bx^3}}$$

input
```
Int[(a*x + b*x^3)^(-3/2),x]
```

output
```
1/(a*Sqrt[a*x + b*x^3]) + (3*((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*
Sqrt[a + b*x^2]*(-((-((Sqrt[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (
a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2
]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3
/4)*Sqrt[a + b*x^2])))/(a*Sqrt[a*x + b*x^3])))/(2*a)
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.72. $\int \frac{1}{(ax+bx^3)^{3/2}}\,dx$

3.72.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2(b x^2+a)}{a^2\sqrt{x(b x^2+a)}}-\frac{b x^2}{a^2\sqrt{(x^2+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{2a^2\sqrt{b x^3+ax}}$
elliptic	$-\frac{2(b x^2+a)}{a^2\sqrt{x(b x^2+a)}}-\frac{b x^2}{a^2\sqrt{(x^2+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{2a^2\sqrt{b x^3+ax}}$
risch	$-\frac{2(b x^2+a)}{a^2\sqrt{x(b x^2+a)}}+\frac{\left(b^2\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{b^2\sqrt{b x^3+ax}}\right.}{}$

input
```
int(1/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^2+a)/a^2/(x*(b*x^2+a))^(1/2)-b*x^2/a^2/((x^2+a/b)*b*x)^(1/2)+3/2/a
^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(
-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*
2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2
),1/2*2^(1/2)))
```

3.72.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx =$$
$$-\frac{3\,(bx^3+ax)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(3\,bx^2+2\,a)}{a^2bx^3+a^3x}$$

input
```
integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-(3*(b*x^3 + a*x)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-
4*a/b, 0, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 + 2*a))/(a^2*b*x^3 + a^3*x)
```

3.72.6 Sympy [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(ax+bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**3)**(-3/2), x)
```

3.72.7 Maxima [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x)^(-3/2), x)
```

3.72.8 Giac [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x)^(-3/2), x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{3}{4};\ -\frac{b\,x^2}{a}\right)}{(b\,x^3+a\,x)^{3/2}}$$

input
```
int(1/(a*x + b*x^3)^(3/2),x)
```

output
```
-(2*x*((b*x^2)/a + 1)^(3/2)*hypergeom([-1/4, 3/2], 3/4, -(b*x^2)/a))/(a*x
+ b*x^3)^(3/2)
```

3.73 $\int \frac{1}{x(ax+bx^3)^{3/2}} \, dx$

3.73.1 Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{1}{x(ax+bx^3)^{3/2}} \, dx = \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}$$

output
```
1/a/x/(b*x^3+a*x)^(1/2)-5/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6*b^(3/4)*(cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/(b*x
^3+a*x)^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.40

$$\int \frac{1}{x(ax+bx^3)^{3/2}} \, dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3ax\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x*(a*x + b*x^3)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/(
3*a*x*Sqrt[x*(a + b*x^2)])
```

3.73.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1929, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax}}dx}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{5\left(-\frac{b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(-\frac{2b\sqrt{x}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}\right)}{2a}+\frac{1}{ax\sqrt{ax+bx^3}}$$

input
```
Int[1/(x*(a*x + b*x^3)^(3/2)),x]
```

output
```
1/(a*x*Sqrt[a*x + b*x^3]) + (5*((-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4
)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x +
 b*x^3])))/(2*a)
```

3.73.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.73.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

method	result
default	$-\frac{bx}{a^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^{3}+ax}}{3a^{2}x^{2}}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6a^{2}\sqrt{b\,x^{3}+ax}}$
elliptic	$-\frac{bx}{a^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^{3}+ax}}{3a^{2}x^{2}}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6a^{2}\sqrt{b\,x^{3}+ax}}$
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3a^{2}x\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{b\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}+3a\left(\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\cdots}}{\cdots}\right.\right.}{3a^{2}}$

input
```
int(1/x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-b*x/a^2/((x^2+a/b)*b*x)^(1/2)-2/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6/a^2*(-a*b
)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(
-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF
(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

3.73.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{5\left(bx^4+ax^2\right)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}(5bx^2+2a)}{3\left(a^2bx^4+a^3x^2\right)}$$

input
```
integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(b*x^4 + a*x^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b
*x^3 + a*x)*(5*b*x^2 + 2*a))/(a^2*b*x^4 + a^3*x^2)
```

3.73.6 Sympy [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(x*(a + b*x**2))**(3/2)), x)
```

3.73.7 Maxima [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

3.73.8 Giac [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^3)^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^3)^(3/2)), x)
```

3.74 $\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx$

3.74.1 Optimal result

Integrand size = 17, antiderivative size = 306

$$\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx = \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} + \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+bx^3}} - \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}$$

output
```
1/a/x^2/(b*x^3+a*x)^(1/2)-21/5*b^(3/2)*x*(b*x^2+a)/a^3/(a^(1/2)+x*b^(1/2))
/(b*x^3+a*x)^(1/2)-7/5*(b*x^3+a*x)^(1/2)/a^2/x^3+21/5*b*(b*x^3+a*x)^(1/2)/
a^3/x+21/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(
1/2))^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(1/2)-21/10*b^(5/4)*(cos(2*arctan(b^(1
/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elli
pticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(
1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5ax^2\sqrt{x(a+bx^2)}}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])/
(5*a*x^2*Sqrt[x*(a + b*x^2)])
```

3.74.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1929, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{x^3\sqrt{bx^3+ax}}dx}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\int \frac{x}{\sqrt{bx^3+ax}}dx}{a}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\sqrt{x}\sqrt{a+bx^2}\int\frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{834}$$

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{761}$$

3.74. $\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx$

$$\frac{7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}\right)}{5a}-\frac{2\sqrt{ax+bx^3}}{5ax^3}\right)}{2a}+$$

$$\frac{1}{ax^2\sqrt{ax+bx^3}}$$

$$\Big\downarrow \text{1510}$$

3.74. $\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx$

$$7\left(-\frac{3b\left(\frac{2b\sqrt{x}\sqrt{a+bx^2}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-}{\sqrt{b}}\right.}{a\sqrt{ax+bx^3}}}{5a}\right.$$

$$\frac{}{2a}$$

$$\frac{1}{ax^2\sqrt{ax+bx^3}}$$

input
```
Int[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

output
```
1/(a*x^2*Sqrt[a*x + b*x^3]) + (7*((-2*Sqrt[a*x + b*x^3])/(5*a*x^3) - (3*b*
((-2*Sqrt[a*x + b*x^3])/(a*x) + (2*b*Sqrt[x]*Sqrt[a + b*x^2]*(-((-((Sqrt[x
]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqr
t[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*Sqr
t[a*x + b*x^3])))/(5*a)))/(2*a)
```

3.74. $\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.74.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2\sqrt{b x^3+ax}}{5a^2x^3}+\frac{16(bx^2+a)b}{5a^3\sqrt{x(bx^2+a)}}+\frac{b^2x^2}{a^3\sqrt{(x^2+\frac{a}{b})bx}}-\frac{21b\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\cdots}\right.}{\cdots}\right.}{10a^3\sqrt{bx^3+ax}}$
elliptic	$-\frac{2\sqrt{b x^3+ax}}{5a^2x^3}+\frac{16(bx^2+a)b}{5a^3\sqrt{x(bx^2+a)}}+\frac{b^2x^2}{a^3\sqrt{(x^2+\frac{a}{b})bx}}-\frac{21b\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\cdots}\right.}{\cdots}\right.}{10a^3\sqrt{bx^3+ax}}$
risch	$-\frac{2(bx^2+a)(-8bx^2+a)}{5a^3x^2\sqrt{x(bx^2+a)}}-\frac{b^2\left(\frac{8\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\cdots}\right.}{\cdots}\right.}{b\sqrt{bx^3+ax}}\cdots\right.}{\cdots}$

input
```
int(1/x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^3+a*x)^(1/2)/a^2/x^3+16/5*(b*x^2+a)/a^3*b/(x*(b*x^2+a))^(1/2)+b^
2*x^2/a^3/((x^2+a/b)*b*x)^(1/2)-21/10*b/a^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/
b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/
(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF
(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.74.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^2 (ax + bx^3)^{3/2}} \, dx = \frac{21 \left(b^2x^5 + abx^3\right)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (21}{5 \left(a^3bx^5 + a^4x^3\right)}$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(b^2*x^5 + a*b*x^3)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrass
PInverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 14*a*b*x^2 - 2*a^2)*sqrt(b*x^3 + a
*x))/(a^3*b*x^5 + a^4*x^3)
```

3.74.6 Sympy [F]

$$\int \frac{1}{x^2 (ax + bx^3)^{3/2}} \, dx = \int \frac{1}{x^2 \left(x \left(a + bx^2\right)\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(b*x**3+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(x*(a + b*x**2))**(3/2)), x)
```

3.74.7 Maxima [F]

$$\int \frac{1}{x^2 (ax + bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + ax)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

3.74.8 Giac [F]

$$\int \frac{1}{x^2 (ax + bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + ax)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax + bx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (b\,x^3 + a\,x)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a*x + b*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^3)^(3/2)), x)
```

3.75 $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}} \, dx$

3.75.1 Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}} \, dx = -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}} - \frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{9a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}$$

output
```
-1/7*x^(25/2)/b/(b*x^3+a*x)^(7/2)-9/35*x^(19/2)/b^2/(b*x^3+a*x)^(5/2)-3/5*
x^(13/2)/b^3/(b*x^3+a*x)^(3/2)-9/2*a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(
1/2))/b^(11/2)-3*x^(7/2)/b^4/(b*x^3+a*x)^(1/2)+9/2*x^(1/2)*(b*x^3+a*x)^(1/
2)/b^5
```

3.75.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}} \, dx = \frac{x^{9/2}\left(\sqrt{b}(a+bx^2)\left(315a^4x+1050a^3bx^3+1218a^2b^2x^5+528ab^3x^7+35b^4x^9\right)-630a(a\right.}{70b^{11/2}\left(x\left(a+bx^2\right)\right)^{9/2}}$$

input
```
Integrate[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(Sqrt[b]*(a + b*x^2)*(315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x
^5 + 528*a*b^3*x^7 + 35*b^4*x^9) - 630*a*(a + b*x^2)^(9/2)*ArcTanh[(Sqrt[b
]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])]))/(70*b^(11/2)*(x*(a + b*x^2))^(9/2))
```

3.75.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1928, 1928, 1928, 1928, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\int \frac{x^{23/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\left(\frac{7\int \frac{x^{17/2}}{(bx^3+ax)^{5/2}}dx}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{x^{11/2}}{(bx^3+ax)^{3/2}}dx}{3b}-\frac{x^{13/2}}{3b(ax+bx^3)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{x^{5/2}}{\sqrt{bx^3+ax}}dx}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1930

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{ax+bx^3}}{2b}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax}}dx}{2b}\right)}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1935

3.75. $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{ax+bx^3}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^3}{bx^3+ax}}d\frac{x^{3/2}}{\sqrt{bx^3+ax}}}{2b}\right)}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 219

3.75. $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{ax+bx^3}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^{7/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{13/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{19/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(25/2)/(b*(a*x + b*x^3)^(7/2)) + (9*(-1/5*x^(19/2)/(b*(a*x + b*x^3)
^(5/2)) + (7*(-1/3*x^(13/2)/(b*(a*x + b*x^3)^(3/2)) + (5*(-(x^(7/2)/(b*Sqr
t[a*x + b*x^3])) + (3*((Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b) - (a*ArcTanh[(Sqr
t[b]*x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(3/2))))/b))/(3*b)))/(5*b)))/(7*b)
```

3.75. $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx$

3.75.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.75.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.33

method	result
default	$-\frac{\sqrt{x(b\,x^2+a)}\,\left(-35x^9 b^{\frac{9}{2}}+315\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a\,b^3 x^6\sqrt{b\,x^2+a}-528b^{\frac{7}{2}}\,a\,x^7+945\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a^2 b^2 x^4\sqrt{b\,x^2+a}-1218b^{\frac{5}{2}}\,a}{70b^{\frac{11}{2}}\sqrt{x}\,(b\,x^2+a)^4}$
risch	$\frac{x^{\frac{3}{2}}\,(b\,x^2+a)}{2b^5\sqrt{x(b\,x^2+a)}}+\frac{\left(-\frac{9a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{11}{2}}}-\frac{a^3\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{112b^7\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)^4}-\frac{53a^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{560b^7\left(x+\frac{\sqrt{-ab}}{b}\right)^3}+571\right.}{}$

input
```
int(x^(29/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/70*(x*(b*x^2+a))^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))*a*b^3*x^6*(b*x^2+a)^(1/2)-528*b^(7/2)*a*x^7+945*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*b^(5/2)*a^2*x^5+945*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))*a^3*b*x^2*(b*x^2+a)^(1/2)-1050*b^(3/2)*a^3*x^3+31
5*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*a^4*x)/x^(
1/2)/(b*x^2+a)^4
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.36

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{315\,(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5)\sqrt{b}\log\left(2\,bx^2-2\,\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}\right.}{140\,(b^{10}x^8+4\,ab^9x^6+6}\right.$$

input
```
integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5
)*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b
^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*
sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3
*b^7*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4
+ 4*a^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2)
)) + (35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 3
15*a^4*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x
^4 + 4*a^3*b^7*x^2 + a^4*b^6)]
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{29}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\,(bx^2+a)^{\frac{7}{2}}} + \frac{9\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}} - \frac{9\,a\log\left(|a|\right)}{4\,b^{\frac{11}{2}}}$$

input
```
integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
 + 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(11/2) - 9/4*a*log(abs(a))/b^(11/2)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{29/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(29/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(29/2)/(a*x + b*x^3)^(9/2), x)
```

3.76 $\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}} \, dx$

3.76.1 Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}} \, dx = -\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}} - \frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}$$

output
```
-1/7*x^(23/2)/b/(b*x^3+a*x)^(7/2)-8/35*x^(17/2)/b^2/(b*x^3+a*x)^(5/2)-16/3
5*x^(11/2)/b^3/(b*x^3+a*x)^(3/2)-64/35*x^(5/2)/b^4/(b*x^3+a*x)^(1/2)+128/3
5*(b*x^3+a*x)^(1/2)/b^5/x^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}} \, dx = \frac{x^{7/2}(128a^4 + 448a^3bx^2 + 560a^2b^2x^4 + 280ab^3x^6 + 35b^4x^8)}{35b^5(x(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b
^4*x^8))/(35*b^5*(x*(a + b*x^2))^(7/2))
```

3.76.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1921, 1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\int \frac{x^{21/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\int \frac{x^{15/2}}{(bx^3+ax)^{5/2}}dx}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\int \frac{x^{9/2}}{(bx^3+ax)^{3/2}}dx}{3b}-\frac{x^{11/2}}{3b(ax+bx^3)^{3/2}}\right)}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{x^{3/2}}{\sqrt{bx^3+ax}}dx}{b}-\frac{x^{5/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{11/2}}{3b(ax+bx^3)^{3/2}}\right)}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\sqrt{ax+bx^3}}{b^2\sqrt{x}}-\frac{x^{5/2}}{b\sqrt{ax+bx^3}}\right)}{3b}-\frac{x^{11/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{17/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(23/2)/(b*(a*x + b*x^3)^(7/2)) + (8*(-1/5*x^(17/2)/(b*(a*x + b*x^3)
^(5/2)) + (6*(-1/3*x^(11/2)/(b*(a*x + b*x^3)^(3/2)) + (4*(-(x^(5/2)/(b*Sqr
t[a*x + b*x^3])) + (2*Sqrt[a*x + b*x^3])/(b^2*Sqrt[x])))/(3*b)))/(5*b)))/(
7*b)
```

3.76.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.76.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{(b\,x^2+a)\,(35x^8b^4+280a\,b^3x^6+560a^2x^4b^2+448a^3b\,x^2+128a^4)\,x^{\frac{9}{2}}}{35b^5\,(b\,x^3+ax)^{\frac{9}{2}}}$	70
default	$\frac{\sqrt{x\,(b\,x^2+a)}\,(35x^8b^4+280a\,b^3x^6+560a^2x^4b^2+448a^3b\,x^2+128a^4)}{35\sqrt{x}\,(b\,x^2+a)^4\,b^5}$	72
risch	$\frac{(b\,x^2+a)\sqrt{x}}{b^5\sqrt{x\,(b\,x^2+a)}}+\frac{(b\,x^2+a)\,(140b^3x^6+350a\,b^2x^4+308a^2b\,x^2+93a^3)\,a\sqrt{x}}{35b^5\,(x^8b^4+4a\,b^3x^6+6a^2x^4b^2+4a^3b\,x^2+a^4)\sqrt{x\,(b\,x^2+a)}}$	128

input
```
int(x^(27/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128
*a^4)*x^(9/2)/b^5/(b*x^3+a*x)^(9/2)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(35\,b^4x^8+280\,ab^3x^6+560\,a^2b^2x^4+448\,a^3bx^2+128\,a^4)\sqrt{bx^3+ax}\sqrt{x}}{35\,(b^9x^9+4\,ab^8x^7+6\,a^2b^7x^5+4\,a^3b^6x^3+a^4b^5x)}$$

input
```
integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a
^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^9*x^9 + 4*a*b^8*x^7 + 6*a^2*b^7*x^5 + 4*a
^3*b^6*x^3 + a^4*b^5*x)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.76.7 Maxima [F]

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{27}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{bx^2+a}}{b^5} - \frac{128\sqrt{a}}{35\,b^5} + \frac{140\,(bx^2+a)^3a - 70\,(bx^2+a)^2a^2 + 28\,(bx^2+a)a^3 - 5\,a^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5}$$

input
```
integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)/b^5 - 128/35*sqrt(a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*
(b*x^2 + a)^2*a^2 + 28*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{27/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(27/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(27/2)/(a*x + b*x^3)^(9/2), x)
```

3.77 $\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx$

3.77.1 Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{21/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2\,(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3\,(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}$$

output
```
-1/7*x^(21/2)/b/(b*x^3+a*x)^(7/2)-1/5*x^(15/2)/b^2/(b*x^3+a*x)^(5/2)-1/3*x
^(9/2)/b^3/(b*x^3+a*x)^(3/2)+arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(1/2))/b^
(9/2)-x^(3/2)/b^4/(b*x^3+a*x)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}\left(-\sqrt{b}x(a+bx^2)\left(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6\right)+210(a+bx^2)^{9/2}\operatorname{arc}\right.}{105b^{9/2}\,(x\,(a+bx^2))^{9/2}}$$

input
```
Integrate[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(-(Sqrt[b]*x*(a + b*x^2)*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4
 + 176*b^3*x^6)) + 210*(a + b*x^2)^(9/2)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + S
qrt[a + b*x^2])]))/(105*b^(9/2)*(x*(a + b*x^2))^(9/2))
```

3.77.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1928, 1928, 1928, 1928, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1928}$$

$$\frac{\int \frac{x^{19/2}}{(bx^3+ax)^{7/2}}dx}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{\frac{\int \frac{x^{13/2}}{\left(bx^3+ax\right)^{5/2}}dx}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{\frac{\frac{\int \frac{x^{7/2}}{\left(bx^3+ax\right)^{3/2}}dx}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{\frac{\frac{\frac{\int \frac{\sqrt{x}}{\sqrt{bx^3+ax}}dx}{b}-\frac{x^{3/2}}{b\sqrt{ax+bx^3}}}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{1-\frac{bx^3}{bx^3+ax}}d\frac{x^{3/2}}{\sqrt{bx^3+ax}}}{b}-\frac{x^{3/2}}{b\sqrt{ax+bx^3}}}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{3/2}}-\frac{x^{3/2}}{b\sqrt{ax+bx^3}}}{b}-\frac{x^{9/2}}{3b\left(ax+bx^3\right)^{3/2}}}{b}-\frac{x^{15/2}}{5b(ax+bx^3)^{5/2}}}{b}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(21/2)/(b*(a*x + b*x^3)^(7/2)) + (-1/5*x^(15/2)/(b*(a*x + b*x^3)^(5
/2)) + (-1/3*x^(9/2)/(b*(a*x + b*x^3)^(3/2)) + (-(x^(3/2)/(b*Sqrt[a*x + b*
x^3])) + ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(3/2))/b)/b)/b
```

3.77.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.77.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{x(b\,x^2+a)}\left(105\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)b^3x^6\sqrt{b\,x^2+a}-176x^7b^{\frac{7}{2}}+315\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a\,b^2x^4\sqrt{b\,x^2+a}-406b^{\frac{5}{2}}a\,x^5+315\ln\left(x\sqrt{b}+\sqrt{\right.\right.}{105b^{\frac{9}{2}}\sqrt{x}\,(b\,x^2+a)^4}$

input
```
int(x^(25/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(x*(b*x^2+a))^(1/2)/b^(9/2)*(105*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*b^3*x
^6*(b*x^2+a)^(1/2)-176*x^7*b^(7/2)+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a*b^2
*x^4*(b*x^2+a)^(1/2)-406*b^(5/2)*a*x^5+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a
^2*b*x^2*(b*x^2+a)^(1/2)-350*b^(3/2)*a^2*x^3+105*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))*a^3*(b*x^2+a)^(1/2)-105*b^(1/2)*a^3*x)/x^(1/2)/(b*x^2+a)^4
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.68

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{b}\log\left(2\,bx^2+2\,\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}+\right.}{210\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+}\right.$$

input
```
integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sq
rt(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*
x^6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(
x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/
105*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(
-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (176*b^4*x^6 + 406*a*
b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8
 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.77.7 Maxima [F]

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{25}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(25/2)/(b*x^3 + a*x)^(9/2), x)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\left(2\left(x^2\left(\frac{88x^2}{b}+\frac{203a}{b^2}\right)+\frac{175a^2}{b^3}\right)x^2+\frac{105a^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}} + \frac{\log(|a|)}{2\,b^{\frac{9}{2}}}$$

input
```
integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/
(b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2) + 1/2*l
og(abs(a))/b^(9/2)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{25/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(25/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(25/2)/(a*x + b*x^3)^(9/2), x)
```

3.78 $\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx$

3.78.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{19/2}}{7b(ax+bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}$$

output
```
-1/7*x^(19/2)/b/(b*x^3+a*x)^(7/2)-6/35*x^(13/2)/b^2/(b*x^3+a*x)^(5/2)-8/35
*x^(7/2)/b^3/(b*x^3+a*x)^(3/2)-16/35*x^(1/2)/b^4/(b*x^3+a*x)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}(a+bx^2)(-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6)}{35b^4(x(a+bx^2))^{9/2}}$$

input
```
Integrate[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(a + b*x^2)*(-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6))
/(35*b^4*(x*(a + b*x^2))^(9/2))
```

3.78.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{x^{17/2}}{\left(bx^3+ax\right)^{7/2}}dx}{7b}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\int \frac{x^{11/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5b}-\frac{x^{13/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{x^{5/2}}{\left(bx^3+ax\right)^{3/2}}dx}{3b}-\frac{x^{7/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{13/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(\frac{4\left(-\frac{2\sqrt{x}}{3b^2\sqrt{ax+bx^3}}-\frac{x^{7/2}}{3b\left(ax+bx^3\right)^{3/2}}\right)}{5b}-\frac{x^{13/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(19/2)/(b*(a*x + b*x^3)^(7/2)) + (6*(-1/5*x^(13/2)/(b*(a*x + b*x^3)
^(5/2)) + (4*(-1/3*x^(7/2)/(b*(a*x + b*x^3)^(3/2)) - (2*Sqrt[x])/(3*b^2*Sq
rt[a*x + b*x^3])))/(5*b)))/(7*b)
```

3.78.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.78.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(35 b^{3} x^{6}+70 a\,b^{2} x^{4}+56 a^{2} b\,x^{2}+16 a^{3}\right) x^{\frac{9}{2}}}{35 b^{4} \left(b\,x^{3}+a x \right)^{\frac{9}{2}}}$	59
default	$-\frac{\sqrt{x \left(b\,x^{2}+a \right)}\,\left(35 b^{3} x^{6}+70 a\,b^{2} x^{4}+56 a^{2} b\,x^{2}+16 a^{3}\right)}{35 \sqrt{x}\,\left(b\,x^{2}+a \right)^{4} b^{4}}$	61

input
```
int(x^(23/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)*x^(9/2)/b^4/
(b*x^3+a*x)^(9/2)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3)\sqrt{bx^3+ax}\sqrt{x}}{35\,(b^8x^9+4\,ab^7x^7+6\,a^2b^6x^5+4\,a^3b^5x^3+a^4b^4x)}$$

input
```
integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^3 + a*x
)*sqrt(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4
*x)
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{23}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{16}{35\sqrt{a}b^4} - \frac{35\left(bx^2+a\right)^3 - 35\left(bx^2+a\right)^2 a + 21\left(bx^2+a\right)a^2 - 5a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
16/35/(sqrt(a)*b^4) - 1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*
x^2 + a)*a^2 - 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \int \frac{x^{23/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

input
```
int(x^(23/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(23/2)/(a*x + b*x^3)^(9/2), x)
```

3.79 $\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx$

3.79.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{21/2}}{7a\,(ax+bx^3)^{7/2}}$$

output
```
1/7*x^(21/2)/a/(b*x^3+a*x)^(7/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{21/2}}{7a\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))
```

3.79.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \; 1920$$

$$\frac{x^{21/2}}{7a\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))
```

3.79.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.79.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{(b\,x^2+a)\,x^{\frac{23}{2}}}{7a(b\,x^3+ax)^{\frac{9}{2}}}$	27
default	$\frac{x^{\frac{13}{2}}\sqrt{x(b\,x^2+a)}}{7a(b\,x^2+a)^4}$	29

input
```
int(x^(21/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*(b*x^2+a)/a*x^(23/2)/(b*x^3+a*x)^(9/2)
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.48 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{bx^3+ax}x^{\frac{13}{2}}}{7\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)}$$

input
```
integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4
+ 4*a^4*b*x^2 + a^5)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.79.7 Maxima [F]

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{21}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^7}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

input
```
integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{21/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{21/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(21/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(21/2)/(a*x + b*x^3)^(9/2), x)
```

3.80 $\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx$

3.80.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{4x^{9/2}}{35b^2\,(ax+bx^3)^{5/2}} - \frac{8x^{3/2}}{105b^3\,(ax+bx^3)^{3/2}}$$

output
```
-1/7*x^(15/2)/b/(b*x^3+a*x)^(7/2)-4/35*x^(9/2)/b^2/(b*x^3+a*x)^(5/2)-8/105
*x^(3/2)/b^3/(b*x^3+a*x)^(3/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{7/2}(-8a^2-28abx^2-35b^2x^4)}{105b^3\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4))/(105*b^3*(x*(a + b*x^2))^(7/2
))
```

3.80.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{4\int \frac{x^{13/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{4\left(\frac{2\int \frac{x^{7/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5b}-\frac{x^{9/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{4\left(-\frac{2x^{3/2}}{15b^2(ax+bx^3)^{3/2}}-\frac{x^{9/2}}{5b(ax+bx^3)^{5/2}}\right)}{7b}-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(15/2)/(b*(a*x + b*x^3)^(7/2)) + (4*(-1/5*x^(9/2)/(b*(a*x + b*x^3)^
(5/2)) - (2*x^(3/2))/(15*b^2*(a*x + b*x^3)^(3/2))))/(7*b)
```

3.80.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.80.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(35 b^{2} x^{4}+28 a b\,x^{2}+8 a^{2}\right) x^{\frac{9}{2}}}{105 b^{3} \left(b\,x^{3}+a x\right)^{\frac{9}{2}}}$	48
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(35 b^{2} x^{4}+28 a b\,x^{2}+8 a^{2}\right)}{105 \sqrt{x}\,\left(b\,x^{2}+a\right)^{4} b^{3}}$	50

input
```
int(x^(19/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(b*x^2+a)*(35*b^2*x^4+28*a*b*x^2+8*a^2)*x^(9/2)/b^3/(b*x^3+a*x)^(9/
2)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(35\,b^2x^4+28\,abx^2+8\,a^2)\sqrt{bx^3+ax}\sqrt{x}}{105\,(b^7x^9+4\,ab^6x^7+6\,a^2b^5x^5+4\,a^3b^4x^3+a^4b^3x)}$$

input
```
integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^
9 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{19}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{8}{105\,a^{\frac{3}{2}}b^3} - \frac{35\,(bx^2+a)^2 - 42\,(bx^2+a)a + 15\,a^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
8/105/(a^(3/2)*b^3) - 1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)
/((b*x^2 + a)^(7/2)*b^3)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{19/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(19/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(19/2)/(a*x + b*x^3)^(9/2), x)
```

3.81 $\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx$

3.81.1 Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{2x^{15/2}}{35a^2\,(ax+bx^3)^{5/2}}$$

output
```
1/7*x^(17/2)/a/(b*x^3+a*x)^(7/2)+2/35*x^(15/2)/a^2/(b*x^3+a*x)^(5/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{7/2}(7ax^5+2bx^7)}{35a^2\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(7*a*x^5 + 2*b*x^7))/(35*a^2*(x*(a + b*x^2))^(7/2))
```

3.81.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{2\int \frac{x^{15/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^{15/2}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5
/2))
```

3.81.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.81.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{19}{2}}\,(2b\,x^2+7a)}{35a^2(b\,x^3+ax)^{\frac{9}{2}}}$	37
default	$\frac{x^{\frac{9}{2}}\,\sqrt{x(b\,x^2+a)}\,(2b\,x^2+7a)}{35a^2(b\,x^2+a)^4}$	39

input
```
int(x^(17/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)*x^(19/2)*(2*b*x^2+7*a)/a^2/(b*x^3+a*x)^(9/2)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(2\,bx^6+7\,ax^4)\sqrt{bx^3+ax}\sqrt{x}}{35\,(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6)}$$

input
```
integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^
3*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{17}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{17/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(17/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(17/2)/(a*x + b*x^3)^(9/2), x)
```

3.82 $\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx$

3.82.1 Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{2x^{5/2}}{35b^2\,(ax+bx^3)^{5/2}}$$

output
```
-1/7*x^(11/2)/b/(b*x^3+a*x)^(7/2)-2/35*x^(5/2)/b^2/(b*x^3+a*x)^(5/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{7/2}(-2a-7bx^2)}{35b^2\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(-2*a - 7*b*x^2))/(35*b^2*(x*(a + b*x^2))^(7/2))
```

3.82.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{2\int \frac{x^{9/2}}{(bx^3+ax)^{7/2}}dx}{7b}-\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{2x^{5/2}}{35b^2\,(ax+bx^3)^{5/2}}-\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(11/2)/(b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^
(5/2))
```

3.82.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.82.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(7b\,x^{2}+2a\right)x^{\frac{9}{2}}}{35b^{2}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	37
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(7b\,x^{2}+2a\right)}{35\sqrt{x}\,\left(b\,x^{2}+a\right)^{4}b^{2}}$	39

input
```
int(x^(15/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)*(7*b*x^2+2*a)*x^(9/2)/b^2/(b*x^3+a*x)^(9/2)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{\sqrt{bx^3+ax}(7\,bx^2+2\,a)\sqrt{x}}{35\,(b^6x^9+4\,ab^5x^7+6\,a^2b^4x^5+4\,a^3b^3x^3+a^4b^2x)}$$

input
```
integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6
*a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.82.7 Maxima [F]

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{15}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{7\,bx^2+2\,a}{35\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{2}{35\,a^{\frac{5}{2}}b^2}$$

input
```
integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2) + 2/35/(a^(5/2)*b^2)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{15/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(15/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(15/2)/(a*x + b*x^3)^(9/2), x)
```

3.83 $\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx$

3.83.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{13/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{4x^{11/2}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{8x^{9/2}}{105a^3\,(ax+bx^3)^{3/2}}$$

output
```
1/7*x^(13/2)/a/(b*x^3+a*x)^(7/2)+4/35*x^(11/2)/a^2/(b*x^3+a*x)^(5/2)+8/105
*x^(9/2)/a^3/(b*x^3+a*x)^(3/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}(a+bx^2)\,(35a^2x^3+28abx^5+8b^2x^7)}{105a^3\,(x\,(a+bx^2))^{9/2}}$$

input
```
Integrate[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(9/2)*(a + b*x^2)*(35*a^2*x^3 + 28*a*b*x^5 + 8*b^2*x^7))/(105*a^3*(x*(a
 + b*x^2))^(9/2))
```

3.83.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{x^{11/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\left(\frac{2\int \frac{x^{9/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{x^{11/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{2x^{9/2}}{15a^2(ax+bx^3)^{3/2}}+\frac{x^{11/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*(x^(11/2)/(5*a*(a*x + b*x^3)^(5/2)
) + (2*x^(9/2))/(15*a^2*(a*x + b*x^3)^(3/2))))/(7*a)
```

3.83.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.83.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{15}{2}}(8b^2x^4+28ab\,x^2+35a^2)}{105a^3(b\,x^3+ax)^{\frac{9}{2}}}$	48
default	$\frac{x^{\frac{5}{2}}\sqrt{x(b\,x^2+a)}\,(8b^2x^4+28ab\,x^2+35a^2)}{105a^3(b\,x^2+a)^4}$	50

input
```
int(x^(13/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(b*x^2+a)*x^(15/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^3+a*x)^(9/
2)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(8\,b^2x^6+28\,abx^4+35\,a^2x^2)\sqrt{bx^3+ax}\sqrt{x}}{105\,(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7)}$$

input
```
integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3
*b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{13}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{13/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(13/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(13/2)/(a*x + b*x^3)^(9/2), x)
```

3.84 $\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx$

3.84.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{7/2}}{7b\,(ax+bx^3)^{7/2}}$$

output
```
-1/7*x^(7/2)/b/(b*x^3+a*x)^(7/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{7/2}}{7b\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(7/2)/(b*(x*(a + b*x^2))^(7/2))
```

3.84.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \; 1920$$

$$-\frac{x^{7/2}}{7b\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
-1/7*x^(7/2)/(b*(a*x + b*x^3)^(7/2))
```

3.84.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.84.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{(b\,x^2+a)x^{\frac{9}{2}}}{7b(b\,x^3+ax)^{\frac{9}{2}}}$	27
default	$-\frac{\sqrt{x(b\,x^2+a)}}{7\sqrt{x}\,(b\,x^2+a)^4 b}$	29

input
```
int(x^(11/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^2+a)/b*x^(9/2)/(b*x^3+a*x)^(9/2)
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\sqrt{bx^3+ax}\sqrt{x}}{7\,(b^5x^9+4\,ab^4x^7+6\,a^2b^3x^5+4\,a^3b^2x^3+a^4bx)}$$

input
```
integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*
a^3*b^2*x^3 + a^4*b*x)
```

3.84.6 Sympy [F]

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(11/2)/(x*(a + b*x**2))**(9/2), x)
```

3.84.7 Maxima [F]

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{1}{7\,a^{\frac{7}{2}}b}$$

input
```
integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/7/((b*x^2 + a)^(7/2)*b) + 1/7/(a^(7/2)*b)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{11/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(11/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(11/2)/(a*x + b*x^3)^(9/2), x)
```

3.85 $\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx$

3.85.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3\,(ax+bx^3)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}$$

output
```
1/7*x^(9/2)/a/(b*x^3+a*x)^(7/2)+6/35*x^(7/2)/a^2/(b*x^3+a*x)^(5/2)+8/35*x^
(5/2)/a^3/(b*x^3+a*x)^(3/2)+16/35*x^(3/2)/a^4/(b*x^3+a*x)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{7/2}(35a^3x+70a^2bx^3+56ab^2x^5+16b^3x^7)}{35a^4\,(x\,(a+bx^2))^{7/2}}$$

input
```
Integrate[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(7/2)*(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7))/(35*a^4*(x
*(a + b*x^2))^(7/2))
```

3.85.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{x^{7/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\int \frac{x^{5/2}}{\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{x^{7/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{x^{3/2}}{\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{x^{5/2}}{3a\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{x^{7/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(\frac{4\left(\frac{2x^{3/2}}{3a^2\sqrt{ax+bx^3}}+\frac{x^{5/2}}{3a\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{x^{7/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}}$$

input
```
Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*(x^(7/2)/(5*a*(a*x + b*x^3)^(5/2))
+ (4*(x^(5/2)/(3*a*(a*x + b*x^3)^(3/2)) + (2*x^(3/2))/(3*a^2*Sqrt[a*x + b*
x^3])))/(5*a)))/(7*a)
```

3.85.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.85.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{11}{2}}\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)}{35a^4\left(b\,x^3+ax\right)^{\frac{9}{2}}}$	59
default	$\frac{\sqrt{x}\,\sqrt{x(b\,x^2+a)}\,\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)}{35\left(b\,x^2+a\right)^4a^4}$	61

input
```
int(x^(9/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)*x^(11/2)*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/
(b*x^3+a*x)^(9/2)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(16\,b^3x^6+56\,ab^2x^4+70\,a^2bx^2+35\,a^3)\sqrt{bx^3+ax}\sqrt{x}}{35\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

input
```
integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*sqrt(b*x^3 + a*x)
*sqrt(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

3.85.6 Sympy [F]

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{9}{2}}}{(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(9/2)/(x*(a + b*x**2))**(9/2), x)
```

3.85.7 Maxima [F]

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{9}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.54

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^
2 + a)^(7/2)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{9/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(9/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(9/2)/(a*x + b*x^3)^(9/2), x)
```

3.86 $\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx$

3.86.1 Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{7/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2\,(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3\,(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}$$

output
```
1/7*x^(7/2)/a/(b*x^3+a*x)^(7/2)+1/5*x^(5/2)/a^2/(b*x^3+a*x)^(5/2)+1/3*x^(3
/2)/a^3/(b*x^3+a*x)^(3/2)-arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(9/
2)+x^(1/2)/a^4/(b*x^3+a*x)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{x}\left(\sqrt{a}(176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6) - 105(a+bx^2)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right.\right.}{105a^{9/2}\,(a+bx^2)^3\sqrt{x\,(a+bx^2)}}$$

input
```
Integrate[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(Sqrt[x]*(Sqrt[a]*(176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6)
- 105*(a + b*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(105*a^(9/2)*(a
 + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

3.86.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1929, 1929, 1929, 1929, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{\int \frac{x^{5/2}}{(bx^3+ax)^{7/2}}dx}{a}+\frac{x^{7/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{\frac{\int \frac{x^{3/2}}{\left(bx^3+ax\right)^{5/2}}dx}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{x}}{\left(bx^3+ax\right)^{3/2}}dx}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax}}dx}{a}+\frac{\sqrt{x}}{a\sqrt{ax+bx^3}}}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\frac{\frac{\frac{\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{\int \frac{1}{1-\frac{ax}{bx^3+ax}}d\frac{\sqrt{x}}{\sqrt{bx^3+ax}}}{a}}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{\frac{\sqrt{x}}{a\sqrt{ax+bx^3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{3/2}}}{a}+\frac{x^{3/2}}{3a\left(ax+bx^3\right)^{3/2}}}{a}+\frac{x^{5/2}}{5a(ax+bx^3)^{5/2}}}{a}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + (x^(5/2)/(5*a*(a*x + b*x^3)^(5/2)) + (
x^(3/2)/(3*a*(a*x + b*x^3)^(3/2)) + (Sqrt[x]/(a*Sqrt[a*x + b*x^3]) - ArcTa
nh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(3/2))/a)/a)/a
```

3.86.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(100) = 200$.

Time = 2.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\sqrt{x(bx^2+a)}\left(105\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)b^3x^6\sqrt{bx^2+a}-105\sqrt{a}\,b^3x^6+315\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)ab^2x^4\sqrt{bx^2+a}-350a^{\frac{3}{2}}b^2x^4+315\right.}{105a^{\frac{9}{2}}\sqrt{x}\,(bx^2+a)^4}$

input
```
int(x^(7/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(x*(b*x^2+a))^(1/2)/a^(9/2)*(105*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x
)*b^3*x^6*(b*x^2+a)^(1/2)-105*a^(1/2)*b^3*x^6+315*ln(2*(a^(1/2)*(b*x^2+a)^
(1/2)+a)/x)*a*b^2*x^4*(b*x^2+a)^(1/2)-350*a^(3/2)*b^2*x^4+315*ln(2*(a^(1/2
)*(b*x^2+a)^(1/2)+a)/x)*a^2*b*x^2*(b*x^2+a)^(1/2)-406*a^(5/2)*b*x^2+105*ln
(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*(b*x^2+a)^(1/2)-176*a^(7/2))/x^(1/2)
/(b*x^2+a)^4
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.77

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{105\,(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)+2}{210\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4}\right.$$

input
```
integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*
sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3) + 2
*(105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 +
a*x)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 +
 a^9*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 +
 a^4*x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (105*a*b
^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt
(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

3.86.6 Sympy [F]

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{(x(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(7/2)/(x*(a + b*x**2))**(9/2), x)
```

3.86.7 Maxima [F]

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/(b*x^3 + a*x)^(9/2), x)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} - \frac{105\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+176\sqrt{-a}}{105\sqrt{-a}a^{\frac{9}{2}}} + \frac{105\,(bx^2+a)^3+35\,(bx^2+a)^2a+21\,(bx^2+a)a^2+15\,a^3}{105\,(bx^2+a)^{\frac{7}{2}}a^4}$$

input
```
integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) - 1/105*(105*sqrt(a)*arcta
n(sqrt(a)/sqrt(-a)) + 176*sqrt(-a))/(sqrt(-a)*a^(9/2)) + 1/105*(105*(b*x^2
 + a)^3 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(
7/2)*a^4)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{7/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

input
```
int(x^(7/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(7/2)/(a*x + b*x^3)^(9/2), x)
```

3.87 $\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

3.87.1 Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}$$

output
```
1/7*x^(5/2)/a/(b*x^3+a*x)^(7/2)+8/35*x^(3/2)/a^2/(b*x^3+a*x)^(5/2)+16/35*x
^(1/2)/a^3/(b*x^3+a*x)^(3/2)+64/35/a^4/x^(1/2)/(b*x^3+a*x)^(1/2)-128/35*(b
*x^3+a*x)^(1/2)/a^5/x^(3/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{5/2}(-35a^4-280a^3bx^2-560a^2b^2x^4-448ab^3x^6-128b^4x^8)}{35a^5\left(x\left(a+bx^2\right)\right)^{7/2}}$$

input
```
Integrate[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(x^(5/2)*(-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*
b^4*x^8))/(35*a^5*(x*(a + b*x^2))^(7/2))
```

3.87.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1921, 1921, 1921, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\int \frac{x^{3/2}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{5/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\int \frac{\sqrt{x}}{(bx^3+ax)^{5/2}}dx}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\int \frac{1}{\sqrt{x}(bx^3+ax)^{3/2}}dx}{3a}+\frac{\sqrt{x}}{3a(ax+bx^3)^{3/2}}\right)}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{x^{3/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{a\sqrt{x}\sqrt{ax+bx^3}}\right)}{3a}+\frac{\sqrt{x}}{3a(ax+bx^3)^{3/2}}\right)}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a\,(ax+bx^3)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{1}{a\sqrt{x}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{a^2x^{3/2}}\right)}{3a}+\frac{\sqrt{x}}{3a\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{x^{3/2}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*(x^(3/2)/(5*a*(a*x + b*x^3)^(5/2))
+ (6*(Sqrt[x]/(3*a*(a*x + b*x^3)^(3/2)) + (4*(1/(a*Sqrt[x]*Sqrt[a*x + b*x^
3]) - (2*Sqrt[a*x + b*x^3])/(a^2*x^(3/2))))/(3*a)))/(5*a)))/(7*a)
```

3.87.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.87.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{x^{\frac{7}{2}}\left(b\,x^{2}+a\right)\left(128x^{8}b^{4}+448a\,b^{3}x^{6}+560a^{2}x^{4}b^{2}+280a^{3}b\,x^{2}+35a^{4}\right)}{35a^{5}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	70
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(128x^{8}b^{4}+448a\,b^{3}x^{6}+560a^{2}x^{4}b^{2}+280a^{3}b\,x^{2}+35a^{4}\right)}{35x^{\frac{3}{2}}\left(b\,x^{2}+a\right)^{4}a^{5}}$	72
risch	$-\frac{b\,x^{2}+a}{a^{5}\sqrt{x}\,\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{\left(b\,x^{2}+a\right)x^{\frac{3}{2}}\left(93b^{3}x^{6}+308a\,b^{2}x^{4}+350a^{2}b\,x^{2}+140a^{3}\right)b}{35\left(x^{8}b^{4}+4a\,b^{3}x^{6}+6a^{2}x^{4}b^{2}+4a^{3}b\,x^{2}+a^{4}\right)a^{5}\sqrt{x\left(b\,x^{2}+a\right)}}$	129

input
```
int(x^(5/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*x^(7/2)*(b*x^2+a)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3
*b*x^2+35*a^4)/a^5/(b*x^3+a*x)^(9/2)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4)\sqrt{bx^3+ax}\sqrt{x}}{35\,(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2)}$$

input
```
integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*
a^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x
^6 + 4*a^8*b*x^4 + a^9*x^2)
```

3.87.6 Sympy [F]

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(5/2)/(x*(a + b*x**2))**(9/2), x)
```

3.87.7 Maxima [F]

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}} + \frac{2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)
*x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^
4)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{5/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(5/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(5/2)/(a*x + b*x^3)^(9/2), x)
```

3.88 $\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$

3.88.1 Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{3/2}}{7a(ax+bx^3)^{7/2}}+\frac{9\sqrt{x}}{35a^2(ax+bx^3)^{5/2}}+\frac{3}{5a^3\sqrt{x}\,(ax+bx^3)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}$$

output
```
1/7*x^(3/2)/a/(b*x^3+a*x)^(7/2)+9/2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^
(1/2))/a^(11/2)+3/5/a^3/(b*x^3+a*x)^(3/2)/x^(1/2)+9/35*x^(1/2)/a^2/(b*x^3+
a*x)^(5/2)+3/a^4/x^(3/2)/(b*x^3+a*x)^(1/2)-9/2*(b*x^3+a*x)^(1/2)/a^5/x^(5/
2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{x(a+bx^2)}\left(-\sqrt{a}(35a^4+528a^3bx^2+1218a^2b^2x^4+1050ab^3x^6+315b^4x^8)+315bx\right.}{70a^{11/2}x^{5/2}(a+bx^2)^4}$$

input
```
Integrate[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
(Sqrt[x*(a + b*x^2)]*(-(Sqrt[a]*(35*a^4 + 528*a^3*b*x^2 + 1218*a^2*b^2*x^4
 + 1050*a*b^3*x^6 + 315*b^4*x^8)) + 315*b*x^2*(a + b*x^2)^(7/2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]))/(70*a^(11/2)*x^(5/2)*(a + b*x^2)^4)
```

3.88.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1929, 1929, 1929, 1929, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$$

$$\Big\downarrow \ 1929$$

$$\frac{9\int \frac{\sqrt{x}}{(bx^3+ax)^{7/2}}dx}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \ 1929$$

$$\frac{9\left(\frac{7\int \frac{1}{\sqrt{x}\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \ 1929$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{x^{3/2}\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \ 1929$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{x^{5/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1931

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{\sqrt{x}\sqrt{bx^3+ax}}dx}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1935

3.88. $\int\frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{1-\frac{ax}{bx^3+ax}}d\frac{\sqrt{x}}{\sqrt{bx^3+ax}}}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{3/2}}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{a}+\frac{1}{ax^{3/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3a\sqrt{x}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{\sqrt{x}}{5a(ax+bx^3)^{5/2}}\right)}{7a}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

3.88. $\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$

input
```
Int[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

output
```
x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*(Sqrt[x]/(5*a*(a*x + b*x^3)^(5/2))
+ (7*(1/(3*a*Sqrt[x]*(a*x + b*x^3)^(3/2)) + (5*(1/(a*x^(3/2)*Sqrt[a*x + b*
x^3]) + (3*(-1/2*Sqrt[a*x + b*x^3]/(a*x^(5/2)) + (b*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[a*x + b*x^3]])/(2*a^(3/2))))/a))/(3*a)))/(5*a)))/(7*a)
```

3.88.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.88.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{x(b\,x^2+a)}\left(315\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^4x^8\sqrt{b\,x^2+a}-315\sqrt{a}\,b^4x^8+945\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\,b^3x^6\sqrt{b\,x^2+a}-1050a^{\frac{3}{2}}b^3x^6+945\ln\right.}{70a^{\frac{11}{2}}x^{\frac{5}{2}}(b\,x^2+a)}$
risch	$-\frac{b\,x^2+a}{2a^5x^{\frac{3}{2}}\sqrt{x(b\,x^2+a)}}+\frac{\left(\frac{9b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{11}{2}}}+\frac{2629b\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{1120a^5\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}-\frac{2629b\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\left(x-\right.}}{1120a^5\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\right.}{}$

input
```
int(x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/70*(x*(b*x^2+a))^(1/2)/a^(11/2)*(315*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)
*b^4*x^8*(b*x^2+a)^(1/2)-315*a^(1/2)*b^4*x^8+945*ln(2*(a^(1/2)*(b*x^2+a)^(
1/2)+a)/x)*a*b^3*x^6*(b*x^2+a)^(1/2)-1050*a^(3/2)*b^3*x^6+945*ln(2*(a^(1/2
)*(b*x^2+a)^(1/2)+a)/x)*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*a^(5/2)*b^2*x^4+3
15*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b*x^2*(b*x^2+a)^(1/2)-528*a^(7/
2)*b*x^2-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.49

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{315\,(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3)\sqrt{a}\log\left(\frac{bx^3+2\,ax+2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)}{140\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8}\right.$$

input
```
integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*
b*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x
^3) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x
^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*
a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9
+ 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 +
a*x)*sqrt(-a)/(a*sqrt(x))) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*
b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11
 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

3.88.6 Sympy [F]

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(x**(3/2)/(x*(a + b*x**2))**(9/2), x)
```

3.88.7 Maxima [F]

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^5} - \frac{\sqrt{bx^2+a}}{2\,a^5x^2} - \frac{140\,(bx^2+a)^3b + 35\,(bx^2+a)^2ab + 14\,(bx^2+a)a^2b + 5\,a^3b}{35\,(bx^2+a)^{\frac{7}{2}}a^5}$$

input
```
integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 +
a)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^
2 + a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{3/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(x^(3/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^3)^(9/2), x)
```

3.89 $\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx$

3.89.1 Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{x}}{7a(ax+bx^3)^{7/2}} + \frac{2}{7a^2\sqrt{x}(ax+bx^3)^{5/2}} + \frac{16}{21a^3x^{3/2}(ax+bx^3)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}} + \frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}$$

output
```
16/21/a^3/x^(3/2)/(b*x^3+a*x)^(3/2)+2/7/a^2/(b*x^3+a*x)^(5/2)/x^(1/2)+1/7*
x^(1/2)/a/(b*x^3+a*x)^(7/2)+32/7/a^4/x^(5/2)/(b*x^3+a*x)^(1/2)-128/21*(b*x
^3+a*x)^(1/2)/a^5/x^(7/2)+256/21*b*(b*x^3+a*x)^(1/2)/a^6/x^(3/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{x}(-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10})}{21a^6(x(a+bx^2))^{7/2}}$$

input
```
Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

output
```
(Sqrt[x]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896
*a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*(x*(a + b*x^2))^(7/2))
```

3.89.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1921, 1921, 1921, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\int \frac{1}{\sqrt{x}\left(bx^3+ax\right)^{7/2}}dx}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\left(\frac{8\int \frac{1}{x^{3/2}\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{1}{5a\sqrt{x}\left(ax+bx^3\right)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\left(\frac{8\left(\frac{2\int \frac{1}{x^{5/2}\left(bx^3+ax\right)^{3/2}}dx}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}\left(ax+bx^3\right)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\left(\frac{8\left(\frac{2\left(\frac{4\int \frac{1}{x^{7/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{5/2}\sqrt{ax+bx^3}}\right)}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}\left(ax+bx^3\right)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{8\left(\frac{2\left(\frac{4\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{bx^3+ax}}dx}{3a}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{a}+\frac{1}{ax^{5/2}\sqrt{ax+bx^3}}\right)}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{10\left(\frac{8\left(\frac{2\left(\frac{4\left(\frac{2b\sqrt{ax+bx^3}}{3a^2x^{3/2}}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{a}+\frac{1}{ax^{5/2}\sqrt{ax+bx^3}}\right)}{a}+\frac{1}{3ax^{3/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5a\sqrt{x}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

output
```
Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + (10*(1/(5*a*Sqrt[x]*(a*x + b*x^3)^(5/2
)) + (8*(1/(3*a*x^(3/2)*(a*x + b*x^3)^(3/2)) + (2*(1/(a*x^(5/2)*Sqrt[a*x +
 b*x^3]) + (4*(-1/3*Sqrt[a*x + b*x^3]/(a*x^(7/2)) + (2*b*Sqrt[a*x + b*x^3]
)/(3*a^2*x^(3/2))))/a))/a))/(5*a)))/(7*a)
```

3.89. $\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx$

3.89.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.89.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{x^{\frac{3}{2}} \left(b \,x^{2}+a \right) \left(-256 b^{5} x^{10}-896 a \,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b \,x^{2}+7 a^{5}\right)}{21 a^{6} \left(b \,x^{3}+a x \right)^{\frac{9}{2}}}$	81
default	$-\frac{\sqrt{x \left(b \,x^{2}+a \right)}\, \left(-256 b^{5} x^{10}-896 a \,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b \,x^{2}+7 a^{5}\right)}{21 x^{\frac{7}{2}} \left(b \,x^{2}+a \right)^{4} a^{6}}$	83
risch	$-\frac{\left(b \,x^{2}+a \right) \left(-14 b \,x^{2}+a \right)}{3 a^{6} x^{\frac{5}{2}} \sqrt{x \left(b \,x^{2}+a \right)}}+\frac{\left(b \,x^{2}+a \right) x^{\frac{3}{2}} \left(158 b^{3} x^{6}+511 a \,b^{2} x^{4}+560 a^{2} b \,x^{2}+210 a^{3}\right) b^{2}}{21 a^{6} \left(x^{8} b^{4}+4 a \,b^{3} x^{6}+6 a^{2} x^{4} b^{2}+4 a^{3} b \,x^{2}+a^{4}\right) \sqrt{x \left(b \,x^{2}+a \right)}}$	139

input
```
int(x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x^(3/2)*(b*x^2+a)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*
a^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/a^6/(b*x^3+a*x)^(9/2)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5)\sqrt{bx^3+ax}\sqrt{x}}{21\,(a^6b^4x^{12}+4\,a^7b^3x^{10}+6\,a^8b^2x^8+4\,a^9bx^6+a^{10}x^4)}$$

input
```
integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 +
70*a^4*b*x^2 - 7*a^5)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^6*b^4*x^12 + 4*a^7*b^3*
x^10 + 6*a^8*b^2*x^8 + 4*a^9*b*x^6 + a^10*x^4)
```

3.89.6 Sympy [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(sqrt(x)/(x*(a + b*x**2))**(9/2), x)
```

3.89.7 Maxima [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\,(bx^2+a)^{\frac{7}{2}}} - \frac{4\left(6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}} - 15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ab^{\frac{3}{2}} + 7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3 a^5}$$

input
```
integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^
3)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) -
15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^3)^(9/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^3)^(9/2), x)
```

3.90 $\int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}}\,dx$

3.90.1 Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx=\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}+\frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{99b^2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}$$

output
```
11/35/a^2/x^(3/2)/(b*x^3+a*x)^(5/2)+33/35/a^3/x^(5/2)/(b*x^3+a*x)^(3/2)-99
/8*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(13/2)+1/7/a/(b*x^3+a*
x)^(7/2)/x^(1/2)+33/5/a^4/x^(7/2)/(b*x^3+a*x)^(1/2)-33/4*(b*x^3+a*x)^(1/2)
/a^5/x^(9/2)+99/8*b*(b*x^3+a*x)^(1/2)/a^6/x^(5/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx=\frac{\sqrt{x\left(a+bx^2\right)}\left(\sqrt{a}(-70a^5+385a^4bx^2+5808a^3b^2x^4+13398a^2b^3x^6+11550ab^4x^8\right.}{280a^{13/2}x^{9/2}\left(a+bx^2\right)^4}$$

input
```
Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

output
```
(Sqrt[x*(a + b*x^2)]*(Sqrt[a]*(-70*a^5 + 385*a^4*b*x^2 + 5808*a^3*b^2*x^4
+ 13398*a^2*b^3*x^6 + 11550*a*b^4*x^8 + 3465*b^5*x^10) - 3465*b^2*x^4*(a +
 b*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(280*a^(13/2)*x^(9/2)*(a
+ b*x^2)^4)
```

3.90.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1929, 1929, 1929, 1929, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{11\int \frac{1}{x^{3/2}(bx^3+ax)^{7/2}}dx}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{11\left(\frac{9\int \frac{1}{x^{5/2}\left(bx^3+ax\right)^{5/2}}dx}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{x^{7/2}\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{x^{9/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\int \frac{1}{x^{5/2}\sqrt{bx^3+ax}}dx}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{ 1931}$$

3.90. $\int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}}\,dx$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{x}\sqrt{bx^3+ax}}dx}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$$\downarrow 1935$$

3.90. $\int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}}\,dx$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax}{bx^3+ax}}d\frac{\sqrt{x}}{\sqrt{bx^3+ax}}}{2a}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

3.90. $\int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}}\,dx$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{3/2}}-\frac{\sqrt{ax+bx^3}}{2ax^{5/2}}\right)}{4a}-\frac{\sqrt{ax+bx^3}}{4ax^{9/2}}\right)}{a}+\frac{1}{ax^{7/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{5/2}\left(ax+bx^3\right)^{3/2}}\right)}{5a}+\frac{1}{5ax^{3/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

3.90. $\int \frac{1}{\sqrt{x}(ax+bx^3)^{9/2}}\,dx$

output
```
1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + (11*(1/(5*a*x^(3/2)*(a*x + b*x^3)^(5
/2)) + (9*(1/(3*a*x^(5/2)*(a*x + b*x^3)^(3/2)) + (7*(1/(a*x^(7/2)*Sqrt[a*x
 + b*x^3]) + (5*(-1/4*Sqrt[a*x + b*x^3]/(a*x^(9/2)) - (3*b*(-1/2*Sqrt[a*x
+ b*x^3]/(a*x^(5/2)) + (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(2
*a^(3/2))))/(4*a)))/a))/(3*a)))/(5*a)))/(7*a)
```

3.90.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.90.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\sqrt{x(b\,x^2+a)}\left(3465\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^5x^{10}\sqrt{b\,x^2+a}-3465\sqrt{a}\,b^5x^{10}+10395\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\,b^4x^8\sqrt{b\,x^2+a}-11550a^{\frac{3}{2}}b\right.}{28}$
risch	$-\frac{(b\,x^2+a)(-19b\,x^2+2a)}{8a^6x^{\frac{7}{2}}\sqrt{x(b\,x^2+a)}}+\frac{\left(-\frac{99b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{8a^{\frac{13}{2}}}-\frac{6311b^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{1120a^6\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{6311b^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2}}{1120a^6\sqrt{-ab}\left(x-\right.}\right.}{}$

input
```
int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(x*(b*x^2+a))^(1/2)/a^(13/2)*(3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)
/x)*b^5*x^10*(b*x^2+a)^(1/2)-3465*a^(1/2)*b^5*x^10+10395*ln(2*(a^(1/2)*(b*
x^2+a)^(1/2)+a)/x)*a*b^4*x^8*(b*x^2+a)^(1/2)-11550*a^(3/2)*b^4*x^8+10395*l
n(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^2*b^3*x^6*(b*x^2+a)^(1/2)-13398*a^(5/
2)*b^3*x^6+3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b^2*x^4*(b*x^2+a)^
(1/2)-5808*a^(7/2)*b^2*x^4-385*a^(9/2)*b*x^2+70*a^(11/2))/x^(9/2)/(b*x^2+a
)^4
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \left[\frac{3465\,(b^6x^{13}+4\,ab^5x^{11}+6\,a^2b^4x^9+4\,a^3b^3x^7+a^4b^2x^5)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}}{x^3}\right.}{560\,(a^7b^4x^{13}+}\right.$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^
4*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x
))/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 580
8*a^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^
4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*
(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x
^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (3465*a*b^5*
x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*
b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11
+ 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

3.90.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{\sqrt{x}\,(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(1/(sqrt(x)*(x*(a + b*x**2))**(9/2)), x)
```

3.90.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{9}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \frac{99\,b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^6} + \frac{350\,(bx^2+a)^3b^2+70\,(bx^2+a)^2ab^2+21\,(bx^2+a)a^2b^2+5\,a^3b^2}{35\,(bx^2+a)^{\frac{7}{2}}a^6} + \frac{19\,(bx^2+a)^{\frac{3}{2}}b^2-21\,\sqrt{bx^2+a}ab^2}{8\,a^6b^2x^4}$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
99/8*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 1/35*(350*(b*x^
2 + a)^3*b^2 + 70*(b*x^2 + a)^2*a*b^2 + 21*(b*x^2 + a)*a^2*b^2 + 5*a^3*b^2
)/((b*x^2 + a)^(7/2)*a^6) + 1/8*(19*(b*x^2 + a)^(3/2)*b^2 - 21*sqrt(b*x^2
+ a)*a*b^2)/(a^6*b^2*x^4)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{\sqrt{x}\,(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)), x)
```

3.91 $\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx$

3.91.1 Optimal result

Integrand size = 19, antiderivative size = 180

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}} + \frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}} - \frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}$$

output
```
1/7/a/x^(3/2)/(b*x^3+a*x)^(7/2)+12/35/a^2/x^(5/2)/(b*x^3+a*x)^(5/2)+8/7/a^
3/x^(7/2)/(b*x^3+a*x)^(3/2)+64/7/a^4/x^(9/2)/(b*x^3+a*x)^(1/2)-384/35*(b*x
^3+a*x)^(1/2)/a^5/x^(11/2)+512/35*b*(b*x^3+a*x)^(1/2)/a^6/x^(7/2)-1024/35*
b^2*(b*x^3+a*x)^(1/2)/a^7/x^(3/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{-7a^6+28a^5bx^2-280a^4b^2x^4-2240a^3b^3x^6-4480a^2b^4x^8-3584ab^5x^{10}-1024b^6}{35a^7x^{3/2}\left(x\left(a+bx^2\right)\right)^{7/2}}$$

input
```
Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

output
```
(-7*a^6 + 28*a^5*b*x^2 - 280*a^4*b^2*x^4 - 2240*a^3*b^3*x^6 - 4480*a^2*b^4
*x^8 - 3584*a*b^5*x^10 - 1024*b^6*x^12)/(35*a^7*x^(3/2)*(x*(a + b*x^2))^(7
/2))
```

3.91.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1921, 1921, 1921, 1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(ax+bx^3\right)^{9/2}} \, dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{12 \int \frac{1}{x^{5/2}\left(bx^3+ax\right)^{7/2}}dx}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\left(\frac{2\int \frac{1}{x^{7/2}\left(bx^3+ax\right)^{5/2}}dx}{a}+\frac{1}{5ax^{5/2}\left(ax+bx^3\right)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\left(\frac{2\left(\frac{8\int \frac{1}{x^{9/2}\left(bx^3+ax\right)^{3/2}}dx}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}\left(ax+bx^3\right)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\int \frac{1}{x^{11/2}\sqrt{bx^3+ax}}dx}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \; 1922$$

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\left(-\frac{4b\int \frac{1}{x^{7/2}\sqrt{bx^3+ax}}dx}{5a}-\frac{\sqrt{ax+bx^3}}{5ax^{11/2}}\right)}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$$\Big\downarrow \; 1922$$

3.91. $\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx$

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\left(-\frac{4b\left(-\frac{2b\int\frac{1}{x^{3/2}\sqrt{bx^3+ax}}\,dx}{3a}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{5a}-\frac{\sqrt{ax+bx^3}}{5ax^{11/2}}\right)}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

$\downarrow$ 1920

3.91. $\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx$

$$\frac{12\left(\frac{2\left(\frac{8\left(\frac{6\left(-\frac{4b\left(\frac{2b\sqrt{ax+bx^3}}{3a^2x^{3/2}}-\frac{\sqrt{ax+bx^3}}{3ax^{7/2}}\right)}{5a}-\frac{\sqrt{ax+bx^3}}{5ax^{11/2}}\right)}{a}+\frac{1}{ax^{9/2}\sqrt{ax+bx^3}}\right)}{3a}+\frac{1}{3ax^{7/2}\left(ax+bx^3\right)^{3/2}}\right)}{a}+\frac{1}{5ax^{5/2}(ax+bx^3)^{5/2}}\right)}{7a}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

input
```
Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

output
```
1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + (12*(1/(5*a*x^(5/2)*(a*x + b*x^3)^(5
/2)) + (2*(1/(3*a*x^(7/2)*(a*x + b*x^3)^(3/2)) + (8*(1/(a*x^(9/2)*Sqrt[a*x
 + b*x^3]) + (6*(-1/5*Sqrt[a*x + b*x^3]/(a*x^(11/2)) - (4*b*(-1/3*Sqrt[a*x
 + b*x^3]/(a*x^(7/2)) + (2*b*Sqrt[a*x + b*x^3])/(3*a^2*x^(3/2))))/(5*a)))/
a))/(3*a)))/a))/(7*a)
```

3.91.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.91.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(1024 b^{6} x^{12}+3584 a\,b^{5} x^{10}+4480 a^{2} b^{4} x^{8}+2240 a^{3} b^{3} x^{6}+280 a^{4} b^{2} x^{4}-28 a^{5} b\,x^{2}+7 a^{6}\right)}{35 \sqrt{x}\, a^{7} \left(b\,x^{3}+a x \right)^{\frac{9}{2}}}$	92
default	$-\frac{\sqrt{x \left(b\,x^{2}+a \right)}\,\left(1024 b^{6} x^{12}+3584 a\,b^{5} x^{10}+4480 a^{2} b^{4} x^{8}+2240 a^{3} b^{3} x^{6}+280 a^{4} b^{2} x^{4}-28 a^{5} b\,x^{2}+7 a^{6}\right)}{35 x^{\frac{11}{2}} \left(b\,x^{2}+a \right)^{4} a^{7}}$	94
risch	$-\frac{\left(b\,x^{2}+a \right)\left(66 b^{2} x^{4}-8 a b\,x^{2}+a^{2}\right)}{5 a^{7} x^{\frac{9}{2}} \sqrt{x \left(b\,x^{2}+a \right)}}-\frac{\left(b\,x^{2}+a \right) x^{\frac{3}{2}} \left(562 b^{3} x^{6}+1792 a\,b^{2} x^{4}+1925 a^{2} b\,x^{2}+700 a^{3}\right) b^{3}}{35 a^{7} \left(x^{8} b^{4}+4 a\,b^{3} x^{6}+6 a^{2} x^{4} b^{2}+4 a^{3} b\,x^{2}+a^{4}\right) \sqrt{x \left(b\,x^{2}+a \right)}}$	150

input
```
int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*b^4*x^8+2240*a^3*b
^3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/x^(1/2)/a^7/(b*x^3+a*x)^(9/2)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{3/2} (ax + bx^3)^{9/2}} \, dx = -\frac{(1024\, b^6x^{12} + 3584\, ab^5x^{10} + 4480\, a^2b^4x^8 + 2240\, a^3b^3x^6 + 280\, a^4b^2x^4 - 28\, a^5bx^2 + 7\, a^6)\sqrt{bx^3 + ax}\sqrt{x}}{35\, (a^7b^4x^{14} + 4\, a^8b^3x^{12} + 6\, a^9b^2x^{10} + 4\, a^{10}bx^8 + a^{11}x^6)}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x
^6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^
7*b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)
```

3.91.6 Sympy [F]

$$\int \frac{1}{x^{3/2} (ax + bx^3)^{9/2}} \, dx = \int \frac{1}{x^{\frac{3}{2}} (x (a + bx^2))^{\frac{9}{2}}} \, dx$$

input
```
integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

output
```
Integral(1/(x**(3/2)*(x*(a + b*x**2))**(9/2)), x)
```

3.91.7 Maxima [F]

$$\int \frac{1}{x^{3/2} (ax + bx^3)^{9/2}} \, dx = \int \frac{1}{(bx^3 + ax)^{\frac{9}{2}} x^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx = -\frac{\left(\left(2\,x^2\left(\frac{281\,b^6x^2}{a^7}+\frac{896\,b^5}{a^6}\right)+\frac{1925\,b^4}{a^5}\right)x^2+\frac{700\,b^3}{a^4}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}} + \frac{4\left(25\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 b^{\frac{5}{2}} - 120\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 ab^{\frac{5}{2}} + 210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 a^2b^{\frac{5}{2}} - 140\left(\sqrt{b}x - \cdots\right.\right.}{5\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5 a^6}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^
3/a^4)*x/(b*x^2 + a)^(7/2) + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/
2) - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2) + 210*(sqrt(b)*x - sqrt
(b*x^2 + a))^4*a^2*b^(5/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/
2) + 33*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{x^{3/2}\,(b\,x^3+a\,x)^{9/2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)), x)
```

3.92 $\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$

3.92.1 Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

output
```
-1/3*a*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(3/2)+1/3*x*(b*x^4+a*x)^(1
/2)/b
```

3.92.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \frac{\sqrt{b}x^2(a+bx^3) - a\sqrt{x}\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x(a+bx^3)}}$$

input
```
Integrate[x^4/Sqrt[a*x + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) +
 Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

3.92.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\int \frac{x}{\sqrt{bx^4+ax}}dx}{2b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax}}d\frac{x^2}{\sqrt{bx^4+ax}}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

input
```
Int[x^4/Sqrt[a*x + b*x^4],x]
```

output
```
(x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])
/(3*b^(3/2))
```

3.92.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.92.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{-x\sqrt{b}\,\sqrt{x(b\,x^3+a)}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)a}{3b^{\frac{3}{2}}}$	45
pseudoelliptic	$-\frac{-x\sqrt{b}\,\sqrt{x(b\,x^3+a)}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)a}{3b^{\frac{3}{2}}}$	45
risch	$\frac{x^2\left(b\,x^3+a\right)}{3b\sqrt{x\left(b\,x^3+a\right)}}-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)}{3b^{\frac{3}{2}}}$	53
elliptic	Expression too large to display	997

input
```
int(x^4/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^(3/2)*(-x*b^(1/2)*(x*(b*x^3+a))^(1/2)+arctanh(1/x^2*(x*(b*x^3+a))^(
1/2)/b^(1/2))*a)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$$
$$= \left[\frac{4\sqrt{bx^4+ax}bx + a\sqrt{b}\log\left(-8\,b^2x^6 - 8\,abx^3 - a^2 + 4\,(2\,bx^4+ax)\sqrt{bx^4+ax}\sqrt{b}\right)}{12\,b^2}, \frac{2\sqrt{bx^4+ax}bx + a\sqrt{}}{}\right.$$

input
```
integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^
2 + 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 +
 a*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)
))/b^2]
```

3.92.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^4}{\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

3.92.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^4 + a*x), x)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \frac{\sqrt{bx^4+axx}}{3\,b} + \frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}b}$$

input
```
integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-
b)*b)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^4}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^4/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^4/(a*x + b*x^4)^(1/2), x)
```

3.93 $\int \frac{x}{\sqrt{ax+bx^4}}\,dx$

3.93.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx=\frac{2\sqrt{x}\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[x/Sqrt[a*x + b*x^4],x]
```

output
```
(2*Sqrt[x]*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt
[b]*Sqrt[x*(a + b*x^3)])
```

3.93.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx$$
$$\Big\downarrow \ 1935$$
$$\frac{2}{3}\int \frac{1}{1-\frac{bx^4}{bx^4+ax}}d\frac{x^2}{\sqrt{bx^4+ax}}$$
$$\Big\downarrow \ 219$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

input
```
Int[x/Sqrt[a*x + b*x^4],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

3.93.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.93.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b}}$	25
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b}}$	25
elliptic	Expression too large to display	979

input
```
int(x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b^(1/2)*arctanh(1/x^2*(x*(b*x^3+a))^(1/2)/b^(1/2))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.94

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\,(2\,bx^4+ax)\sqrt{bx^4+ax}\sqrt{b}\right)}{6\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^4+ax}\sqrt{-b}x}{2\,bx^3+a}\right)}{3\,b}\right]$$

input
```
integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x
)*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2
*b*x^3 + a))/b]
```

3.93.6 Sympy [F]

$$\int \frac{x}{\sqrt{ax + bx^4}}\, dx = \int \frac{x}{\sqrt{x\,(a + bx^3)}}\, dx$$

input
```
integrate(x/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(x*(a + b*x**3)), x)
```

3.93.7 Maxima [F]

$$\int \frac{x}{\sqrt{ax + bx^4}}\, dx = \int \frac{x}{\sqrt{bx^4 + ax}}\, dx$$

input
```
integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^4 + a*x), x)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{ax + bx^4}}\, dx = -\frac{2\,\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}$$

input
```
integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx=\int \frac{x}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x/(a*x + b*x^4)^(1/2), x)
```

3.94 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$

3.94.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

output
```
-2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

3.94.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{x(a+bx^3)}}{3ax^2}$$

input
```
Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)
```

3.94.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$$
$$\Big\downarrow \text{ 1920}$$
$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

input
```
Int[1/(x^2*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

3.94.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.94.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
trager	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
elliptic	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
pseudoelliptic	$-\frac{2\sqrt{x\left(b\,x^{3}+a\right)}}{3 a\,x^{2}}$	20
gosper	$-\frac{2\left(b\,x^{3}+a\right)}{3 x a \sqrt{b\,x^{4}+a x}}$	27
risch	$-\frac{2\left(b\,x^{3}+a\right)}{3 a x \sqrt{x\left(b\,x^{3}+a\right)}}$	27

input
```
int(1/x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{bx^4+ax}}{3\,ax^2}$$

input
```
integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^4 + a*x)/(a*x^2)
```

3.94.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^2\sqrt{x\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**2/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\left(bx^4+ax\right)}{3\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

input
```
integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}$$

input
```
integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b + a/x^3)/a
```

3.94.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx=-\frac{2\,\sqrt{b\,x^4+a\,x}}{3\,a\,x^2}$$

input
```
int(1/(x^2*(a*x + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^4)^(1/2))/(3*a*x^2)
```

3.95 $\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx$

3.95.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}$$

output
```
-2/9*(b*x^4+a*x)^(1/2)/a/x^5+4/9*b*(b*x^4+a*x)^(1/2)/a^2/x^2
```

3.95.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx=-\frac{2(a-2bx^3)\sqrt{x(a+bx^3)}}{9a^2x^5}$$

input
```
Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)
```

3.95.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}dx$$
$$\Big\downarrow \text{1922}$$
$$-\frac{2b\int \frac{1}{x^2\sqrt{bx^4+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$
$$\Big\downarrow \text{1920}$$
$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

input
```
Int[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

3.95.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.95.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

method	result	size
trager	$-\frac{2(-2b\,x^3+a)\sqrt{b\,x^4+ax}}{9x^5a^2}$	28
pseudoelliptic	$-\frac{2(-2b\,x^3+a)\sqrt{x(b\,x^3+a)}}{9x^5a^2}$	28
gosper	$-\frac{2(b\,x^3+a)(-2b\,x^3+a)}{9x^4a^2\sqrt{b\,x^4+ax}}$	35
risch	$-\frac{2(b\,x^3+a)(-2b\,x^3+a)}{9a^2x^4\sqrt{x(b\,x^3+a)}}$	35
default	$-\frac{2\sqrt{b\,x^4+ax}}{9a\,x^5}+\frac{4b\sqrt{b\,x^4+ax}}{9a^2x^2}$	41
elliptic	$-\frac{2\sqrt{b\,x^4+ax}}{9a\,x^5}+\frac{4b\sqrt{b\,x^4+ax}}{9a^2x^2}$	41

input
```
int(1/x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-2*b*x^3+a)/x^5/a^2*(b*x^4+a*x)^(1/2)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \frac{2\,\sqrt{bx^4+ax}(2\,bx^3-a)}{9\,a^2x^5}$$

input
```
integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)
```

3.95.6 Sympy [F]

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^5\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**5/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \frac{2\,(2\,b^2x^7+abx^4-a^2x)}{9\,\sqrt{bx^3+a}a^2x^{\frac{11}{2}}}$$

input
```
integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2\,\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\,\sqrt{b+\frac{a}{x^3}}b}{3\,a^2}$$

input
```
integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2
```

3.95.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x}\,(a-2\,b\,x^3)}{9\,a^2\,x^5}$$

input
```
int(1/(x^5*(a*x + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^4)^(1/2)*(a - 2*b*x^3))/(9*a^2*x^5)
```

3.96 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$

3.96.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}$$

output
```
-2/15*(b*x^4+a*x)^(1/2)/a/x^8+8/45*b*(b*x^4+a*x)^(1/2)/a^2/x^5-16/45*b^2*(
b*x^4+a*x)^(1/2)/a^3/x^2
```

3.96.2 Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{x(a+bx^3)}(3a^2-4abx^3+8b^2x^6)}{45a^3x^8}$$

input
```
Integrate[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

3.96.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\int \frac{1}{x^5\sqrt{bx^4+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{x^2\sqrt{bx^4+ax}}dx}{3a}-\frac{2\sqrt{ax+bx^4}}{9ax^5}\right)}{5a}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{4b\left(\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}\right)}{5a}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

input
```
Int[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(15*a*x^8) - (4*b*((-2*Sqrt[a*x + b*x^4])/(9*a*x^5)
 + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)))/(5*a)
```

3.96.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.96.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
trager	$-\frac{2\left(8 b^{2} x^{6}-4 a b \,x^{3}+3 a^{2}\right) \sqrt{b \,x^{4}+a x}}{45 x^{8} a^{3}}$	41
pseudoelliptic	$-\frac{2\left(8 b^{2} x^{6}-4 a b \,x^{3}+3 a^{2}\right) \sqrt{x \left(b \,x^{3}+a \right)}}{45 x^{8} a^{3}}$	41
gosper	$-\frac{2\left(b \,x^{3}+a \right) \left(8 b^{2} x^{6}-4 a b \,x^{3}+3 a^{2}\right)}{45 x^{7} a^{3} \sqrt{b \,x^{4}+a x}}$	48
risch	$-\frac{2\left(b \,x^{3}+a \right) \left(8 b^{2} x^{6}-4 a b \,x^{3}+3 a^{2}\right)}{45 a^{3} x^{7} \sqrt{x \left(b \,x^{3}+a \right)}}$	48
default	$-\frac{2 \sqrt{b \,x^{4}+a x}}{15 a \,x^{8}}+\frac{8 b \sqrt{b \,x^{4}+a x}}{45 a^{2} x^{5}}-\frac{16 b^{2} \sqrt{b \,x^{4}+a x}}{45 a^{3} x^{2}}$	63
elliptic	$-\frac{2 \sqrt{b \,x^{4}+a x}}{15 a \,x^{8}}+\frac{8 b \sqrt{b \,x^{4}+a x}}{45 a^{2} x^{5}}-\frac{16 b^{2} \sqrt{b \,x^{4}+a x}}{45 a^{3} x^{2}}$	63

input
```
int(1/x^8/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(8*b^2*x^6-4*a*b*x^3+3*a^2)/x^8/a^3*(b*x^4+a*x)^(1/2)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,(8\,b^2x^6-4\,abx^3+3\,a^2)\sqrt{bx^4+ax}}{45\,a^3x^8}$$

input
```
integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)
```

3.96.6 Sympy [F]

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^8\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**8/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,(8\,b^3x^{10}+4\,ab^2x^7-a^2bx^4+3\,a^3x)}{45\,\sqrt{bx^3+a}a^3x^{\frac{17}{2}}}$$

input
```
integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
-2/45*(8*b^3*x^10 + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^
3*x^(17/2))
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{b+\frac{a}{x^3}}b^2}{3\,a^3} - \frac{2\left(3\,\left(b+\frac{a}{x^3}\right)^{\frac{5}{2}}-10\,\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}b\right)}{45\,a^3}$$

input
```
integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b + a/x^3)*b^2/a^3 - 2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^
(3/2)*b)/a^3
```

3.96.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{b\,x^4+a\,x}\,\left(3\,a^2-4\,a\,b\,x^3+8\,b^2\,x^6\right)}{45\,a^3\,x^8}$$

input
```
int(1/(x^8*(a*x + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^6 - 4*a*b*x^3))/(45*a^3*x^8)
```

3.97 $\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx$

3.97.1 Optimal result

Integrand size = 17, antiderivative size = 224

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
1/2*(b*x^4+a*x)^(1/2)/b-1/12*a^(2/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^
(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.97.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.29

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \frac{x\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[x^3/Sqrt[a*x + b*x^4],x]
```

output
```
(x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((
b*x^3)/a)]))/(2*b*Sqrt[x*(a + b*x^3)])
```

3.97.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1930, 1917, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^4+ax}}dx}{4b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{4b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{851}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

input
```
Int[x^3/Sqrt[a*x + b*x^4],x]
```

output
```
Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) +
b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

3.97.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.97.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.07

method	result
default	$\frac{\sqrt{b x^4+ax}}{2b} - \frac{a\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b} - \frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-ab^2)^{\frac{1}{3}}\left(x \cdots\right.}{b\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b} - \cdots\right.}}\cdots}{2\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\cdots}{\cdots}\right.\cdots}$
elliptic	$\frac{\sqrt{b x^4+ax}}{2b} - \frac{a\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b} - \frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-ab^2)^{\frac{1}{3}}\left(x \cdots\right.}{b\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b} - \cdots\right.}}\cdots}{2\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\cdots}{\cdots}\right.\cdots}$
risch	$\frac{x(b x^3+a)}{2b\sqrt{x(b x^3+a)}} - \frac{a\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b} - \frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-ab^2 \cdots}{b\left(-\frac{(-ab^2 \cdots}{2b}\right.\cdots}}\cdots}{2\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+ i \cdots\right.}$

input
```
int(x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^4+a*x)^(1/2)/b-1/2*a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.97.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a*x)*x^2/(b*x^3 + a), x)
```

3.97.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^3}{\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(x**3/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x*(a + b*x**3)), x)
```

3.97.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(b*x^4 + a*x), x)
```

3.97.8 Giac [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(b*x^4 + a*x), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^3/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^4)^(1/2), x)
```

3.98 $\int \frac{1}{\sqrt{ax+bx^4}}\,dx$

3.98.1 Optimal result

Integrand size = 13, antiderivative size = 197

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
1/3*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a
^(1/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx=\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[1/Sqrt[a*x + b*x^4],x]
```

output
```
(2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/S
qrt[x*(a + b*x^3)]
```

3.98.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1917, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{766}$$

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

input
```
Int[1/Sqrt[a*x + b*x^4],x]
```

output
```
(x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sq
rt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/
(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

3.98.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.98.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.41

method	result
default	$\frac{2\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-a\,b^2)^{\frac{1}{3}}\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\cdots\right.}{b\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\cdots\right.}}}{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)(-a\,b\cdots}$
elliptic	$\frac{2\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-a\,b^2)^{\frac{1}{3}}\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\cdots\right.}{b\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\cdots\right.}}}{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)(-a\,b\cdots}$

input
```
int(1/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^
2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*b/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.98.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = -\frac{2\,\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

input
```
integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

3.98.6 Sympy [F]

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{ax+bx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**4), x)
```

3.98.7 Maxima [F]

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^4 + a*x), x)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{1}{3}\sqrt{bx^4+ax}x - \frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

input
```
integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^4 + a*x)*x - 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{2\,x\,\sqrt{\frac{b\,x^3}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^4+a\,x}}$$

input
```
int(1/(a*x + b*x^4)^(1/2),x)
```

output
```
(2*x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/6, 1/2], 7/6, -(b*x^3)/a))/(a*x +
b*x^4)^(1/2)
```

3.99 $\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$

3.99.1 Optimal result

Integrand size = 17, antiderivative size = 225

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
-2/5*(b*x^4+a*x)^(1/2)/a/x^3-2/15*b*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/
3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5x^2\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(
5*x^2*Sqrt[x*(a + b*x^3)])
```

3.99.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1931, 1917, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{2b\int \frac{1}{\sqrt{bx^4+ax}}dx}{5a}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

$$\Big\downarrow \text{1917}$$

$$-\frac{2b\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{5a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

$$\Big\downarrow \text{851}$$

$$-\frac{4b\sqrt{x}\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{5a\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

$$\Big\downarrow \text{766}$$

$$-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

input
```
Int[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*
(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b
*x^4])
```

3.99.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.99.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 696, normalized size of antiderivative = 3.09

method	result
default	$-\frac{2\sqrt{b\,x^4+a x}}{5 a\,x^3}-\frac{4 b^2\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{\left(-a\,b^2\cdots\right.}{b\left(-\frac{\left(-a\,b\cdots\right.}{2\cdots}\right.}}\cdots}{5 a\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\cdots\right.}$
elliptic	$-\frac{2\sqrt{b\,x^4+a x}}{5 a\,x^3}-\frac{4 b^2\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{\left(-a\,b^2\cdots\right.}{b\left(-\frac{\left(-a\,b\cdots\right.}{2\cdots}\right.}}\cdots}{5 a\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\cdots\right.}$
risch	$-\frac{2\left(b\,x^3+a\right)}{5 a\,x^2\sqrt{x\left(b\,x^3+a\right)}}-\frac{4 b^2\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{\cdots}{b\left(\cdots\right.}}\cdots}{5 a\left(-\frac{3\left(-a\,b\cdots\right.}{2 b}\cdots\right.}$

input
```
int(1/x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^4+a*x)^(1/2)/a/x^3-4/5*b^2/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

3.99.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \frac{2\left(2\sqrt{a}bx^3\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^4+ax}a\right)}{5\,a^2x^3}$$

input
```
integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(2*sqrt(a)*b*x^3*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^4 + a*
x)*a)/(a^2*x^3)
```

3.99.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^3\sqrt{x\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x*(a + b*x**3))), x)
```

3.99.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

3.99.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^4)^(1/2)), x)
```

3.100 $\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$

3.100.1 Optimal result

Integrand size = 17, antiderivative size = 503

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = -\frac{5(1+\sqrt{3})\,ax(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}+\frac{x^2\sqrt{ax+bx^4}}{4b}$$
$$+\frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$
$$+\frac{5(1-\sqrt{3})\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{3})\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
-5/8*a*x*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*
x^4+a*x)^(1/2)+1/4*x^2*(b*x^4+a*x)^(1/2)/b+5/8*3^(1/4)*a^(4/3)*x*(a^(1/3)+
b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^4+a*x)^(1/2
)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+
5/48*a^(4/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/
2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*
x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.13

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\frac{x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[x^5/Sqrt[a*x + b*x^4],x]
```

output
```
(x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6,
-((b*x^3)/a)]))/(4*b*Sqrt[x*(a + b*x^3)])
```

3.100.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.412$, Rules used = {1930, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\int \frac{x^2}{\sqrt{bx^4+ax}}dx}{8b}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{8b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{4b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{x^2\sqrt{ax+bx^4}}{4b}-$$

$$\frac{5a\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{4b\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{ 2420}$$

3.100. $\int \frac{x^5}{\sqrt{ax+bx^4}}dx$

$$\frac{\frac{x^2\sqrt{ax+bx^4}}{4b}-\frac{5a\sqrt{x}\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\cdots\right)}{4b\sqrt{ax+bx^4}}}{}$$

input
```
Int[x^5/Sqrt[a*x + b*x^4],x]
```

output
```
(x^2*Sqrt[a*x + b*x^4])/(4*b) - (5*a*Sqrt[x]*Sqrt[a + b*x^3]*((((1 + Sqrt[
3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4
)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^
(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)
*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (
1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3]
)/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*b*Sqrt[a*x + b*x^4])
```

3.100. $\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.100.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 1079, normalized size of antiderivative = 2.15

method	result	size
default	Expression too large to display	1079
elliptic	Expression too large to display	1079
risch	Expression too large to display	1086

input
```
int(x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(b*x^4+a*x)^(1/2)/b-5/8*a/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*...
```

3.100.5 Fricas [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a*x)*x^4/(b*x^3 + a), x)
```

3.100.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^5}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**5/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**5/sqrt(x*(a + b*x**3)), x)
```

3.100.7 Maxima [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(b*x^4 + a*x), x)
```

3.100.8 Giac [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(b*x^4 + a*x), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^5}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^5/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^5/(a*x + b*x^4)^(1/2), x)
```

3.101 $\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$

3.101.1 Optimal result

Integrand size = 17, antiderivative size = 474

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \frac{\left(1+\sqrt{3}\right)x(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}} - \frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
x*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4+a*x
)^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2
))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)/b^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^
(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^4+a*x)^
(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)
```

3.101.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \frac{2x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[x^2/Sqrt[a*x + b*x^4],x]
```

output
```
(2*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]
)/(5*Sqrt[x*(a + b*x^3)])
```

3.101.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^4}}dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{\sqrt{x}\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{837}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{766}$$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x\cdots}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{\sqrt{ax+bx^4}}$$

$$\Big\downarrow \text{2420}$$

3.101. $\int \frac{x^2}{\sqrt{ax+bx^4}}dx$

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}} - \cdots \right.}{\sqrt{ax+bx^4}}$$

input
```
Int[x^2/Sqrt[a*x + b*x^4],x]
```

output
```
(2*Sqrt[x]*Sqrt[a + b*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(
a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a
^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3
]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(
1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[
a + b*x^3])))/Sqrt[a*x + b*x^4]
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.101.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 1054, normalized size of antiderivative = 2.22

method	result	size
default	Expression too large to display	1054
elliptic	Expression too large to display	1054

input
```
int(x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(
-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-
a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a...
```

3.101.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a*x)*x/(b*x^3 + a), x)
```

3.101.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(x**2/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x*(a + b*x**3)), x)
```

3.101.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^4 + a*x), x)
```

3.101.8 Giac [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^4 + a*x), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^2}{\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(x^2/(a*x + b*x^4)^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^4)^(1/2), x)
```

3.102 $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

3.102.1 Optimal result

Integrand size = 17, antiderivative size = 497

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax} - \frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

output
```
2*b^(1/3)*x*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4
+a*x)^(1/2)-2*(b*x^4+a*x)^(1/2)/a/x-2*3^(1/4)*b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipt
icE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/3*b^(1/
3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/a^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)/a)])/S
qrt[x*(a + b*x^3)]
```

3.102.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1931, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$$

$$\Big\downarrow \; 1931$$

$$\frac{2b\int \frac{x^2}{\sqrt{bx^4+ax}}dx}{a} - \frac{2\sqrt{ax+bx^4}}{ax}$$

$$\Big\downarrow \; 1938$$

$$\frac{2b\sqrt{x}\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{a\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax}$$

$$\Big\downarrow \; 851$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{a\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax}$$

$$\Big\downarrow \; 837$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}} - \frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax}$$

$$\Big\downarrow \; 25$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}} - \frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax}$$

$$\Big\downarrow \; 766$$

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\Bigg(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}} - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\Bigg(\arccos\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}}}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{a\sqrt{ax+bx^4}}$$

$$\frac{2\sqrt{ax+bx^4}}{ax}$$

$$\Big\downarrow \; 2420$$

3.102. $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

$$\frac{4b\sqrt{x}\sqrt{a+bx^3}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\cdots\right)}{a\sqrt{ax+bx^4}}$$

$$\frac{2\sqrt{ax+bx^4}}{ax}$$

input
```
Int[1/(x*Sqrt[a*x + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x + b*x^4])/(a*x) + (4*b*Sqrt[x]*Sqrt[a + b*x^3]*((((1 + Sqrt[3
])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)
*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(
1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 +
 Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*
Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1
 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])
/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(a*Sqrt[a*x + b*x^4])
```

3.102. $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.102.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.18

method	result	size
default	Expression too large to display	1083
risch	Expression too large to display	1083
elliptic	Expression too large to display	1083

input
```
int(1/x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^3+a)/a/(x*(b*x^3+a))^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(
1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1...
```

3.102.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \frac{2\,\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

3.102.6 Sympy [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x\sqrt{x\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x/(b*x**4+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x*(a + b*x**3))), x)
```

3.102.7 Maxima [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

3.102.8 Giac [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^4+a\,x}}\,dx$$

input
```
int(1/(x*(a*x + b*x^4)^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^4)^(1/2)), x)
```

3.103 $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.103.1 Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3}-\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2}+\frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a}-\frac{63b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{11/2}}$$

output
```
-63/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)+63/64*b
^4*(b*x^(1/2)+a*x)^(1/2)/a^5+21/40*b^2*x*(b*x^(1/2)+a*x)^(1/2)/a^3-9/20*b*
x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/2)+a*x)^(1/2)/a-21/32*b^
3*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4
```

3.103.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{\sqrt{b\sqrt{x}+ax}\left(315b^4-210ab^3\sqrt{x}+168a^2b^2x-144a^3bx^{3/2}+128a^4x^2\right)}{320a^5}-\frac{63b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{64a^{11/2}}$$

input
```
Integrate[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(315*b^4 - 210*a*b^3*Sqrt[x] + 168*a^2*b^2*x - 144*
a^3*b*x^(3/2) + 128*a^4*x^2))/(320*a^5) - (63*b^5*ArcTanh[(Sqrt[a]*Sqrt[b*
Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(64*a^(11/2))
```

3.103.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1924, 1134, 1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^{5/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\int \frac{x^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{10a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\int \frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}\right)}{10a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int \frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)}{8a}\right)}{10a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

$$\downarrow 1160$$

3.103. $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

$$\downarrow 1091$$

3.103. $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

$\downarrow$ 219

3.103. $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

input
```
Int[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (9*b*((x^(3/2)*Sqrt[b*Sqrt[x] + a*x
])/(4*a) - (7*b*((x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*S
qrt[x] + a*x])/(2*a) - (3*b*(Sqrt[b*Sqrt[x] + a*x]/a - (b*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/(10*a)
)
```

3.103. $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.103.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{4a} - \frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a} - \frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{5a}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(544(b\sqrt{x}+ax)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}b-256x(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{9}{2}}-880(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{5}{2}}b^2+1300\sqrt{b\sqrt{x}+ax}\,\sqrt{x}\,a^{\frac{5}{2}}b^3+650\sqrt{\right.}{640\sqrt{\sqrt{}}}$

input
```
int(x^2/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^2*(b*x^(1/2)+a*x)^(1/2)/a-9/5*b/a*(1/4*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)
/a-7/8*b/a*(1/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+
a*x)^(1/2)/a-3/4*b/a*((b*x^(1/2)+a*x)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^
(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2))))))
```

3.103.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x^2}{5a}-\frac{9bx^{\frac{3}{2}}}{40a^2}+\frac{21b^2x}{80a^3}-\frac{21b^3\sqrt{x}}{64a^4}+\frac{63b^4}{128a^5}\right)-\frac{63b^5\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{256a^5} & \text{fo} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{11}{2}}}{11b^6} & \text{fo} \\ \tilde{\infty}x^3 & \text{o} \end{cases}\right.$$

input
```
integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**2/(5*a) - 9*b*x**(3/2)/(40*a**2) +
21*b**2*x/(80*a**3) - 21*b**3*sqrt(x)/(64*a**4) + 63*b**4/(128*a**5)) - 63
*b**5*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sq
rt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*
(sqrt(x) + b/(2*a))**2), True))/(256*a**5), Ne(a, 0)), (2*(b*sqrt(x))**(11
/2)/(11*b**6), Ne(b, 0)), (zoo*x**3, True))
```

3.103.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a*x + b*sqrt(x)), x)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a}-\frac{9b}{a^2}\right)+\frac{21b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)\sqrt{x}+\frac{315b^4}{a^5}\right) + \frac{63b^5\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{128a^{\frac{11}{2}}}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a - 9*b/a^2) + 21*
b^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(ab
s(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(11/2)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/2))^(1/2), x)
```

3.104 $\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.104.1 Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}$$

output
```
-5/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)+5/4*b^2*(b
*x^(1/2)+a*x)^(1/2)/a^3+2/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/6*b*x^(1/2)*(b*x^(
1/2)+a*x)^(1/2)/a^2
```

3.104.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(15b^2-10ab\sqrt{x}+8a^2x\right)}{12a^3} - \frac{5b^3\arctan\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{4a^{7/2}}$$

input
```
Integrate[x/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(15*b^2 - 10*a*b*Sqrt[x] + 8*a^2*x))/(12*a^3) - (5*
b^3*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(4*a^(7/2))
```

3.104.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1924, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \ 1924$$

$$2\int \frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \ 1134$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int \frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)$$

$$\Big\downarrow \ 1134$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)$$

$$\Big\downarrow \ 1160$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)}{6a}\right)$$

$$\Big\downarrow \ 1091$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)}{6a}\right)$$

$\downarrow$ 219

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)$$

input
```
Int[x/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(2*a) - (3*b*(Sqrt[b*Sqrt[x] + a*x]/a - (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt
[b*Sqrt[x] + a*x]])/a^(3/2)))/(4*a)))/(6*a))
```

3.104.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.104.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(16\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{5}{2}}-36\sqrt{b\sqrt{x}+ax}\,\sqrt{x}\,a^{\frac{5}{2}}b-18\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b^{2}+48\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}b^{2}-24a\ln\left(\frac{2a\sqrt{x}+2\cdots}{}\right.\right.}{24a^{\frac{9}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}}$

input
```
int(x/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/3*b/a*(1/2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a
-3/4*b/a*((b*x^(1/2)+a*x)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/
2)+(b*x^(1/2)+a*x)^(1/2))))
```

3.104.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x}{3a}-\frac{5b\sqrt{x}}{12a^2}+\frac{5b^2}{8a^3}\right)-\frac{5b^3\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{16a^3} & \text{for } a\neq 0\\ \frac{2\left(b\sqrt{x}\right)^{\frac{7}{2}}}{7b^4} & \text{for } b\neq 0\\ \tilde{\infty}x^2 & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x/(3*a) - 5*b*sqrt(x)/(12*a**2) + 5*b*
*2/(8*a**3)) - 5*b**3*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a
*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) +
b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(16*a**3), Ne(a, 0)), (2*(
b*sqrt(x))**(7/2)/(7*b**4), Ne(b, 0)), (zoo*x**2, True))
```

3.104.7 Maxima [F]

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a*x + b*sqrt(x)), x)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a}-\frac{5b}{a^2}\right)+\frac{15b^2}{a^3}\right) + \frac{5b^3\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{8a^{\frac{7}{2}}}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3
) + 5/8*b^3*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) +
b))/a^(7/2)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x/(a*x + b*x^(1/2))^(1/2), x)
```

3.105 $\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.105.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

output
```
-2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)+2*(b*x^(1/2)+a
*x)^(1/2)/a
```

3.105.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{3/2}}$$

input
```
Integrate[1/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])
/(b + a*Sqrt[x])])/a^(3/2)
```

3.105.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1916, 1919, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1916}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}b+ax}}dx}{2a}$$

$$\Big\downarrow \text{1919}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

input
```
Int[1/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x
] + a*x]])/a^(3/2)
```

3.105.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1916
```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Simp[a*((2*n - j - 2)/(b*(n -
2)))   Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
 LtQ[2*(n - 1), j, n]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

3.105.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}$	50
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(b\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\sqrt{a}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\sqrt{a}\right)}{\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}}$	83

input
```
int(1/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(1/2)+a*x)^(1/2)/a-b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2
)+a*x)^(1/2))
```

3.105.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} -\frac{b\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0 \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise} \end{cases}\right)}{2a} + \frac{\sqrt{ax+b\sqrt{x}}}{a} & \text{for } a\neq 0 \\ \frac{2\left(b\sqrt{x}\right)^{\frac{3}{2}}}{3b^2} & \text{for } b\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}\right)$$

input
```
integrate(1/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((-b*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(
x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a
))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(2*a) + sqrt(a*x + b*sqrt(x))/a,
 Ne(a, 0)), (2*(b*sqrt(x))**(3/2)/(3*b**2), Ne(b, 0)), (zoo*x, True))
```

3.105.7 Maxima [F]

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x + b*sqrt(x)), x)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{b\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{ax+b\sqrt{x}}}{a}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
b*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(3/2
) + 2*sqrt(a*x + b*sqrt(x))/a
```

3.105.9 Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4x\left(\frac{3\sqrt{b}\sqrt{b+a\sqrt{x}}}{2a\sqrt{x}} + \frac{b^{3/2}\operatorname{asin}\left(\frac{\sqrt{a}\,x^{1/4}\,1\mathrm{i}}{\sqrt{b}}\right)3\mathrm{i}}{2a^{3/2}x^{3/4}}\right)\sqrt{\frac{a\sqrt{x}}{b}+1}}{3\sqrt{a\,x+b\sqrt{x}}}$$

input
```
int(1/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
(4*x*((3*b^(1/2)*(b + a*x^(1/2))^(1/2))/(2*a*x^(1/2)) + (b^(3/2)*asin((a^(
1/2)*x^(1/4)*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/4)))*((a*x^(1/2))/b + 1)^(1/
2))/(3*(a*x + b*x^(1/2))^(1/2))
```

3.106 $\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx$

3.106.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

output
```
-4*(b*x^(1/2)+a*x)^(1/2)/b/x^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

input
```
Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

3.106.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

$$\Big\downarrow \ \text{1920}$$

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

input
```
Int[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

3.106.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.106.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\,\left(4\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}\sqrt{a}-2\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}x-2\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}x-\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)abx+\ln\left(\frac{2a\sqrt{}}{}\right.\right.}{\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,b^{2}x\sqrt{a}}$

input
```
int(1/x/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(b*x^(1/2)+a*x)^(1/2)/b/x^(1/2)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))
```

3.106.6 Sympy [F]

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)
```

3.106.7 Maxima [F]

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4}{\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}}$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/2))^(1/2)), x)
```

3.107 $\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx$

3.107.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}$$

output
```
-4/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)+16/15*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x-3
2/15*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(3b^2-4ab\sqrt{x}+8a^2x\right)}{15b^3x^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/
2))
```

3.107.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

input
```
Int[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x]
)/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)
```

3.107.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.107.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(60\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{5}{2}}a^{\frac{5}{2}}-30\sqrt{b\sqrt{x}+ax}\,x^{\frac{7}{2}}a^{\frac{7}{2}}-30x^{\frac{7}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{7}{2}}-15x^{\frac{7}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+\cdots}{2\sqrt{a}}\right.\right.}{15\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b^{4}\cdots}$

input
```
int(1/x^2/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)-8/5*a/b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b
/x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(4\,abx-\left(8\,a^2x+3\,b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{15\,b^3x^2}$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)
```

3.107.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^2\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)
```

3.107.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(20\,a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+15\,\sqrt{a}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+3\,b^2\right)}{15\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5}$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqr
t(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x
 + b*sqrt(x)))^5
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^2\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/2))^(1/2)), x)
```

3.108 $\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$

3.108.1 Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}}+\frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{315b^5\sqrt{x}}$$

output
```
-4/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)+32/63*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2
-64/105*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)+256/315*a^3*(b*x^(1/2)+a*x)^
(1/2)/b^4/x-512/315*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\,(35b^4-40ab^3\sqrt{x}+48a^2b^2x-64a^3bx^{3/2}+128a^4x^2)}{315b^5x^{5/2}}$$

input
```
Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a
^3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

3.108.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

input
```
Int[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x]
)/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*S
qrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x]
)))/(5*b)))/(7*b)))/(9*b)
```

3.108.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.108.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{16a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}\right)}{9b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(1260\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{9}{2}}-630\sqrt{b\sqrt{x}+ax}\,x^{\frac{11}{2}}a^{\frac{11}{2}}-315x^{\frac{11}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{5}b-630x^{\frac{11}{2}}a^{\frac{1}{2}}\right.}{}$

input
```
int(1/x^3/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)-16/9*a/b*(-2/7*(b*x^(1/2)+a*x)^(1/2)/
b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)-4/5*a/b*(-2/3*(b*x^(1/
2)+a*x)^(1/2)/b/x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2))))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(64\,a^3bx^2+40\,ab^3x-\left(128\,a^4x^2+48\,a^2b^2x+35\,b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{315\,b^5x^3}$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*s
qrt(x))*sqrt(a*x + b*sqrt(x))/(b^5*x^3)
```

3.108.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)
```

3.108.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$$
$$= \frac{4\left(1008\,a^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+1680\,a^{\frac{3}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+1080\,ab^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\right.\right.}{315\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^9}$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)
*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(
x) - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*
x + b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/2))^(1/2)), x)
```

3.109 $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

3.109.1 Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}+\frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{1001b^5x^{3/2}}+\frac{2048a^5\sqrt{b\sqrt{x}+ax}}{3003b^6x}-\frac{4096a^6\sqrt{b\sqrt{x}+ax}}{3003b^7\sqrt{x}}$$

output

```
-4/13*(b*x^(1/2)+a*x)^(1/2)/b/x^(7/2)+48/143*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x
^3-160/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)+1280/3003*a^3*(b*x^(1/2)+
a*x)^(1/2)/b^4/x^2-512/1001*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)+2048/300
3*a^5*(b*x^(1/2)+a*x)^(1/2)/b^6/x-4096/3003*a^6*(b*x^(1/2)+a*x)^(1/2)/b^7/
x^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(231b^6-252ab^5\sqrt{x}+280a^2b^4x-320a^3b^3x^{3/2}+384a^4b^2x^2-512a^5bx^{5/2}+1024a^6x^3\right)}{3003b^7x^{7/2}}$$

input
```
Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 3
20*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/
(3003*b^7*x^(7/2))
```

3.109.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{12a\int \frac{1}{x^{7/2}\sqrt{\sqrt{x}b+ax}}dx}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{12a\left(-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$\downarrow$ 1922

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$\downarrow$ 1922

3.109. $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$$\Big\downarrow \; 1920$$

3.109. $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

input
```
Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) - (12*a*((-4*Sqrt[b*Sqrt[x] + a*
x])/(11*b*x^3) - (10*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((
-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*
b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x
] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))/(11*b)))/(13*b)
```

3.109.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.109.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{13b\,x^{\frac{7}{2}}}-\frac{24a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{11b\,x^{3}}-\frac{10a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{\cdots}}{3\cdots}\right)}{5b}\right)}{7b}\right)}{9b}\right)}{11b}\right)}{13b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(12012\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{13}{2}}a^{\frac{13}{2}}-6006\sqrt{b\sqrt{x}+ax}\,x^{\frac{15}{2}}a^{\frac{15}{2}}-3003x^{\frac{15}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{7}b-6006\cdots\right.}{\cdots}$

input
```
int(1/x^4/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/13*(b*x^(1/2)+a*x)^(1/2)/b/x^(7/2)-24/13*a/b*(-2/11*(b*x^(1/2)+a*x)^(1/
2)/b/x^3-10/11*a/b*(-2/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)-8/9*a/b*(-2/7*(b*
x^(1/2)+a*x)^(1/2)/b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)-4/5
*a/b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b/x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/
2))))))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(512\,a^5bx^3+320\,a^3b^3x^2+252\,ab^5x-(1024\,a^6x^3+384\,a^4b^2x^2+280\,a^2b^4x+231\,b^6)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3003\,b^7x^4}$$

input
```
integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 38
4*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b
^7*x^4)
```

3.109.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)
```

3.109.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$$
$$= \frac{4\left(27456\,a^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^6+72072\,a^{\frac{5}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+80080\,a^2b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax\right.\right.}{3}$$

input
```
integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^6 + 72072*a^(5
/2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)
*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 48048*a^(3/2)*b^3*(sqrt(a)*sqrt(x) -
 sqrt(a*x + b*sqrt(x)))^3 + 16380*a*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sq
rt(x)))^2 + 3003*sqrt(a)*b^5*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 2
31*b^6)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^13
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^4\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(1/2))^(1/2)), x)
```

3.110 $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.110.1 Optimal result

Integrand size = 19, antiderivative size = 197

$$\begin{aligned}\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = & -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} \\ & -\frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} \\ & +\frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2}-\frac{693b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{13/2}}\end{aligned}$$

output
```
-693/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-4*x^3/
a/(b*x^(1/2)+a*x)^(1/2)+693/64*b^4*(b*x^(1/2)+a*x)^(1/2)/a^6+231/40*b^2*x*
(b*x^(1/2)+a*x)^(1/2)/a^4-99/20*b*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3+22/5*x
^2*(b*x^(1/2)+a*x)^(1/2)/a^2-231/32*b^3*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^5
```

3.110.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.70

$$\begin{aligned}\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = & \frac{\sqrt{b\sqrt{x}+ax}\left(3465b^5+1155ab^4\sqrt{x}-462a^2b^3x+264a^3b^2x^{3/2}-176a^4bx^2+128a^5x^5\right.}{320a^6\left(b+a\sqrt{x}\right)} \\ & -\frac{693b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{64a^{13/2}}\end{aligned}$$

input
```
Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(3465*b^5 + 1155*a*b^4*Sqrt[x] - 462*a^2*b^3*x + 26
4*a^3*b^2*x^(3/2) - 176*a^4*b*x^2 + 128*a^5*x^(5/2)))/(320*a^6*(b + a*Sqrt
[x])) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])
/(64*a^(13/2))
```

3.110.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {1924, 1124, 25, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^{7/2}}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int -\frac{-x^{5/2}a^5+bx^2a^4-b^2x^{3/2}a^3+b^3xa^2-b^4\sqrt{x}a+b^5}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^6}+\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\int \frac{-x^{5/2}a^5+bx^2a^4-b^2x^{3/2}a^3+b^3xa^2-b^4\sqrt{x}a+b^5}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^6}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{19bx^2a^5-10b^2x^{3/2}a^4+10b^3xa^3-10b^4\sqrt{x}a^2+10b^5a}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{5a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{19bx^2a^5-10b^2x^{3/2}a^4+10b^3xa^3-10b^4\sqrt{x}a^2+10b^5a}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \textbf{2192}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\int \frac{-213b^2x^{3/2}a^5+80b^3xa^4-80b^4\sqrt{x}a^3+80b^5a^2}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \textbf{27}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\int \frac{-213b^2x^{3/2}a^5+80b^3xa^4-80b^4\sqrt{x}a^3+80b^5a^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \textbf{2192}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{\int \frac{15\left(103b^3xa^5-32b^4\sqrt{x}a^4+32b^5a^3\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{3a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \textbf{27}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\int \frac{103b^3xa^5-32b^4\sqrt{x}a^4+32b^5a^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \textbf{2192}$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{\int \frac{a^4b^4\left(128b-437a\sqrt{x}\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right.$$

3.110. $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

$\Big\downarrow$ 27

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{1}{4}a^3b^4\int\frac{128b-437a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right.$$

$\Big\downarrow$ 1160

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{1}{4}a^3b^4\left(\frac{693}{2}b\int\frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-437\sqrt{ax+b\sqrt{x}}\right)+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}\cdots}{a^6}\right.$$

$\Big\downarrow$ 1091

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{1}{4}a^3b^4\left(693b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-437\sqrt{ax+b\sqrt{x}}\right)+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\cdots}{a^6}\right.$$

$\Big\downarrow$ 219

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}+\frac{\frac{5\left(\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}+\frac{1}{4}a^3b^4\left(\frac{693b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-437\sqrt{ax+b\sqrt{x}}\right)\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{\cdots}}}{8a}}{10a}}{a^6}\right.$$

input
```
Int[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((2*b^5*Sqrt[x])/(a^6*Sqrt[b*Sqrt[x] + a*x]) - (-1/5*(a^4*x^2*Sqrt[b*Sqr
t[x] + a*x]) + ((19*a^4*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/4 + (-71*a^4*b^2*
x*Sqrt[b*Sqrt[x] + a*x] + (5*((103*a^4*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/
2 + (a^3*b^4*(-437*Sqrt[b*Sqrt[x] + a*x] + (693*b*ArcTanh[(Sqrt[a]*Sqrt[x]
)/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]))/4))/(2*a))/(8*a))/(10*a))/a^6)
```

3.110. $\int\frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.110.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{2x^3}{5a\sqrt{b\sqrt{x}+ax}} - \frac{11b\left(\frac{x^{\frac{5}{2}}}{4a\sqrt{b\sqrt{x}+ax}} - \frac{9b\left(\frac{x^2}{3a\sqrt{b\sqrt{x}+ax}} - \frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}} - \frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}} - \frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} - \frac{b\left(-\frac{1}{a\sqrt{b\sqrt{}}}\right.}{}\right.}{}\right.}{6a}\right.}{8a}\right.}{5a}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(352\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}a^{\frac{11}{2}}b-256\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^2a^{\frac{13}{2}}-528\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x\,a^{\frac{9}{2}}b^2+4060\sqrt{b\sqrt{x}+ax}\,x^{\frac{3}{2}}a^{\frac{9}{2}}b^3-3\right.}{}$

3.110. $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

input
```
int(x^3/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^3/a/(b*x^(1/2)+a*x)^(1/2)-11/5*b/a*(1/4*x^(5/2)/a/(b*x^(1/2)+a*x)^(1
/2)-9/8*b/a*(1/3*x^2/a/(b*x^(1/2)+a*x)^(1/2)-7/6*b/a*(1/2*x^(3/2)/a/(b*x^(
1/2)+a*x)^(1/2)-5/4*b/a*(x/a/(b*x^(1/2)+a*x)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*
x^(1/2)+a*x)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(b+2*a*x^(1/2
))/(b*x^(1/2)+a*x)^(1/2))+1/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2
)+a*x)^(1/2))))))
```

3.110.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.110.6 Sympy [F]

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

3.110.7 Maxima [F]

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a^2}-\frac{19\,b}{a^3}\right)+\frac{71\,b^2}{a^4}\right)\sqrt{x}-\frac{515\,b^3}{a^5}\right)\sqrt{x}+\frac{2185\,b^4}{a^6}\right.$$
$$+\frac{693\,b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{128\,a^{\frac{13}{2}}}+\frac{4\,b^6}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{13}{2}}}$$

input
```
integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a^2 - 19*b/a^3) +
71*b^2/a^4)*sqrt(x) - 515*b^3/a^5)*sqrt(x) + 2185*b^4/a^6) + 693/128*b^5*l
og(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(13/2)
 + 4*b^6/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(13/2)
)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^(1/2))^(3/2), x)
```

3.111 $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.111.1 Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}}+\frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} -\frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3}+\frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2}-\frac{35b^3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{9/2}}$$

output
```
-35/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-4*x^2/a/(
b*x^(1/2)+a*x)^(1/2)+35/4*b^2*(b*x^(1/2)+a*x)^(1/2)/a^4+14/3*x*(b*x^(1/2)+
a*x)^(1/2)/a^2-35/6*b*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

3.111.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(105b^3+35ab^2\sqrt{x}-14a^2bx+8a^3x^{3/2}\right)}{12a^4\left(b+a\sqrt{x}\right)} -\frac{35b^3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{4a^{9/2}}$$

input
```
Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(105*b^3 + 35*a*b^2*Sqrt[x] - 14*a^2*b*x + 8*a^3*x^
(3/2)))/(12*a^4*(b + a*Sqrt[x])) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x]
 + a*x])/(b + a*Sqrt[x])])/(4*a^(9/2))
```

3.111.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1924, 1124, 25, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^{5/2}}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int -\frac{-x^{3/2}a^3+bxa^2-b^2\sqrt{x}a+b^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^4}+\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\int \frac{-x^{3/2}a^3+bxa^2-b^2\sqrt{x}a+b^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^4}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{11bxa^3-6b^2\sqrt{x}a^2+6b^3a}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{3a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{11bxa^3-6b^2\sqrt{x}a^2+6b^3a}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\int \frac{3a^2b^2(8b-19a\sqrt{x})}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{3}{4}ab^2\int \frac{8b-19a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{3}{4}ab^2\left(\frac{35}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-19\sqrt{ax+b\sqrt{x}}\right)+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{3}{4}ab^2\left(35b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-19\sqrt{ax+b\sqrt{x}}\right)+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}+\frac{3}{4}ab^2\left(\frac{35b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-19\sqrt{ax+b\sqrt{x}}\right)}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

input
```
Int[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((2*b^3*Sqrt[x])/(a^4*Sqrt[b*Sqrt[x] + a*x]) - (-1/3*(a^2*x*Sqrt[b*Sqrt[
x] + a*x]) + ((11*a^2*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/2 + (3*a*b^2*(-19*S
qrt[b*Sqrt[x] + a*x] + (35*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*
x]])/Sqrt[a]))/4)/(6*a))/a^4)
```

3.111. $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.111.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{2x^2}{3a\sqrt{b\sqrt{x}+ax}}-\frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{\cdots}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{4a}\right)}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(16x\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}-60\sqrt{b\sqrt{x}+ax}\,x^{\frac{3}{2}}a^{\frac{9}{2}}b+32\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}b-150\sqrt{b\sqrt{x}+ax}\,x\,a^{\frac{7}{2}}b^2+240x\sqrt{\sqrt{x}\left(a\cdots\right.}\right.}{\cdots}$

input
```
int(x^2/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^2/a/(b*x^(1/2)+a*x)^(1/2)-7/3*b/a*(1/2*x^(3/2)/a/(b*x^(1/2)+a*x)^(1/
2)-5/4*b/a*(x/a/(b*x^(1/2)+a*x)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*x^(1/2)+a*x)^
(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)
+a*x)^(1/2))+1/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2))
)))
```

3.111.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.111.6 Sympy [F]

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)
```

3.111.7 Maxima [F]

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a^2}-\frac{11\,b}{a^3}\right)+\frac{57\,b^2}{a^4}\right) + \frac{35\,b^3\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{8\,a^{\frac{9}{2}}} + \frac{4\,b^4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{9}{2}}}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a^2 - 11*b/a^3) + 57*b^2/
a^4) + 35/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)
)) - b))/a^(9/2) + 4*b^4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)
)) + b)*a^(9/2))
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/2))^(3/2), x)
```

3.112 $\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.112.1 Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{6b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}$$

output
```
-6*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-4*x/a/(b*x^(1/
2)+a*x)^(1/2)+6*(b*x^(1/2)+a*x)^(1/2)/a^2
```

3.112.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{2\left(3b+a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{a^2\left(b+a\sqrt{x}\right)}-\frac{6b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{5/2}}$$

input
```
Integrate[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(2*(3*b + a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(a^2*(b + a*Sqrt[x])) - (6*b*A
rcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/a^(5/2)
```

3.112.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1924, 1124, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^{3/2}}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int -\frac{b-a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^2}+\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{\int \frac{b-a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^2}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{\frac{3}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-\sqrt{ax+b\sqrt{x}}}{a^2}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{3b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-\sqrt{ax+b\sqrt{x}}}{a^2}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-\sqrt{ax+b\sqrt{x}}}{a^2}\right)$$

input
```
Int[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((2*b*Sqrt[x])/(a^2*Sqrt[b*Sqrt[x] + a*x]) - (-Sqrt[b*Sqrt[x] + a*x] + (
3*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a])/a^2)
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(59) = 118$.

Time = 2.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{2x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}\right)}{a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(6x\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{5}{2}}-3x\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+12\sqrt{x}\,\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{3}{2}}b-6\sqrt{x}\,\ln\left(\frac{2a\sqrt{\cdot}}{\cdots}\right.\right.}{a^{\frac{5}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+\cdots}}$

input
```
int(x/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/a/(b*x^(1/2)+a*x)^(1/2)-3*b/a*(-x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-1/2*b/
a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))
+1/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2)))
```

3.112.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.112.6 Sympy [F]

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

3.112.7 Maxima [F]

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{3\,b\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{5}{2}}} + \frac{2\,\sqrt{ax+b\sqrt{x}}}{a^2} + \frac{4\,b^2}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{5}{2}}}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
3*b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(
5/2) + 2*sqrt(a*x + b*sqrt(x))/a^2 + 4*b^2/((sqrt(a)*(sqrt(a)*sqrt(x) - sq
rt(a*x + b*sqrt(x))) + b)*a^(5/2))
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x/(a*x + b*x^(1/2))^(3/2), x)
```

3.113 $\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.113.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

output
```
4*x^(1/2)/b/(b*x^(1/2)+a*x)^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{b\sqrt{x}+ax}}{b\left(b+a\sqrt{x}\right)}$$

input
```
Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]
```

output
```
(4*Sqrt[b*Sqrt[x] + a*x])/(b*(b + a*Sqrt[x]))
```

3.113.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[(b*Sqrt[x] + a*x)^(-3/2),x]
```

output
```
(4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

3.113.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.

Time = 2.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

method	result
derivativedivides	$-\frac{2}{a\sqrt{b\sqrt{x}+ax}}+\frac{2b+4a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}$
default	$\sqrt{b\sqrt{x}+ax}\left(2\sqrt{b\sqrt{x}+ax}\,x\,a^{\frac{5}{2}}+x\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{2}b+2x\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{5}{2}}-x\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}}{2\sqrt{a}}\right.$

input
```
int(1/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(b*x^(1/2)+a*x)^(1/2)+2/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\sqrt{ax+b\sqrt{x}}\left(a\sqrt{x}-b\right)}{a^2bx-b^3}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a^2*b*x - b^3)
```

3.113.6 Sympy [F]

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*sqrt(x))**(-3/2), x)
```

3.113.7 Maxima [F]

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*sqrt(x))^(-3/2), x)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)\sqrt{a}}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\,x\left(\frac{b}{a\,\sqrt{x}}+1\right)}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}\left(\sqrt{\frac{b}{a\,\sqrt{x}}+1}+1\right)}$$

input
```
int(1/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
-(4*x*(b/(a*x^(1/2)) + 1))/((a*x + b*x^(1/2))^(3/2)*((b/(a*x^(1/2)) + 1)^(
1/2) + 1))
```

3.114 $\int \frac{1}{x(b\sqrt{x}+ax)^{3/2}}\,dx$

3.114.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}-\frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x}+\frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}$$

output
```
4/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)-16/3*(b*x^(1/2)+a*x)^(1/2)/b^2/x+32/3*a*
(b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(b^2-4ab\sqrt{x}-8a^2x\right)}{3b^3\left(b+a\sqrt{x}\right)x}$$

input
```
Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(b^2 - 4*a*b*Sqrt[x] - 8*a^2*x))/(3*b^3*(b + a*S
qrt[x])*x)
```

3.114.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+b\sqrt{x}\right)^{3/2}}dx$$
$$\Big\downarrow \ 1921$$
$$\frac{4\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$
$$\Big\downarrow \ 1922$$
$$\frac{4\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{b}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$
$$\Big\downarrow \ 1920$$
$$\frac{4\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{b}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) + (4*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*
x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/b
```

3.114.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.114.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{4}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{16a\left(b+2a\sqrt{x}\right)}{3b^{3}\sqrt{b\sqrt{x}+ax}}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\,\left(24\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{5}{2}}a^{\frac{7}{2}}-6\sqrt{b\sqrt{x}+ax}\,x^{\frac{7}{2}}a^{\frac{9}{2}}-3x^{\frac{7}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{4}b-6x^{\frac{7}{2}}a^{\frac{9}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\right.}{}$

input
```
int(1/x/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+16/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)
+a*x)^(1/2)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(4\,a^2bx-b^3-\left(8\,a^3x-5\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3\left(a^2b^3x^2-b^5x\right)}$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))
/(a^2*b^3*x^2 - b^5*x)
```

3.114.6 Sympy [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)
```

3.114.7 Maxima [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

3.114.8 Giac [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/2))^(3/2)), x)
```

3.115 $\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.115.1 Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}-\frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{35b^5\sqrt{x}}$$

output
```
4/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-32/7*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2+192/3
5*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)-256/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4
/x+512/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^4-8ab^3\sqrt{x}+16a^2b^2x-64a^3bx^{3/2}-128a^4x^2\right)}{35b^5\left(b+a\sqrt{x}\right)x^2}$$

input
```
Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^4 - 8*a*b^3*Sqrt[x] + 16*a^2*b^2*x - 64*a^3
*b*x^(3/2) - 128*a^4*x^2))/(35*b^5*(b + a*Sqrt[x])*x^2)
```

3.115.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1921, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) + (8*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*
x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*S
qrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*
b)))/(7*b)))/b
```

3.115.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.115.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{4}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{16a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a(b+2a\sqrt{x})}{3b^{3}\sqrt{b\sqrt{x}+ax}}\right)}{5b}\right)}{7b}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(560\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{11}{2}}-210\sqrt{b\sqrt{x}+ax}\,x^{\frac{11}{2}}a^{\frac{13}{2}}-210x^{\frac{11}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{13}{2}}-105x^{\frac{11}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}}{2\sqrt{a}}\right.\right.}{}$

input
```
int(1/x^2/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/7/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-16/7*a/b*(-2/5/b/x/(b*x^(1/2)+a*x)^(1
/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+8/3*a/b^3*(b+2*a*x^(1/2)
)/(b*x^(1/2)+a*x)^(1/2)))
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(64\,a^4bx^2-24\,a^2b^3x-5\,b^5-\left(128\,a^5x^2-80\,a^3b^2x-13\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\left(a^2b^5x^3-b^7x^2\right)}$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x -
 13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)
```

3.115.6 Sympy [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)
```

3.115.7 Maxima [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

3.115.8 Giac [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/2))^(3/2)), x)
```

3.116 $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.116.1 Optimal result

Integrand size = 19, antiderivative size = 195

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{77b^5x^{3/2}}-\frac{2048a^4\sqrt{b\sqrt{x}+ax}}{231b^6x}+\frac{4096a^5\sqrt{b\sqrt{x}+ax}}{231b^7\sqrt{x}}$$

output
```
4/b/x^(5/2)/(b*x^(1/2)+a*x)^(1/2)-48/11*(b*x^(1/2)+a*x)^(1/2)/b^2/x^3+160/
33*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)-1280/231*a^2*(b*x^(1/2)+a*x)^(1/2)/
b^4/x^2+512/77*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)-2048/231*a^4*(b*x^(1/
2)+a*x)^(1/2)/b^6/x+4096/231*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(21b^6-28ab^5\sqrt{x}+40a^2b^4x-64a^3b^3x^{3/2}+128a^4b^2x^2-512a^5bx^{5/2}-1024a^6x^3\right)}{231b^7\left(b+a\sqrt{x}\right)x^3}$$

input
```
Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(21*b^6 - 28*a*b^5*Sqrt[x] + 40*a^2*b^4*x - 64*a
^3*b^3*x^(3/2) + 128*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) - 1024*a^6*x^3))/(231
*b^7*(b + a*Sqrt[x])*x^3)
```

3.116.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\int \frac{1}{x^{7/2}\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int\frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

3.116. $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1920

3.116. $\int\frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) + (12*((-4*Sqrt[b*Sqrt[x] + a*x])/(11*
b*x^3) - (10*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((-4*Sqrt[
b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2
)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x]
)/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))/(11*b)))/b
```

3.116.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.116.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{4}{11 b\, x^{\frac{5}{2}}\sqrt{b\sqrt{x}+a x}}-\frac{24 a \left(-\frac{2}{9 b\, x^{2}\sqrt{b\sqrt{x}+a x}}-\frac{10 a \left(-\frac{2}{7 b\, x^{\frac{3}{2}}\sqrt{b\sqrt{x}+a x}}-\frac{8 a \left(-\frac{2}{5 b x \sqrt{b\sqrt{x}+a x}}-\frac{6 a \left(-\frac{2}{3 b \sqrt{x}\, \sqrt{b\sqrt{x}+a x}}+\frac{8 a}{3 b^{3}\cdots}\right)}{5 b}\right)}{7 b}\right)}{9 b}\right)}{11 b}$
default	$\frac{\sqrt{b\sqrt{x}+a x}\,\left(8716 \left(b\sqrt{x}+a x\right)^{\frac{3}{2}} x^{6} a^{\frac{13}{2}} b-4620 \sqrt{b\sqrt{x}+a x}\, x^{7} a^{\frac{15}{2}} b-4620 x^{7} a^{\frac{15}{2}} \sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\, b-512 \left(b\sqrt{x}+a x\right)^{\frac{3}{2}} x^{5} a^{\frac{9}{2}} b^{3}\cdots\right)}{\cdots}$

input
```
int(1/x^3/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/11/b/x^(5/2)/(b*x^(1/2)+a*x)^(1/2)-24/11*a/b*(-2/9/b/x^2/(b*x^(1/2)+a*x
)^(1/2)-10/9*a/b*(-2/7/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-8/7*a/b*(-2/5/b/x/(
b*x^(1/2)+a*x)^(1/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+8/3*a/b
^3*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2)))))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(512\,a^6bx^3-192\,a^4b^3x^2-68\,a^2b^5x-21\,b^7-\left(1024\,a^7x^3-640\,a^5b^2x^2-104\,a^3b^4x-49\,ab^6\right)\sqrt{x}\right)\sqrt{ax+}}{231\left(a^2b^7x^4-b^9x^3\right)}$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^
7*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*
sqrt(x))/(a^2*b^7*x^4 - b^9*x^3)
```

3.116.6 Sympy [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)
```

3.116.7 Maxima [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

3.116.8 Giac [F]

$$\int \frac{1}{x^3 \left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/2))^(3/2)), x)
```

3.117 $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.117.1 Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} -\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} +\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{231b^6\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{256a^{13/2}}$$

output
```
231/256*b^6*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-231/25
6*b^5*(b*x^(1/2)+a*x)^(1/2)/a^6-77/160*b^3*x*(b*x^(1/2)+a*x)^(1/2)/a^4+33/
80*b^2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3-11/30*b*x^2*(b*x^(1/2)+a*x)^(1/2)
/a^2+1/3*x^(5/2)*(b*x^(1/2)+a*x)^(1/2)/a+77/128*b^4*x^(1/2)*(b*x^(1/2)+a*x
)^(1/2)/a^5
```

3.117.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.62

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-3465b^5+2310ab^4\sqrt{x}-1848a^2b^3x+1584a^3b^2x^{3/2}-1408a^4bx^2+1280a\right.}{3840a^6} +\frac{231b^6\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{256a^{13/2}}$$

input
```
Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-3465*b^5 + 2310*a*b^4*Sqrt[x] - 1848*a^2*b^3*x +
1584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1280*a^5*x^(5/2)))/(3840*a^6) + (2
31*b^6*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(256*a^(1
3/2))
```

3.117.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1924, 1134, 1134, 1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\int \frac{x^{5/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{12a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\int \frac{x^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{10a}\right)}{12a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\int\frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}\right)}{10a}\right)}{12a}\right)$$

$$\Big\downarrow \text{ 1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int\frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)}{8a}\right)}{10a}\right)}{12a}\right)$$

$$\Big\downarrow \text{ 1134}$$

3.117. $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)}{12a}\right)$$

$$\Big\downarrow \text{ 1160}$$

3.117. $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a} - \frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a} - \frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a} - \frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a} - \frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a} - \frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a} - \frac{b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}\,d\sqrt{x}}{2a}\right)}{4a}\right.}{6a}\right.}{8a}\right.}{10a}\right.}{12a}\right.$$

3.117. $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

↓ 1091

3.117. $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right.}{4a}\right.}{6a}\right.}{8a}\right.}{10a}\right.}{12a}\right.$$

3.117. $\int\frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

↓ 219

3.117. $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)}{12a}\right)$$

3.117. $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

input
```
Int[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(6*a) - (11*b*((x^2*Sqrt[b*Sqrt[x] + a*
x])/(5*a) - (9*b*((x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(4*a) - (7*b*((x*Sqrt[b*
Sqrt[x] + a*x])/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(2*a) - (3*b
*(Sqrt[b*Sqrt[x] + a*x]/a - (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/(10*a)))/(12*a))
```

3.117.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.117.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}}{3a}-\frac{11b\left(\frac{x^{2}\sqrt{b\sqrt{x}+ax}}{5a}-\frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{4a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{\cdots}}{\sqrt{a}}\cdots\right)}{\cdots}\right)}{4a}\cdots\right)}{6a}\cdots\right)}{8a}\cdots\right)}{10a}\cdots\right)}{6a}\cdots$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(2560x^{\frac{3}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{11}{2}}+8544a^{\frac{7}{2}}\sqrt{x}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}b^{2}-5376a^{\frac{9}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}bx+16860a^{\frac{5}{2}}\sqrt{x}\sqrt{b\sqrt{x}+ax}\,b^{4}\cdots\right)}{\cdots}$

3.117. $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

input
```
int(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(5/2)*(b*x^(1/2)+a*x)^(1/2)/a-11/6*b/a*(1/5*x^2*(b*x^(1/2)+a*x)^(1/2
)/a-9/10*b/a*(1/4*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a-7/8*b/a*(1/3*x*(b*x^(1/2
)+a*x)^(1/2)/a-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a-3/4*b/a*((b*x^
(1/2)+a*x)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a
*x)^(1/2))))))
```

3.117.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.117.6 Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x^{\frac{5}{2}}}{6a} - \frac{11bx^2}{60a^2} + \frac{33b^2x^{\frac{3}{2}}}{160a^3} - \frac{77b^3x}{320a^4} + \frac{77b^4\sqrt{x}}{256a^5} - \frac{231b^5}{512a^6}\right) + \frac{231b^6\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+\ldots}\right.}{\sqrt{\ldots}} \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\ldots\right.}{\sqrt{a\left(\sqrt{x}+\ldots\right.}} \end{cases}\right.}{\ldots} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{13}{2}}}{13b^7} \\ \tilde{\infty}x^{\frac{7}{2}} \end{cases}\right.$$

input
```
integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**(5/2)/(6*a) - 11*b*x**2/(60*a**2) +
 33*b**2*x**(3/2)/(160*a**3) - 77*b**3*x/(320*a**4) + 77*b**4*sqrt(x)/(256
*a**5) - 231*b**5/(512*a**6)) + 231*b**6*Piecewise((log(2*sqrt(a)*sqrt(a*x
 + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2
*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(1024*a
**6), Ne(a, 0)), (2*(b*sqrt(x))**(13/2)/(13*b**7), Ne(b, 0)), (zoo*x**(7/2
), True))
```

3.117.7 Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{3840}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\left(8\sqrt{x}\left(\frac{10\sqrt{x}}{a}-\frac{11\,b}{a^2}\right)+\frac{99\,b^2}{a^3}\right)\sqrt{x}-\frac{231\,b^3}{a^4}\right)\sqrt{x}+\frac{115}{a}\right.\right.$$
$$-\frac{231\,b^6\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{512\,a^{\frac{13}{2}}}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2)
 + 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 34
65*b^5/a^6) - 231/512*b^6*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x +
b*sqrt(x))) + b))/a^(13/2)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{5/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^(5/2)/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^(5/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.118 $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.118.1 Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{35b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{9/2}}$$

output
```
35/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-35/32*b^3
*(b*x^(1/2)+a*x)^(1/2)/a^4-7/12*b*x*(b*x^(1/2)+a*x)^(1/2)/a^2+1/2*x^(3/2)*
(b*x^(1/2)+a*x)^(1/2)/a+35/48*b^2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

3.118.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-105b^3+70ab^2\sqrt{x}-56a^2bx+48a^3x^{3/2}\right)}{96a^4} + \frac{35b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{32a^{9/2}}$$

input
```
Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-105*b^3 + 70*a*b^2*Sqrt[x] - 56*a^2*b*x + 48*a^3*
x^(3/2)))/(96*a^4) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b +
a*Sqrt[x])])/(32*a^(9/2))
```

3.118.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1924, 1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\int \frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int \frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)}{8a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)}{8a}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)}{6a}\right)}{8a}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)}{6a}\right)}{8a}\right)$$

$$\Big\downarrow \text{219}$$

3.118. $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)}{8a}\right)$$

input
```
Int[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(4*a) - (7*b*((x*Sqrt[b*Sqrt[x] + a*x])
/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(2*a) - (3*b*(Sqrt[b*Sqrt[x
] + a*x]/a - (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2))
)/(4*a)))/(6*a)))/(8*a))
```

3.118.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.118.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{2a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)}{4a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(96(b\sqrt{x}+ax)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}+348\sqrt{x}\,\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}b^{2}-208(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{5}{2}}b+174\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b^{3}-384a^{\frac{3}{2}}\sqrt{\sqrt{x}\right.}{192\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{11}{2}}}$

input
```
int(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a-7/4*b/a*(1/3*x*(b*x^(1/2)+a*x)^(1/2)/a
-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a-3/4*b/a*((b*x^(1/2)+a*x)^(1/
2)/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2)))))
```

3.118.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x^{\frac{3}{2}}}{4a} - \frac{7bx}{24a^2} + \frac{35b^2\sqrt{x}}{96a^3} - \frac{35b^3}{64a^4}\right) + \frac{35b^4\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a} \neq 0 \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise} \end{cases}\right)}{128a^4} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{9}{2}}}{9b^5} \\ \tilde{\infty}x^{\frac{5}{2}} \end{cases}\right.$$

input
```
integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**(3/2)/(4*a) - 7*b*x/(24*a**2) + 35*
b**2*sqrt(x)/(96*a**3) - 35*b**3/(64*a**4)) + 35*b**4*Piecewise((log(2*sqr
t(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((s
qrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), T
rue))/(128*a**4), Ne(a, 0)), (2*(b*sqrt(x))**(9/2)/(9*b**5), Ne(b, 0)), (z
oo*x**(5/2), True))
```

3.118.7 Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{96}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{6\sqrt{x}}{a}-\frac{7b}{a^2}\right)+\frac{35b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right) - \frac{35b^4\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{64a^{\frac{9}{2}}}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/
a^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x)
 - sqrt(a*x + b*sqrt(x))) + b))/a^(9/2)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{3/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^(3/2)/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.119 $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.119.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}$$

output
```
3/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-3/2*b*(b*x^
(1/2)+a*x)^(1/2)/a^2+x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a
```

3.119.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{(-3b+2a\sqrt{x})\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{2a^{5/2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
((-3*b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (3*b^2*ArcTanh[(Sqr
t[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(2*a^(5/2))
```

3.119.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1924, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)$$

input
```
Int[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

3.119. $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}dx$

output
```
2*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(2*a) - (3*b*(Sqrt[b*Sqrt[x] + a*x]/a -
 (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)))/(4*a))
```

3.119.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.119. $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$

3.119.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{2a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(4\sqrt{x}\,\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}+2\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b+4\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a\,b^{2}-8\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{3}{2}}b-b^{2}\cdots}{4\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{7}{2}}}$

input
```
int(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a-3/2*b/a*((b*x^(1/2)+a*x)^(1/2)/a-1/2*b/a^(
3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2)))
```

3.119.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$$
$$=2\left(\begin{cases}\left(\frac{\sqrt{x}}{2a}-\frac{3b}{4a^2}\right)\sqrt{ax+b\sqrt{x}}+\frac{3b^2\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{8a^2} & \text{for } a\neq 0\\ \frac{2\left(b\sqrt{x}\right)^{\frac{5}{2}}}{5b^3} & \text{for } b\neq 0\\ \tilde{\infty}x^{\frac{3}{2}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise(((sqrt(x)/(2*a) - 3*b/(4*a**2))*sqrt(a*x + b*sqrt(x)) + 3*b**2
*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a)
, Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt
(x) + b/(2*a))**2), True))/(8*a**2), Ne(a, 0)), (2*(b*sqrt(x))**(5/2)/(5*b
**3), Ne(b, 0)), (zoo*x**(3/2), True))
```

3.119.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a}-\frac{3\,b}{a^2}\right) - \frac{3\,b^2\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{4\,a^{\frac{5}{2}}}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(2*sqrt
(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(5/2)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{\sqrt{x}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.120 $\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}$$

output
```
4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(4*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/Sqrt[a]
```

3.120.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1919, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1919}$$

$$2\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1091}$$

$$4\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

3.120.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

3.120.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{\sqrt{a}}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)\right)}{\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,b\sqrt{a}}$

input
```
int(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2))/a^(1/2)
```

3.120.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } a\neq 0\wedge \frac{b^2}{a}\neq 0 \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{for } a\neq 0 \\ \frac{2\sqrt{b\sqrt{x}}}{b} & \text{for } b\neq 0 \\ \tilde{\infty}\sqrt{x} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a
), Ne(a, 0) & Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/
sqrt(a*(sqrt(x) + b/(2*a))**2), Ne(a, 0)), (2*sqrt(b*sqrt(x))/b, Ne(b, 0))
, (zoo*sqrt(x), True))
```

3.120.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{2\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{\sqrt{a}}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/sqrt(
a)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.121 $\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx$

3.121.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}$$

output
```
-4/3*(b*x^(1/2)+a*x)^(1/2)/b/x+8/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(b-2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{3b^2x}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*(b - 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

3.121.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

input
```
Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sq
rt[x])
```

3.121.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.121.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(6x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}+6x^{\frac{5}{2}}a^{\frac{5}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}+3x^{\frac{5}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{2}b-3x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+2\sqrt{\cdots}}{\cdots}\right.\right.}{3\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,b^{3}x^{\frac{5}{2}}\sqrt{a}}$

input
```
int(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(b*x^(1/2)+a*x)^(1/2)/b/x+8/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx=\frac{4\,\sqrt{ax+b\sqrt{x}}\,(2\,a\sqrt{x}-b)}{3\,b^{2}x}$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)
```

3.121.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{1}{x^{\frac{3}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.121.7 Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(3\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)}{3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}}$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqr
t(x) - sqrt(a*x + b*sqrt(x)))^3
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.122 $\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx$

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}+\frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} -\frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x}+\frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}$$

output
```
-4/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2+24/35*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)
-32/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x+64/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^4
/x^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^3-6ab^2\sqrt{x}+8a^2bx-16a^3x^{3/2}\right)}{35b^4x^2}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^3 - 6*a*b^2*Sqrt[x] + 8*a^2*b*x - 16*a^3*x^
(3/2)))/(35*b^4*x^2)
```

3.122.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \ 1922$$

$$-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

$$\Big\downarrow \ 1922$$

$$-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

$$\Big\downarrow \ 1922$$

$$-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

$$\Big\downarrow \ 1920$$

$$-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

input
```
Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5
*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[
x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)
```

3.122.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.122.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{12a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(70x^{\frac{9}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}+70x^{\frac{9}{2}}a^{\frac{9}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}-140x^{\frac{7}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{7}{2}}+35x^{\frac{9}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+}{2\sqrt{a}}\right.\right.}{35\sqrt{\sqrt{x}}}$

input
```
int(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2-12/7*a/b*(-2/5*(b*x^(1/2)+a*x)^(1/2)/b/x^
(3/2)-4/5*a/b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b/x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/
b^2/x^(1/2)))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(8\,a^2bx+5\,b^3-2\left(8\,a^3x+3\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\,b^4x^2}$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-4/35*(8*a^2*b*x + 5*b^3 - 2*(8*a^3*x + 3*a*b^2)*sqrt(x))*sqrt(a*x + b*sqr
t(x))/(b^4*x^2)
```

3.122.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.122.7 Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(70\,a^{\frac{3}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}+84\,ab\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{2}+35\,\sqrt{a}b^{2}\left(\sqrt{\right.\right.}{35\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{7}}$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqr
t(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x)
- sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))
)^7
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.123 $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

3.123.1 Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}+\frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}+\frac{1024a^5\sqrt{b\sqrt{x}+ax}}{693b^6\sqrt{x}}$$

output
```
-4/11*(b*x^(1/2)+a*x)^(1/2)/b/x^3+40/99*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2
)-320/693*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2+128/231*a^3*(b*x^(1/2)+a*x)^(1
/2)/b^4/x^(3/2)-512/693*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x+1024/693*a^5*(b*x^
(1/2)+a*x)^(1/2)/b^6/x^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(63b^5-70ab^4\sqrt{x}+80a^2b^3x-96a^3b^2x^{3/2}+128a^4bx^2-256a^5x^{5/2}\right)}{693b^6x^3}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(63*b^5 - 70*a*b^4*Sqrt[x] + 80*a^2*b^3*x - 96*a
^3*b^2*x^(3/2) + 128*a^4*b*x^2 - 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

3.123.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

input
```
Int[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) - (10*a*((-4*Sqrt[b*Sqrt[x] + a*x])/
(9*b*x^(5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqr
t[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*
x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))
/(11*b)
```

3.123. $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

3.123.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.123.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{11b\,x^{3}}-\frac{20a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}\right)}{9b}\right)}{11b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(1386x^{\frac{13}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{13}{2}}+1386x^{\frac{13}{2}}a^{\frac{13}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}-2772x^{\frac{11}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{11}{2}}+693x^{\frac{13}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}}\right.\right.}{}$

input
```
int(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/11*(b*x^(1/2)+a*x)^(1/2)/b/x^3-20/11*a/b*(-2/9*(b*x^(1/2)+a*x)^(1/2)/b/
x^(5/2)-8/9*a/b*(-2/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)
+a*x)^(1/2)/b/x^(3/2)-4/5*a/b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b/x+4/3*a*(b*x^(
1/2)+a*x)^(1/2)/b^2/x^(1/2)))))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(128\,a^4bx^2+80\,a^2b^3x+63\,b^5-2\left(128\,a^5x^2+48\,a^3b^2x+35\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{693\,b^6x^3}$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^
2*x + 35*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^6*x^3)
```

3.123.6 Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.123.7 Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(3696\,a^{\frac{5}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+7920\,a^2b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+6930\,a\right.}{}$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2
*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)
*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a
*x + b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(
x))) + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.124 $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.124.1 Optimal result

Integrand size = 21, antiderivative size = 171

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{315b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{11/2}}$$

output
```
315/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)-4*x^(5/
2)/a/(b*x^(1/2)+a*x)^(1/2)-315/32*b^3*(b*x^(1/2)+a*x)^(1/2)/a^5-21/4*b*x*(
b*x^(1/2)+a*x)^(1/2)/a^3+9/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+105/16*b^2*
x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4
```

3.124.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-315b^4-105ab^3\sqrt{x}+42a^2b^2x-24a^3bx^{3/2}+16a^4x^2\right)}{32a^5\left(b+a\sqrt{x}\right)} + \frac{315b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{32a^{11/2}}$$

input
```
Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-315*b^4 - 105*a*b^3*Sqrt[x] + 42*a^2*b^2*x - 24*a
^3*b*x^(3/2) + 16*a^4*x^2))/(32*a^5*(b + a*Sqrt[x])) + (315*b^4*ArcTanh[(S
qrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(32*a^(11/2))
```

3.124.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {1924, 1124, 2192, 27, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^3}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int \frac{x^2a^4-bx^{3/2}a^3+b^2xa^2-b^3\sqrt{x}a+b^4}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{\frac{\int \frac{-15bx^{3/2}a^4+8b^2xa^3-8b^3\sqrt{x}a^2+8b^4a}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{-15bx^{3/2}a^4+8b^2xa^3-8b^3\sqrt{x}a^2+8b^4a}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{\frac{\frac{\int \frac{3\left(41b^2xa^4-16b^3\sqrt{x}a^3+16b^4a^2\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{3a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{\frac{\frac{\int \frac{41b^2xa^4-16b^3\sqrt{x}a^3+16b^4a^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \ 2192$$

$$2\left(\frac{\frac{\frac{\frac{\int \frac{a^3b^3(64b-187a\sqrt{x})}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{\frac{\frac{\frac{1}{4}a^2b^3\int \frac{64b-187a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \ 1160$$

$$2\left(\frac{\frac{\frac{\frac{1}{4}a^2b^3\left(\frac{315}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-187\sqrt{ax+b\sqrt{x}}\right)+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \ 1091$$

$$2\left(\frac{\frac{\frac{\frac{1}{4}a^2b^3\left(315b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-187\sqrt{ax+b\sqrt{x}}\right)+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \ 219$$

3.124. $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

$$2\left(\frac{\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}+\frac{\frac{\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}+\frac{1}{4}a^2b^3\left(\frac{315b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-187\sqrt{ax+b\sqrt{x}}\right)}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

input
```
Int[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((-2*b^4*Sqrt[x])/(a^5*Sqrt[b*Sqrt[x] + a*x]) + ((a^3*x^(3/2)*Sqrt[b*Sqr
t[x] + a*x])/4 + (-5*a^3*b*x*Sqrt[b*Sqrt[x] + a*x] + ((41*a^3*b^2*Sqrt[x]*
Sqrt[b*Sqrt[x] + a*x])/2 + (a^2*b^3*(-187*Sqrt[b*Sqrt[x] + a*x] + (315*b*A
rcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]))/4)/(2*a))/(8*a)
)/a^5)
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.124.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{2a\sqrt{b\sqrt{x}+ax}}-\frac{9b\left(\frac{x^{2}}{3a\sqrt{b\sqrt{x}+ax}}-\frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\cdots\right)}{2a}\right)}{2a}\cdots\right)}{4a}\cdots\right)}{6a}\cdots\right)}{4a}\cdots$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(32x^{\frac{3}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{11}{2}}+276x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}b^{2}-48a^{\frac{9}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}bx+690x\sqrt{b\sqrt{x}+ax}\,a^{\frac{7}{2}}b^{3}-768x\,a^{\frac{7}{2}}\sqrt{\cdots}\right.}{\cdots}$

input
```
int(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(5/2)/a/(b*x^(1/2)+a*x)^(1/2)-9/4*b/a*(1/3*x^2/a/(b*x^(1/2)+a*x)^(1/
2)-7/6*b/a*(1/2*x^(3/2)/a/(b*x^(1/2)+a*x)^(1/2)-5/4*b/a*(x/a/(b*x^(1/2)+a*
x)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2
)+a*x)^(1/2)+1/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))+1/a^(3/2)*ln((1/
2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2))))))
```

3.124. $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.124.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.124.6 Sympy [F]

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)
```

3.124.7 Maxima [F]

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{1}{32}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{2\sqrt{x}}{a^2}-\frac{5b}{a^3}\right)+\frac{41b^2}{a^4}\right)\sqrt{x}-\frac{187b^3}{a^5}\right) - \frac{315b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{64a^{\frac{11}{2}}} - \frac{4b^5}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{11}{2}}}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/32*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(2*sqrt(x)/a^2 - 5*b/a^3) + 41*b^
2/a^4)*sqrt(x) - 187*b^3/a^5) - 315/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqr
t(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2) - 4*b^5/((sqrt(a)*(sqrt(a)*sq
rt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(11/2))
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{5/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^(5/2)/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^(5/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.125 $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.125.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}$$

output
```
15/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)-4*x^(3/2)/
a/(b*x^(1/2)+a*x)^(1/2)-15/2*b*(b*x^(1/2)+a*x)^(1/2)/a^3+5*x^(1/2)*(b*x^(1
/2)+a*x)^(1/2)/a^2
```

3.125.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-15b^2-5ab\sqrt{x}+2a^2x\right)}{2a^3\left(b+a\sqrt{x}\right)}+\frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{2a^{7/2}}$$

input
```
Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-15*b^2 - 5*a*b*Sqrt[x] + 2*a^2*x))/(2*a^3*(b + a*
Sqrt[x])) + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x]
)])/(2*a^(7/2))
```

3.125.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1924, 1124, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^2}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int \frac{xa^2-b\sqrt{x}a+b^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{\frac{\int \frac{ab\left(4b-7a\sqrt{x}\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{1}{4}b\int \frac{4b-7a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{\frac{1}{4}b\left(\frac{15}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-7\sqrt{ax+b\sqrt{x}}\right)+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{\frac{1}{4}b\left(15b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-7\sqrt{ax+b\sqrt{x}}\right)+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$\downarrow$ 219

$$2\left(\frac{\frac{1}{4}b\left(\frac{15b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-7\sqrt{ax+b\sqrt{x}}\right)+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

input
```
Int[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((-2*b^2*Sqrt[x])/(a^3*Sqrt[b*Sqrt[x] + a*x]) + ((a*Sqrt[x]*Sqrt[b*Sqrt[
x] + a*x])/2 + (b*(-7*Sqrt[b*Sqrt[x] + a*x] + (15*b*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]))/4)/a^3)
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

3.125. $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.125.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{a\sqrt{b\sqrt{x}+ax}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{2a}$
default	$\sqrt{b\sqrt{x}+ax}\left(4x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}+10x\sqrt{b\sqrt{x}+ax}\,a^{\frac{7}{2}}b-32x\,a^{\frac{7}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b+16x\,a^{3}\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\sqrt{a}+b}{2\sqrt{a}}\right.\right.$

input
```
int(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^(3/2)/a/(b*x^(1/2)+a*x)^(1/2)-5/2*b/a*(x/a/(b*x^(1/2)+a*x)^(1/2)-3/2*b/a
*(-x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b
/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))+1/a^(3/2)*ln((1/2*b+a*x^(1/2))/a
^(1/2)+(b*x^(1/2)+a*x)^(1/2))))
```

3.125.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.125.6 Sympy [F]

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

3.125.7 Maxima [F]

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a^2}-\frac{7\,b}{a^3}\right) - \frac{15\,b^2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{4\,a^{\frac{7}{2}}} - \frac{4\,b^3}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{7}{2}}}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a^2 - 7*b/a^3) - 15/4*b^2*log(abs(-2*
sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(7/2) - 4*b^3/((
sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(7/2))
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{3/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^(3/2)/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.126 $\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.126.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

output
```
4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)-4*x^(1/2)/a/(b*x^
(1/2)+a*x)^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{a\left(b+a\sqrt{x}\right)}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{3/2}}$$

input
```
Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(a*(b + a*Sqrt[x])) + (4*ArcTanh[(Sqrt[a]*Sqrt[
b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/a^(3/2)
```

3.126.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1924, 1124, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1924}$$

$$2\int \frac{x}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1124}$$

$$2\left(\frac{\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a}-\frac{2\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{ 1091}$$

$$2\left(\frac{2\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}-\frac{2\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{2\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}\right)$$

input
```
Int[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((-2*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (2*ArcTanh[(Sqrt[a]*Sqrt[x])/S
qrt[b*Sqrt[x] + a*x]])/a^(3/2))
```

3.126.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 2.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

method	result
derivativedivides	$-\frac{2\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{a}+\frac{2\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{b\sqrt{x}+ax}\left(2x\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{5}{2}}-x\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+4\sqrt{x}\,\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}b-2\sqrt{x}\,\ln\left(\frac{2a\cdots}{\cdots}\right.\cdots}{a^{\frac{3}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}\cdots\right.}\cdots}$

input
```
int(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(
b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))+2/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/
2)+(b*x^(1/2)+a*x)^(1/2))
```

3.126.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.126.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

3.126.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{2\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{a^{\frac{3}{2}}} - \frac{4\,b}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(3/
2) - 4*b/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(3/2))
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{\sqrt{x}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.127 $\int \frac{1}{\sqrt{x}(b\sqrt{x}+ax)^{3/2}} dx$

3.127.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}} dx = -\frac{4(b+2a\sqrt{x})}{b^2\sqrt{b\sqrt{x}+ax}}$$

output
```
-4*(b+2*a*x^(1/2))/b^2/(b*x^(1/2)+a*x)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}} dx = -\frac{4(b+2a\sqrt{x})\sqrt{b\sqrt{x}+ax}}{b^2\left(b+a\sqrt{x}\right)\sqrt{x}}$$

input
```
Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^2*(b + a*Sqrt[x])*Sqrt[x])
```

3.127.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1919, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1919}$$

$$2\int \frac{1}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1088}$$

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

3.127.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, 
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

3.127.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{4(b+2a\sqrt{x})}{b^2\sqrt{b\sqrt{x}+ax}}$	25
default	$-\frac{4\sqrt{b\sqrt{x}+ax}\left(x(b\sqrt{x}+ax)^{\frac{3}{2}}a^2+2\sqrt{x}\,(b\sqrt{x}+ax)^{\frac{3}{2}}ab-(\sqrt{x}\,(a\sqrt{x}+b))^{\frac{3}{2}}a^2x+(b\sqrt{x}+ax)^{\frac{3}{2}}b^2\right)}{\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,b^3x(a\sqrt{x}+b)^2}$	111

input
```
int(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(b+2*a*x^(1/2))/b^2/(b*x^(1/2)+a*x)^(1/2)
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\left(abx-(2a^2x-b^2)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{a^2b^2x^2-b^4x}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b
^4*x)
```

3.127.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\sqrt{x}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.127.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(\frac{2\,a\sqrt{x}}{b^2}+\frac{1}{b}\right)}{\sqrt{ax+b\sqrt{x}}}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.128 $\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.128.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}$$

output
```
4/b/x/(b*x^(1/2)+a*x)^(1/2)-24/5*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)+32/5*a*
(b*x^(1/2)+a*x)^(1/2)/b^3/x-64/5*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(b^3-2ab^2\sqrt{x}+8a^2bx+16a^3x^{3/2}\right)}{5b^4\left(b+a\sqrt{x}\right)x^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3
/2)))/(5*b^4*(b + a*Sqrt[x])*x^(3/2))
```

3.128.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x*Sqrt[b*Sqrt[x] + a*x]) + (6*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2
)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x]
)/(3*b^2*Sqrt[x])))/(5*b)))/b
```

3.128.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.128.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{4}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{12a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a\left(b+2a\sqrt{x}\right)}{3b^{3}\sqrt{b\sqrt{x}+ax}}\right)}{5b}$
default	$\frac{2\sqrt{b\sqrt{x}+ax}\left(10x^{\frac{9}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{11}{2}}+10x^{\frac{9}{2}}a^{\frac{11}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}-30x^{\frac{7}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}+10x^{\frac{7}{2}}a^{\frac{9}{2}}\left(\sqrt{x}\left(a\sqrt{x}+b\right)\right)^{\frac{3}{2}}+10x\right.}{}$

input
```
int(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5/b/x/(b*x^(1/2)+a*x)^(1/2)-12/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1
/2)+8/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\left(8\,a^3bx^2-3\,ab^3x-\left(16\,a^4x^2-10\,a^2b^2x-b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{5\left(a^2b^4x^3-b^6x^2\right)}$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*
sqrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)
```

3.128.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.128.7 Maxima [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

3.128.8 Giac [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{3/2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.129 $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}-\frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{63b^5x}-\frac{1024a^4\sqrt{b\sqrt{x}+ax}}{63b^6\sqrt{x}}$$

output
```
4/b/x^2/(b*x^(1/2)+a*x)^(1/2)-40/9*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2)+320/6
3*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2-128/21*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(
3/2)+512/63*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x-1024/63*a^4*(b*x^(1/2)+a*x)^(1
/2)/b^6/x^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(7b^5-10ab^4\sqrt{x}+16a^2b^3x-32a^3b^2x^{3/2}+128a^4bx^2+256a^5x^{5/2}\right)}{63b^6\left(b+a\sqrt{x}\right)x^{5/2}}$$

input
```
Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^
3*b^2*x^(3/2) + 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*(b + a*Sqrt[x])*
x^(5/2))
```

3.129.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1921, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1922

$$\frac{10\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1920

$$\frac{10\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) + (10*((-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(
5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt
[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*
a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))/b
```

3.129. $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.129.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.129.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{4}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{20a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a\left(b+2a\sqrt{x}\right)}{3b^{3}\sqrt{b\sqrt{x}+ax}}\right)}{5b}\right)}{7b}\right)}{9b}$
default	$\frac{4\sqrt{b\sqrt{x}+ax}\left(126x^{\frac{13}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{15}{2}}+126x^{\frac{13}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{15}{2}}-315x^{\frac{11}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{13}{2}}+63x^{\frac{11}{2}}\left(\sqrt{x}\left(a\sqrt{x}+b\right)\right)^{\frac{3}{2}}\cdots}{\cdots}$

input
```
int(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9/b/x^2/(b*x^(1/2)+a*x)^(1/2)-20/9*a/b*(-2/7/b/x^(3/2)/(b*x^(1/2)+a*x)^
(1/2)-8/7*a/b*(-2/5/b/x/(b*x^(1/2)+a*x)^(1/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x
^(1/2)+a*x)^(1/2)+8/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\left(128\,a^5bx^3-48\,a^3b^3x^2-17\,ab^5x-\left(256\,a^6x^3-160\,a^4b^2x^2-26\,a^2b^4x-7\,b^6\right)\cdots\right.}{63\left(a^2b^6x^4-b^8x^3\right)}$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4
*b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x
^4 - b^8*x^3)
```

3.129.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{5}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.129.7 Maxima [F]

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

3.129.8 Giac [F]

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{5/2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.130 $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

3.130.1 Optimal result

Integrand size = 21, antiderivative size = 223

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} +\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}}+\frac{2560a^3\sqrt{b\sqrt{x}+ax}}{429b^5x^2} -\frac{1024a^4\sqrt{b\sqrt{x}+ax}}{143b^6x^{3/2}}+\frac{4096a^5\sqrt{b\sqrt{x}+ax}}{429b^7x}-\frac{8192a^6\sqrt{b\sqrt{x}+ax}}{429b^8\sqrt{x}}$$

output
```
4/b/x^3/(b*x^(1/2)+a*x)^(1/2)-56/13*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(7/2)+672/
143*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^3-2240/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4
/x^(5/2)+2560/429*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^2-1024/143*a^4*(b*x^(1/2
)+a*x)^(1/2)/b^6/x^(3/2)+4096/429*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x-8192/429
*a^6*(b*x^(1/2)+a*x)^(1/2)/b^8/x^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(33b^7-42ab^6\sqrt{x}+56a^2b^5x-80a^3b^4x^{3/2}+128a^4b^3x^2-256a^5b^2x^{5/2}+1024a^6bx^3+2048a^7\right.}{429b^8\left(b+a\sqrt{x}\right)x^{7/2}}$$

input
```
Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a
^3*b^4*x^(3/2) + 128*a^4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 +
2048*a^7*x^(7/2)))/(429*b^8*(b + a*Sqrt[x])*x^(7/2))
```

3.130.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{14\int \frac{1}{x^4\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\int \frac{1}{x^{7/2}\sqrt{\sqrt{x}b+ax}}dx}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1922

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1922

3.130. $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int\frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\downarrow 1922$$

3.130. $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

$$14\Bigg(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}\,dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^{2}}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^{3}}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

3.130. $\int\frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

$$\Big\downarrow \; 1920$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

3.130. $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

output
```
4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) + (14*((-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^
(7/2)) - (12*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) - (10*a*((-4*Sqrt[b*
Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2)
 - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[
x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))
/(7*b)))/(9*b)))/(11*b)))/(13*b)))/b
```

3.130.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.130.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{4}{13b\,x^{3}\sqrt{b\sqrt{x}+ax}}-\frac{28a\left(-\frac{2}{11b\,x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{12a\left(-\frac{2}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{10a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6}{}\right.}{}\right.}{9b}\right.}{11b}\right.}{13b}$
default	$\frac{2\sqrt{b\sqrt{x}+ax}\left(2574x^{\frac{17}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{19}{2}}+2574x^{\frac{17}{2}}a^{\frac{19}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}-6006x^{\frac{15}{2}}(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{17}{2}}+858x^{\frac{15}{2}}a^{\frac{17}{2}}(\sqrt{x}\,(a\sqrt{\right.}{}$

input
```
int(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/13/b/x^3/(b*x^(1/2)+a*x)^(1/2)-28/13*a/b*(-2/11/b/x^(5/2)/(b*x^(1/2)+a*
x)^(1/2)-12/11*a/b*(-2/9/b/x^2/(b*x^(1/2)+a*x)^(1/2)-10/9*a/b*(-2/7/b/x^(3
/2)/(b*x^(1/2)+a*x)^(1/2)-8/7*a/b*(-2/5/b/x/(b*x^(1/2)+a*x)^(1/2)-6/5*a/b*
(-2/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+8/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)
+a*x)^(1/2))))))
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\left(1024\,a^7bx^4 - 384\,a^5b^3x^3 - 136\,a^3b^5x^2 - 75\,ab^7x - (2048\,a^8x^4 - 1280\,a^6b^2x^3 \cdots\right.}{429\left(a^2b^8x^5 - b^{10}x^4\right)}$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (
2048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)
*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)
```

3.130.6 Sympy [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**(7/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.130.7 Maxima [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

3.130.8 Giac [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{7/2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.131 $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

3.131.1 Optimal result

Integrand size = 19, antiderivative size = 301

$$\begin{aligned}\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx &= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}\\ &\quad+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}\\ &\quad+\frac{442b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{25/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+884/24035*b^5*x^(2/3)*(b*x^(1/3)+
a*x)^(1/2)/a^5-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+476/19665
*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^3-28/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/
2)/a^2+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/9*x^4*(b*x^(1/3)+a*x)^(1
/2)+442/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^
(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/
(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(25/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.131.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.51

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$$
$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}(-9945b^6+3978ab^5x^{2/3}-3094a^2b^4x^{4/3}+2618a^3b^3x^2-2310a^4b^2x^{8/3}+2090a\right.}{216315a^6\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-9945*b^6 + 3978*a*b^5*
x^(2/3) - 3094*a^2*b^4*x^(4/3) + 2618*a^3*b^3*x^2 - 2310*a^4*b^2*x^(8/3) +
 2090*a^5*b*x^(10/3) + 24035*a^6*x^4) + 9945*b^6*Hypergeometric2F1[-1/2, 1
/4, 5/4, -((a*x^(2/3))/b)]))/(216315*a^6*Sqrt[1 + (a*x^(2/3))/b])
```

3.131.3 Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1927, 1930, 1930, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{ax+b\sqrt[3]{x}}\,dx$$
$$\Big\downarrow \text{1924}$$
$$3\int x^{11/3}\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}$$
$$\Big\downarrow \text{1927}$$
$$3\left(\frac{2}{27}b\int \frac{x^4}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{27}x^4\sqrt{ax+b\sqrt[3]{x}}\right)$$
$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\int \frac{x^{10/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{23a}\right)+\frac{2}{27}x^4\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\int \frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19a}\right)}{23a}\right)+\frac{2}{27}x^4\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int \frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{19a}\right)}{23a}\right)+\frac{2}{27}x^4\sqrt{ax+b\right.$$

$$\Big\downarrow \text{1930}$$

3.131. $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)}{19a}\right)}{23a}\right.\right.$$

↓ 1930

3.131. $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int \frac{}{\sqrt{}}}{}\right.}{11a}\right.}{15a}\right.}{19a}\right.}{23a}\right.\right.$$

3.131. $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

$\downarrow$ 1930

3.131. $\int x^3 \sqrt{b\sqrt[3]{x}+ax}\,dx$

$$\Bigg(\Bigg(\Bigg(21b \left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a} - \frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a} - \frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a} - \frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a} - \frac{5b\left(\frac{2\sqrt{}}{}\right.}{}\right.}{}\right.}{15a}\right.}{19a}\right.$$

3.131. $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

↓ 1917

3.131. $\int x^3 \sqrt{b\sqrt[3]{x}+ax}\,dx$

$5b\left(2\sqrt{\right.$

$9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\right.$

$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$

$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\right.$

$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{\quad}{19a}\right.$

$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{\quad}{23a}\right.\right.$

3.131. $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

$\downarrow$ 266

3.131. $\int x^3 \sqrt{b\sqrt[3]{x}+ax}\,dx$

$$5b\left(\frac{2\sqrt{}}{}\right.$$

$$9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{}{}\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{}{}\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{}{15}\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{}{19a}\right.$$

$$3\left(\frac{2}{27}b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{}{23a}\right.\right.$$

3.131. $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

$\downarrow$ 761

3.131. $\int x^3 \sqrt{b\sqrt[3]{x} + ax}\, dx$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

3.131. $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

input
```
Int[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^4*Sqrt[b*x^(1/3) + a*x])/27 + (2*b*((2*x^(10/3)*Sqrt[b*x^(1/3) + a
*x])/(23*a) - (21*b*((2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(19*a) - (17*b*((2*
x^2*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x
])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[
b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*
x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*Ellip
ticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3)
+ a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a)))/(19*a)))/(23*a)))/
27)
```

3.131.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.131.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-\frac{6188b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^4}+$ 88
default	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-\frac{6188b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^4}+$ 88

input
```
int(x^3*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a-28/
1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+476/19665*b^3*x^2*(b*x^(1/3)+a*
x)^(1/2)/a^3-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+884/24035*b
^5*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a
^6+442/14421*b^7/a^7*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/
2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a
/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^
(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

3.131.5 Fricas [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))*x^3, x)
```

3.131.6 Sympy [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^3\sqrt{ax+b\sqrt[3]{x}}\,dx$$

input
```
integrate(x**3*(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*x + b*x**(1/3)), x)
```

3.131.7 Maxima [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

3.131.8 Giac [F]

$$\int x^3 \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^3\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

input
```
int(x^3*(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^3*(a*x + b*x^(1/3))^(1/2), x)
```

3.132 $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

3.132.1 Optimal result

Integrand size = 19, antiderivative size = 411

$$\begin{aligned}\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx &= \frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}\\
&\quad-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}\\
&\quad-\frac{44b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad+\frac{22b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
44/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(9/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1
/3)+a*x)^(1/2)-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+220/4641*b^3*x
*(b*x^(1/3)+a*x)^(1/2)/a^3-60/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+4
/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)-44/22
1*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(
1/2)+b^(1/2))^2)^(1/2)/a^(19/4)/(b*x^(1/3)+a*x)^(1/2)+22/221*b^(21/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(
1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2
)^(1/2)/a^(19/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.132.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.33

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(-385b^4+110ab^3x^{2/3}-90a^2b^2x^{4/3}+78a^3bx^2+663a^4x^{8/3}\right)+385b^4\,\mathrm{Hyper}\right.}{4641a^4\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-385*b^4 + 110*
a*b^3*x^(2/3) - 90*a^2*b^2*x^(4/3) + 78*a^3*b*x^2 + 663*a^4*x^(8/3)) + 385
*b^4*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(4641*a^4*Sqrt[
1 + (a*x^(2/3))/b])
```

3.132.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1924, 1927, 1930, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+b\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{8/3}\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{21}b\int \frac{x^3}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17a}\right)+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)}{17a}\right)+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int\frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{13a}\right)}{17a}\right)+\frac{2}{21}x^3\sqrt{ax+b\sqrt[3]{x}}$$

$\downarrow$ 1930

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)}{13a}\right)}{17a}\right)\right)$$

$$\downarrow 1938$$

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{\ldots}}{5a\sqrt{ax+b\sqrt[3]{x}}}\ldots\right.}{9a}\ldots\right.}{13a}\ldots\right.}{17a}\ldots\right.\right.\right.$$

$\downarrow$ 266

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{\right.}}{5a\sqrt{ax+b\sqrt[3]{x}}}}{9a}}{13a}}{17a}\right.\right.$$

$\downarrow$ 834

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}}{\sqrt{a}}\right.}{5a\sqrt{ax}}\right.}{9a}\right.}{13a}\right.}{17a}\right.\right.$$

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$\Big\downarrow 27$$

3.132. $\int x^2 \sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+\cdots}}}{\sqrt{a}}\cdots\right.}{5a\sqrt{ax+\cdots}}\cdots\right.}{9a}\cdots\right.}{13a}\cdots\right.}{17a}\cdots\right.\right.$$

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

$\downarrow$ 761

3.132. $\int x^2 \sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\Bigg(\frac{2}{21}b\Bigg(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\Bigg(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\Bigg(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\Bigg(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\Bigg(\frac{\sqrt[4]{b}(\sqrt{a}x^{2/3}+\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}}{17a}$$

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

$\downarrow$ 1510

3.132. $\int x^2 \sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{21}b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

input
```
Int[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^3*Sqrt[b*x^(1/3) + a*x])/21 + (2*b*((2*x^(7/3)*Sqrt[b*x^(1/3) + a*
x])/(17*a) - (15*b*((2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*((2*x
*Sqrt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5
*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-(((x^(1/6)*Sqrt[b + a*x^(4/3)])
/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/
4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(
2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*S
qrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/(17*a
)))/21)
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.132. $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

3.132.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^2}+\frac{220b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^3}-\frac{44b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^4}+\frac{22b^5\sqrt{-}}{}$
default	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^2}+\frac{220b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^3}-\frac{44b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^4}+\frac{22b^5\sqrt{-}}{}$

input
```
int(x^2*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^3*(b*x^(1/3)+a*x)^(1/2)+4/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a-60/1
547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+220/4641*b^3*x*(b*x^(1/3)+a*x)^(
1/2)/a^3-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+22/221*b^5/a^5*(-a*b
)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a
*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^
(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/
a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.132.5 Fricas [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x^2\,dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))*x^2, x)
```

3.132.6 Sympy [F]

$$\int x^2 \sqrt{b \sqrt[3]{x} + ax}\, dx = \int x^2 \sqrt{ax + b \sqrt[3]{x}}\, dx$$

input
```
integrate(x**2*(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*x + b*x**(1/3)), x)
```

3.132.7 Maxima [F]

$$\int x^2 \sqrt{b \sqrt[3]{x} + ax}\, dx = \int \sqrt{ax + bx^{\frac{1}{3}}}x^2\, dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

3.132.8 Giac [F]

$$\int x^2 \sqrt{b \sqrt[3]{x} + ax}\, dx = \int \sqrt{ax + bx^{\frac{1}{3}}}x^2\, dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^2\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

input
```
int(x^2*(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^2*(a*x + b*x^(1/3))^(1/2), x)
```

3.133 $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

3.133.1 Optimal result

Integrand size = 17, antiderivative size = 213

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2}+\frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{6b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{13/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-36/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/
2)/a^2+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/5*x^2*(b*x^(1/3)+a*x)^(1/2
)-6/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(13/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.133.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$$
$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(45b^3-18ab^2x^{2/3}+14a^2bx^{4/3}+77a^3x^2\right)-45b^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4}\right.\right.}{385a^3\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(45*b^3 - 18*a*b^2*x^(2/
3) + 14*a^2*b*x^(4/3) + 77*a^3*x^2) - 45*b^3*Hypergeometric2F1[-1/2, 1/4,
5/4, -((a*x^(2/3))/b)]))/(385*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

3.133.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1924, 1927, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+b\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{5/3}\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{15}b\int \frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$\big\downarrow$ 1930

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right)+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$\big\downarrow$ 1930

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{7a}\right)}{11a}\right)+\frac{2}{15}x^2\sqrt{ax+b\sqrt[3]{x}}\right)$$

$\big\downarrow$ 1917

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)+\frac{2}{15}x^2\sqrt{\right.$$

$\big\downarrow$ 266

3.133. $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)+\frac{2}{15}x^2\sqrt{ax\right)$$

$\Big\downarrow$ 761

$$3\left(\frac{2}{15}b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(\right.}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}\right.}{7a}}{11a}\right.\right.$$

input
```
Int[x*Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^2*Sqrt[b*x^(1/3) + a*x])/15 + (2*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*
x])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt
[b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a
*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*Elli
pticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3)
 + a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/15)
```

3.133. $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

3.133.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.133.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^2}+\frac{12b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^3}-\frac{6b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\cdots}}{\cdots}$
default	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^2}+\frac{12b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^3}-\frac{6b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\cdots}}{\cdots}$

input
```
int(x*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^2*(b*x^(1/3)+a*x)^(1/2)+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a-36/38
5*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^
3-6/77*b^4/a^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1
/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b
)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))
*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

3.133.5 Fricas [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))*x, x)
```

3.133.6 Sympy [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x\sqrt{ax+b\sqrt[3]{x}}\,dx$$

input
```
integrate(x*(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(a*x + b*x**(1/3)), x)
```

3.133.7 Maxima [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

3.133.8 Giac [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

input
```
int(x*(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x*(a*x + b*x^(1/3))^(1/2), x)
```

3.134 $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$

3.134.1 Optimal result

Integrand size = 15, antiderivative size = 323

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx = -\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}$$
$$+\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}}$$
$$-\frac{2b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-4/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(3/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3
)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/3*x*(b*x^(1/3)+a*x)^
(1/2)+4/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(
1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1
/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)-2/5*b^(9/4)*x^
(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2
^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))
^2)^(1/2)/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.134.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)\sqrt{1+\frac{ax^{2/3}}{b}}-b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)\right)}{3a\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))*Sqrt[1 + (a*x^(2/3))/b]
- b*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(3*a*Sqrt[1 + (a
*x^(2/3))/b])
```

3.134.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1910, 1924, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+b\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{9}b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}dx+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{1924}$$

$$\frac{2}{3}b\int \frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$\downarrow$ 1938

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$\downarrow$ 266

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$\downarrow$ 834

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$\downarrow$ 27

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$\downarrow$ 761

3.134. $\int\sqrt{b\sqrt[3]{x}+ax}\,dx$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}\cdots}{\sqrt{ax^{4/3}\cdots}}}{\sqrt{\cdots}}\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\right.$$

$$\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

$$\Big\downarrow \; 1510$$

$$\frac{2}{3}b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x\cdots\right.}{\cdots}\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\right.$$

$$\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

input
```
Int[Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*x*Sqrt[b*x^(1/3) + a*x])/3 + (2*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5
*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])
/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/
4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(
2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*S
qrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/3
```

3.134. $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.134.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x \cdots\right.}{\cdots}}\right.}{\cdots}\right.}{5a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x \cdots\right.}{\cdots}}\right.}{\cdots}\right.}{5a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int((b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(b*x^(1/3)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a-2/5/a^2
*b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x
^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(
1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*
b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((
x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.134.5 Fricas [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3)), x)
```

3.134.6 Sympy [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+b\sqrt[3]{x}}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3)), x)
```

3.134.7 Maxima [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3)), x)
```

3.134.8 Giac [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3)), x)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\frac{6\,x\,\sqrt{a\,x+b\,x^{1/3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{7\,\sqrt{\frac{a\,x^{2/3}}{b}+1}}$$

input
```
int((a*x + b*x^(1/3))^(1/2),x)
```

output
```
(6*x*(a*x + b*x^(1/3))^(1/2)*hypergeom([-1/2, 7/4], 11/4, -(a*x^(2/3))/b))
/(7*((a*x^(2/3))/b + 1)^(1/2))
```

3.135 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$

3.135.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = 2\sqrt{b\sqrt[3]{x}+ax} + \frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
2*(b*x^(1/3)+a*x)^(1/2)+2*b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+
a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^(1/3)+a*x)^(1/2
)
```

3.135.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x,x]
```

output
```
(6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((a*x^(2/3))/b
)])/Sqrt[1 + (a*x^(2/3))/b]
```

3.135.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1924, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{3}b\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1917}$$

$$3\left(\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x,x]
```

output
```
3*((2*Sqrt[b*x^(1/3) + a*x])/3 + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sq
rt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))
^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[b
*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]))
```

3.135.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.135. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.135.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$	132
default	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$	132

input
```
int((b*x^(1/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(1/3)+a*x)^(1/2)+2*b/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(
-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-
x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/
a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

3.135.5 Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x, x)
```

3.135.6 Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x, x)
```

3.135.7 Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

3.135.8 Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x, x)
```

3.136 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

3.136.1 Optimal result

Integrand size = 19, antiderivative size = 325

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{12a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} + \frac{6a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
12/5*a^(3/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+
a*x)^(1/2)-6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(1
/3)-12/5*a^(5/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/
3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)+6/5*a^(5/4)*x^(
1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)
*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^
(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^
2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.136.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}x}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^(2/3)
)/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x)
```

3.136.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1924, 1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^2}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 1926$$

$$3\left(\frac{2}{5}a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \; 1931$$

$$3\left(\frac{2}{5}a\left(\frac{a\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \; 1938$$

$$3\left(\frac{2}{5}a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \; 266$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \; 834$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \; 27$$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)$$

$$\Big\downarrow \; 761$$

3.136. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{\ldots}}{\ldots}\right.\right.$$

$\downarrow$ 1510

$$3\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+\ldots}{\left(\sqrt{a}x^{2/3}+\ldots\right.}}}{\sqrt[4]{a}\sqrt{\ldots}}}{\ldots}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.\right.$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(5*x) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*
x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-(-((x^(1/6)*Sqrt[b + a*x^(4
/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*
Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^
(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (
b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a
]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3
/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x])))/5)
```

3.136. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.136. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.136.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\cdots}\right.\cdots}{\cdots}\cdots\right)}{5b\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\cdots}\right.\cdots}{\cdots}\cdots\right)}{5b\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*(b+a*x^(2/3))*a/b/(x^(1/3)*(b+a*x^(2/3))
)^(1/2)+6/5*a/b*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*
b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*E
llipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.136.5 Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^2, x)
```

3.136.6 Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^2}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**2, x)
```

3.136.7 Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

3.136.8 Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = \int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^2}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^2,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^2, x)
```

3.137 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$

3.137.1 Optimal result

Integrand size = 19, antiderivative size = 188

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}+\frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}+\frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20
/77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^(11/4)*x^(1/6)*(cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3
)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(9/
4)/(b*x^(1/3)+a*x)^(1/2)
```

3.137.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=-\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{2},-\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{11\sqrt{1+\frac{ax^{2/3}}{b}}x^2}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((a*x^(2/3
))/b)])/(11*Sqrt[1 + (a*x^(2/3))/b]*x^2)
```

3.137.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1924, 1926, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^3}dx$$

$$\Big\downarrow \text{ 1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1926}$$

$$3\left(\frac{2}{11}a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{2}{11}a\left(-\frac{5a\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$$\Big\downarrow \text{1917}$$

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax}}{7b}\right.\right.$$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

3.137. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(11*x^2) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/
(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)*
(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3
))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*b)
))/11)
```

3.137.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.137. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.137.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^2}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^2x^{\frac{2}{3}}}+\frac{10a^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}}{77b^2\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^2}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^2x^{\frac{2}{3}}}+\frac{10a^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}}{77b^2\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20
/77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^2/b^2*(-a*b)^(1/2)*((x^(
1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2)
)*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1
/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))
```

3.137.5 Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^3, x)
```

3.137.6 Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^3}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**3, x)
```

3.137.7 Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

3.137.8 Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^3}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^3,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^3, x)
```

3.138 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

3.138.1 Optimal result

Integrand size = 19, antiderivative size = 413

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx
&= -\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} \\
&\quad + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} \\
&\quad + \frac{308a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{154a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

output
```
-308/1105*a^(9/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x
^(1/3)+a*x)^(1/2)-6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^
(1/2)/b/x^(7/3)+44/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*
(b*x^(1/3)+a*x)^(1/2)/b^3/x+308/1105*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)
+308/1105*a^(17/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2
)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^
(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)-154/1105*a^(
17/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
)),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+
b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{17}{4},-\frac{1}{2},-\frac{13}{4},-\frac{ax^{2/3}}{b}\right)}{17\sqrt{1+\frac{ax^{2/3}}{b}}x^3}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-17/4, -1/2, -13/4, -((a*x^(2/
3))/b)])/(17*Sqrt[1 + (a*x^(2/3))/b]*x^3)
```

3.138.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1924, 1926, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^4}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{17}a\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{17}a\left(-\frac{11a\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)$$

$$\Big\downarrow \text{1931}$$

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{a\cdots}}{1\cdots}$$

$\downarrow$ 1938

3.138. $\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right.\right.$$

$\downarrow$ 266

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right.\right.$$

$\downarrow$ 834

3.138. $\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax \cdots}}{9b \cdots}}{13b} \cdots$$

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$\downarrow 27$$

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\cdots}}{9bx^{5/3}}\cdots\right)}{13b}\cdots\right)\right)$$

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$\downarrow$ 761

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}}{5b}}{9b}}{13b}$$

3.138. $\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$\downarrow$ 1510

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

$$3\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\cdots}}{\cdots}\cdots}{b\sqrt{ax+b\sqrt[3]{x}}}\cdots}{5b}\cdots}{9b}\cdots}{13b}\cdots$$

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(17*x^3) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/
(13*b*x^(7/3)) - (11*a*((-2*Sqrt[b*x^(1/3) + a*x])/(9*b*x^(5/3)) - (7*a*((
-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^
(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3
)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sq
rt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1
/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^
(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*
x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4
)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x])))/(5*b)))/(9*b)))/(13*b
)))/17)
```

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.138. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

3.138.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{17x^{3}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{221b\,x^{\frac{7}{3}}}+\frac{44a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^{2}x^{\frac{5}{3}}}-\frac{308a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^{3}x}+\frac{308\left(b+a\,x^{\frac{2}{3}}\right)a^{4}}{1105b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-$ $154a^{4}\sqrt{-}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{17x^{3}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{221b\,x^{\frac{7}{3}}}+\frac{44a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^{2}x^{\frac{5}{3}}}-\frac{308a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^{3}x}+\frac{308\left(b+a\,x^{\frac{2}{3}}\right)a^{4}}{1105b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-$ $154a^{4}\sqrt{-}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+4
4/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*x^(1/3)+a*x)^(
1/2)/b^3/x+308/1105*(b+a*x^(2/3))*a^4/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-15
4/1105*a^4/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1
/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b
)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3
)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*El
lipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.138.5 Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^4, x)
```

3.138.6 Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^4}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**4, x)
```

3.138.7 Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

3.138.8 Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^4}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^4,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^4, x)
```

3.139 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

3.139.1 Optimal result

Integrand size = 19, antiderivative size = 276

$$\begin{aligned}\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}}+\frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}}\\ &\quad-\frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}+\frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}-\frac{2652a^5\sqrt{b\sqrt[3]{x}+ax}}{33649b^5x^{2/3}}\\ &\quad-\frac{1326a^{23/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+
68/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x
)^(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/336
49*a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^(23/4)*x^(1/6)*(cos(
2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^
(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b
^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{23}{4},-\frac{1}{2},-\frac{19}{4},-\frac{ax^{2/3}}{b}\right)}{23\sqrt{1+\frac{ax^{2/3}}{b}}\,x^4}$$

input
```
Integrate[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

output
```
(-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-23/4, -1/2, -19/4, -((a*x^(2/
3))/b)])/(23*Sqrt[1 + (a*x^(2/3))/b]*x^4)
```

3.139.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1924, 1926, 1931, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^5}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{13/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{23}a\int \frac{1}{x^{10/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{23}a\left(-\frac{17a\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\int \frac{1}{x^2\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)$$

$$\Big\downarrow \text{1931}$$

3.139. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\right.$$

$\downarrow$ 1931

3.139. $\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{a}}{19}\right.\right.$$

3.139. $\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

$\downarrow$ 1917

3.139. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\,\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\cdots}}{15bx^{8/\cdots}}\cdots\right)}{19b}\cdots\right.\right.$$

3.139. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

$$\Big\downarrow \; 266$$

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\cdots\right)}{19b}\cdots\right)\right)$$

3.139. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

↓ 761

3.139. $\int \frac{\sqrt{b \sqrt[3]{x} + ax}}{x^5} \, dx$

$$3\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\cdots}{11b}}{15b}}{19b}\right.\right.$$

3.139. $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

input
```
Int[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(23*x^4) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/
(19*b*x^(10/3)) - (17*a*((-2*Sqrt[b*x^(1/3) + a*x])/(15*b*x^(8/3)) - (13*a
*((-2*Sqrt[b*x^(1/3) + a*x])/(11*b*x^2) - (9*a*((-2*Sqrt[b*x^(1/3) + a*x])
/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)
*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/
3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^
(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]))))/(7*b
)))/(11*b)))/(15*b)))/(19*b)))/23)
```

3.139.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.139.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^{4}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^{2}x^{\frac{8}{3}}}-\frac{884a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^{3}x^{2}}+\frac{7956a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{168245b^{4}x^{\frac{4}{3}}}-\frac{2652a^{5}\sqrt{b\,x^{\frac{1}{3}}}}{33649b^{5}x}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^{4}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^{2}x^{\frac{8}{3}}}-\frac{884a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^{3}x^{2}}+\frac{7956a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{168245b^{4}x^{\frac{4}{3}}}-\frac{2652a^{5}\sqrt{b\,x^{\frac{1}{3}}}}{33649b^{5}x}$

input
```
int((b*x^(1/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+
68/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x
)^(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/336
49*a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^5/b^5*(-a*b)^(1/2)*(
(x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x
)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))
```

3.139.5 Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(a*x + b*x^(1/3))/x^5, x)
```

3.139.6 Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^5}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a*x + b*x**(1/3))/x**5, x)
```

3.139.7 Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

3.139.8 Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = \int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^5}\,dx$$

input
```
int((a*x + b*x^(1/3))^(1/2)/x^5,x)
```

output
```
int((a*x + b*x^(1/3))^(1/2)/x^5, x)
```

3.140 $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

3.140.1 Optimal result

Integrand size = 19, antiderivative size = 298

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{884b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
2/9*x^3*(b*x^(1/3)+a*x)^(3/2)+1768/100947*b^6*(b*x^(1/3)+a*x)^(1/2)/a^5-17
68/168245*b^5*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+1768/216315*b^4*x^(4/3)*(b
*x^(1/3)+a*x)^(1/2)/a^3-136/19665*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+8/1311
*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a+4/69*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)
-884/100947*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x
^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.48

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(3315b^4-7293ab^3x^{2/3}+12155a^2b^2x^{4/3}-17765a^3bx^2\right.\right.}{216315a^5\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]*(3315*
b^4 - 7293*a*b^3*x^(2/3) + 12155*a^2*b^2*x^(4/3) - 17765*a^3*b*x^2 + 24035
*a^4*x^(8/3)) - 3315*b^6*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b
)]))/(216315*a^5*Sqrt[1 + (a*x^(2/3))/b])
```

3.140.3 Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1927, 1927, 1930, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{8/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{9}b\int x^3\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{27}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\int \frac{x^{10/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{23}x^{10/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{27}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \; 1930$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\int \frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19a}\right)+\frac{2}{23}x^{10/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{27}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \; 1930$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int \frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{19a}\right)+\frac{2}{23}x^{10/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{27}x^3(\right.$$

$$\Big\downarrow \; 1930$$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int \frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)}{19a}\right)\right)\right)+\frac{2}{23}x^{10/3}\sqrt{a$$

$$\Big\downarrow \; 1930$$

3.140. $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right)}{15a}\right.}{19a}\right.\right.\right.$$

$$\downarrow 1930$$

3.140. $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{\cdots}{\sqrt[3]{\cdots}\sqrt{\cdots}}}{\cdots}\right)}{7a}\right)}{11a}\right)}{15a}\right)}{19a}\right.\right.\right.$$

3.140. $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$\downarrow$ 1917

3.140. $\int x^2 \left(b\sqrt[3]{x} + ax\right)^{3/2} \, dx$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{11a}\right.}{15a}\right.}{19a}\right.\right.\right.$$

3.140. $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$\downarrow$ 266

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\cdots}{\cdots}\right.}{\cdots}\right.}{11a}\right.}{15a}\right.}{19a}\right.\right.\right.$$

3.140. $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

↓ 761

3.140. $\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{9}b\left(\frac{2}{23}b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{\cdots}}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{19a}\right.\right.\right.$$

3.140. $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

input
```
Int[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*((2*x^3*(b*x^(1/3) + a*x)^(3/2))/27 + (2*b*((2*x^(10/3)*Sqrt[b*x^(1/3) +
 a*x])/23 + (2*b*((2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(19*a) - (17*b*((2*x^2
*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/
(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[b*x
^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(
2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*Elliptic
F[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a
*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a)))/(19*a)))/23))/9)
```

3.140.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.140.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.66

method	result
default	$\frac{\frac{1126x^{\frac{11}{3}}a^{6}b^{2}}{3933}+\frac{104x^{\frac{13}{3}}a^{7}b}{207}-\frac{16a^{5}b^{3}x^{3}}{19665}-\frac{3536x^{\frac{5}{3}}a^{3}b^{5}}{1514205}+\frac{272x^{\frac{7}{3}}a^{4}b^{4}}{216315}+\frac{2x^{5}a^{8}}{9}-\frac{884b^{7}\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}}{100947}}{a^{6}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$\frac{2a\,x^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{58b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207}+\frac{8b^{2}x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a}-\frac{136b^{3}x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^{2}}+\frac{1768b^{4}x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^{3}}-$ 1

input
```
int(x^2*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1514205*(216755*x^(11/3)*a^6*b^2+380380*x^(13/3)*a^7*b-616*a^5*b^3*x^3-1
768*x^(5/3)*a^3*b^5+952*x^(7/3)*a^4*b^4+168245*x^5*a^8-6630*b^7*(-a*b)^(1/
2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1
/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+5304*a^2*b^6*x+13260*x^
(1/3)*a*b^7)/a^6/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

3.140.5 Fricas [F]

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x^3 + b*x^(7/3))*sqrt(a*x + b*x^(1/3)), x)
```

3.140.6 Sympy [F]

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x^2\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2*(a*x + b*x**(1/3))**(3/2), x)
```

3.140.7 Maxima [F]

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

3.140.8 Giac [F]

$$\int x^2 \left(b \sqrt[3]{x}+a x\right)^{3/2} dx=\int \left(a x+b x^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 dx$$

input
```
integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(b \sqrt[3]{x}+a x\right)^{3/2} dx=\int x^2 \left(a x+b x^{1/3}\right)^{3/2} dx$$

input
```
int(x^2*(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^2*(a*x + b*x^(1/3))^(3/2), x)
```

3.141 $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

3.141.1 Optimal result

Integrand size = 17, antiderivative size = 408

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx = -\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3} -\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax} +\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{88b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^{21/4}\left(\sqrt{b}+\right.}{}$$

output
```
2/7*x^2*(b*x^(1/3)+a*x)^(3/2)-88/1105*b^5*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x
^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+88/3315*b^4*x^(1/3)*(b*x^(1/
3)+a*x)^(1/2)/a^3-88/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^2+24/1547*b^2*x^(5
/3)*(b*x^(1/3)+a*x)^(1/2)/a+12/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)+88/1105
*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*a
rctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1
/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)-44/1105*b^(21/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(
1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2
)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.141.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.30

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(77b^2-143abx^{2/3}+221a^2x^{4/3}\right)-77b^4\,\text{Hypergeom}\right.}{1547a^3\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[x*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b
]*(77*b^2 - 143*a*b*x^(2/3) + 221*a^2*x^(4/3)) - 77*b^4*Hypergeometric2F1[
-3/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(1547*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

3.141.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1924, 1927, 1927, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+b\sqrt[3]{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int x^{5/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{7}b\int x^2\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{21}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{17}x^{7/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{21}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\downarrow 1930$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int\frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)+\frac{2}{17}x^{7/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{21}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\downarrow 1930$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int\frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{13a}\right)+\frac{2}{17}x^{7/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{21}x^2(ax$$

$$\downarrow 1930$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)}{13a}\right)+\frac{2}{17}x^{7/3}\sqrt{ax-}$$

$$\downarrow 1938$$

3.141. $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{13a}\right)+\frac{2}{17}$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{13a}\right)+\frac{2}{17}$$

$$\Big\downarrow \text{834}$$

3.141. $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\cdots}{\sqrt{b}\sqrt{\cdots}}\cdots}{\cdots}\cdots\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\cdots\right.}{9a}\cdots\right.}{13a}\cdots\right.\right.\right.$$

$\downarrow$ 27

3.141. $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}\cdots}{\sqrt{ax^{4/\cdots}}}\cdots}{\cdots}\cdots\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\cdots\right.}{9a}\cdots\right.}{13a}\cdots\right.\right.\right.$$

$\downarrow$ 761

3.141. $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^{2}}}\cdots}{2a^{3/4}\cdots}\cdots\right)}{5a\cdots}\right)}{9a}\cdots\right)}{13a}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 1510$$

3.141. $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$3\left(\frac{2}{7}b\left(\frac{2}{17}b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\cdots}{2a^{3/4}\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.\right.$$

3.141. $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

input
```
Int[x*(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*((2*x^2*(b*x^(1/3) + a*x)^(3/2))/21 + (2*b*((2*x^(7/3)*Sqrt[b*x^(1/3) +
a*x])/17 + (2*b*((2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*((2*x*Sq
rt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5*a)
 - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(S
qrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b
+ a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^
(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*
(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3
))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt
[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/17))/7)
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.141.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

method	result
default	$\dfrac{\frac{622x^{\frac{8}{3}}a^4b^2}{1547}+\frac{80x^{\frac{10}{3}}a^5b}{119}-\frac{16a^3b^3x^2}{4641}-\frac{88b^6\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{1105}+\frac{44b^6\sqrt{a\,x}}{}}{a^4\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$ $44b^5\sqrt{}$
derivativedivides	$\frac{2a\,x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{46b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119}+\frac{24b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a}-\frac{88b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^2}+\frac{88b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315a^3}-$

input
```
int(x*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/23205/a^4*(4665*x^(8/3)*a^4*b^2+7800*x^(10/3)*a^5*b-40*a^3*b^3*x^2-924*b
^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1
/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*b^6*((a*x^(1/3)+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+3315*a^6*x^4+308*x^(2/3)*a*b^5+88*x^(4/3)
*a^2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

3.141.5 Fricas [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x^2 + b*x^(4/3))*sqrt(a*x + b*x^(1/3)), x)
```

3.141.6 Sympy [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x*(a*x + b*x**(1/3))**(3/2), x)
```

3.141.7 Maxima [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

3.141.8 Giac [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,dx$$

input
```
int(x*(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x*(a*x + b*x^(1/3))^(3/2), x)
```

3.142 $\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

3.142.1 Optimal result

Integrand size = 15, antiderivative size = 208

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{4b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
2/5*x*(b*x^(1/3)+a*x)^(3/2)-8/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^2+24/385*b^2*
x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a+12/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)+4/77
*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*a
rctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1
/2)+b^(1/2))^2)^(1/2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.142.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.51

$$\int (b\sqrt[3]{x}+ax)^{3/2}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-\left(\left(5b-11ax^{2/3}\right)\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\right)+5b^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\ldots\right.\right.}{55a^2\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(-((5*b - 11*a*x^(2/3))*(b + a*x^(2/3))^2*Sqrt[1
+ (a*x^(2/3))/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/
b)]))/(55*a^2*Sqrt[1 + (a*x^(2/3))/b])
```

3.142.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {1910, 1924, 1927, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+b\sqrt[3]{x}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1910}$$

$$\frac{2}{5}b\int \sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}dx+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{ 1924}$$

$$\frac{6}{5}b\int x\sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\int \frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{1930}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{1917}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)+\frac{2}{11}x^{4/3}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

3.142. $\int\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

$$\frac{6}{5}b\left(\frac{2}{11}b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right.\right.}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}\right.}{7a}\right.\right.$$

$$\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x*(b*x^(1/3) + a*x)^(3/2))/5 + (6*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/
11 + (2*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[b*x^(1/
3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)
]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*
ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a*x]*
Sqrt[b + a*x^(4/3)])))/(7*a)))/11))/5
```

3.142.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.142.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{262 a^{3} b^{2} x^{\frac{5}{3}}}{385}+\frac{56 a^{4} b\, x^{\frac{7}{3}}}{55}+\frac{4 b^{4} \sqrt{-a b}\, \sqrt{\frac{a\, x^{\frac{1}{3}}+\sqrt{-a b}}{\sqrt{-a b}}}\, \sqrt{-\frac{2 \left(a\, x^{\frac{1}{3}}-\sqrt{-a b}\right)}{\sqrt{-a b}}}\, \sqrt{-\frac{x^{\frac{1}{3}} a}{\sqrt{-a b}}}\, F\left(\sqrt{\frac{a\, x^{\frac{1}{3}}+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{77}-\frac{16 a^{2} b^{3} x}{385}+\frac{2 a^{5} x^{3}}{5}}{a^{3} \sqrt{x^{\frac{1}{3}} \left(b+a\, x^{\frac{2}{3}}\right)}}$
derivativedivides	$\frac{2 a\, x^{2} \sqrt{b\, x^{\frac{1}{3}}+a x}}{5}+\frac{34 b\, x^{\frac{4}{3}} \sqrt{b\, x^{\frac{1}{3}}+a x}}{55}+\frac{24 b^{2} x^{\frac{2}{3}} \sqrt{b\, x^{\frac{1}{3}}+a x}}{385 a}-\frac{8 b^{3} \sqrt{b\, x^{\frac{1}{3}}+a x}}{77 a^{2}}+\frac{4 b^{4} \sqrt{-a b}\, \sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-a b}}{a}\right) a}{\sqrt{-a b}}}}{}$

input
```
int((b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/385*(131*a^3*b^2*x^(5/3)+196*a^4*b*x^(7/3)+10*b^4*(-a*b)^(1/2)*((a*x^(1/
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-8*a^2*b^3*x+77*a^5*x^3-20*a*b^4*x^(
1/3))/a^3/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

3.142.5 Fricas [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2), x)
```

3.142.6 Sympy [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2), x)
```

3.142.7 Maxima [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2), x)
```

3.142.8 Giac [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2} dx = \int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2), x)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.19

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2} dx = \frac{2\,x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{9}{4};\ \frac{13}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{3\,\left(\frac{a\,x^{2/3}}{b}+1\right)^{3/2}}$$

input
```
int((a*x + b*x^(1/3))^(3/2),x)
```

output
```
(2*x*(a*x + b*x^(1/3))^(3/2)*hypergeom([-3/2, 9/4], 13/4, -(a*x^(2/3))/b))
/(3*((a*x^(2/3))/b + 1)^(3/2))
```

3.143 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

3.143.1 Optimal result

Integrand size = 19, antiderivative size = 319

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx = \frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{8b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
2/3*(b*x^(1/3)+a*x)^(3/2)+8/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(1/2)/(x^(1/3)*a
^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4/5*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)-
8/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^(1/3)+a*x)^(1/2)+4/5*b^(9/4)*x^(1/6)*
(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2)
)*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.19

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\frac{2b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

output
```
(2*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((a*
x^(2/3))/b)])/Sqrt[1 + (a*x^(2/3))/b]
```

3.143.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1924, 1927, 1910, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1927}$$

$$3\left(\frac{2}{3}b\int \sqrt{\sqrt[3]{x}b+ax}d\sqrt[3]{x}+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1910}$$

$$3\left(\frac{2}{3}b\left(\frac{2}{5}b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1938}$$

$$3\left(\frac{2}{3}b\left(\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{834}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\right)+\frac{2}{9}\left(ax+b\sqrt[3]{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{5}\sqrt[3]{}\right.\right.$$

$$\Big\downarrow \text{1510}$$

3.143. $\displaystyle\int\frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

$$3\left(\frac{2}{3}b\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{\right.}}}}{\sqrt[4]{a}\sqrt{a}}\right.}{5\sqrt{ax+b\sqrt[3]{x}}}\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

output
```
3*((2*(b*x^(1/3) + a*x)^(3/2))/9 + (2*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])
/5 + (4*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/
(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(
b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4
)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2
/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sq
rt[b + a*x^(4/3)])))/(5*Sqrt[b*x^(1/3) + a*x])))/3)
```

3.143.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.143. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.143. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

3.143.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2ax\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{22b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\cdots}\right.}{\cdots}\right.}{5a\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{\frac{8b^3\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{4b^3\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\cdots}{5}\cdots}{a\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*a*x*(b*x^(1/3)+a*x)^(1/2)+22/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)+4/5*b^
2/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^
(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1
/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b
)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x
^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.143.5 Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x, x)
```

3.143.6 Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x, x)
```

3.143.7 Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

3.143.8 Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x, x)
```

3.144 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx$

3.144.1 Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = 4a\sqrt{b\sqrt[3]{x}+ax}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-2*(b*x^(1/3)+a*x)^(3/2)/x+4*a*(b*x^(1/3)+a*x)^(1/2)+4*a^(3/4)*b^(3/4)*x^(
1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)
*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^
(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^
2)^(1/2)/(b*x^(1/3)+a*x)^(1/2)
```

3.144.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = -\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{2/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^(2/3
))/b)])/(Sqrt[1 + (a*x^(2/3))/b]*x^(2/3))
```

3.144.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1924, 1926, 1927, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(2a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{\sqrt[3]{x}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{1927}$$

$$3\left(2a\left(\frac{2}{3}b\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{1917}$$

$$3\left(2a\left(\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(2a\left(\frac{4b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3\sqrt{ax+b\sqrt[3]{x}}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(2a\left(\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}+\frac{2}{3}\sqrt{ax+b\sqrt[3]{x}}\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(3*x) + 2*a*((2*Sqrt[b*x^(1/3) + a*x])/3 +
 (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(
b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/
3)])))
```

3.144.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.144.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

method	result
default	$\frac{4x^{\frac{1}{3}}\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b+2x^{\frac{4}{3}}a^{2}-2b^{2}}{x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{x^{\frac{2}{3}}}+2a\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\cdots\right.}{\sqrt{b\,x^{\frac{1}{3}}+ax}}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/x^(1/3)*(2*x^(1/3)*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^
(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*b+x^(4/3)*a^2-b^2)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

3.144.5 Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

3.144.6 Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**2,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**2, x)
```

3.144.7 Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

3.144.8 Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^2,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

3.145 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

3.145.1 Optimal result

Integrand size = 19, antiderivative size = 350

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx = \frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} - \frac{8a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} + \frac{4a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-2/3*(b*x^(1/3)+a*x)^(3/2)/x^2+8/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3
)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-4/5*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5
*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-8/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(
1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ell
ipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2
)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^
(1/3)+a*x)^(1/2)+4/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(
a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(
2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.145.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.18

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{3\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{5/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^(2/
3))/b)])/(3*Sqrt[1 + (a*x^(2/3))/b]*x^(5/3))
```

3.145.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1924, 1926, 1926, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \; 1924$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{7/3}}d\sqrt[3]{x}$$

$\Big\downarrow$ 1926

$$3\left(\frac{2}{3}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{4/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$\Big\downarrow$ 1926

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$\Big\downarrow$ 1931

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{a\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$\Big\downarrow$ 1938

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$\Big\downarrow$ 266

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^2}\right)$$

$\Big\downarrow$ 834

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right) -$$

$\Big\downarrow$ 27

3.145. $\displaystyle\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5x}\right)-\frac{2}{}\right.$$

$\downarrow$ 761

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\right.\right.\right.$$

$\downarrow$ 1510

$$3\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

3.145. $\int\frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(9*x^2) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])
/(5*x) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^
(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2
/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b]
 + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2]
)/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^
(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*Arc
Tan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b
*Sqrt[b*x^(1/3) + a*x])))/5))/3)
```

3.145.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.145. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.145.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{3x^{\frac{5}{3}}}-\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15x}-\frac{8\left(b+a\,x^{\frac{2}{3}}\right)a^{2}}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{4a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x}{\sqrt{}}}\cdots}{\cdots}$
default	$-\frac{2\left(-12a^{2}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+6a^{2}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{}}{\sqrt{-a}}}\cdots\right.}{\cdots}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*b*(b*x^(1/3)+a*x)^(1/2)/x^(5/3)-22/15*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5*(
b+a*x^(2/3))*a^2/b/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+4/5/b*a^2*(-a*b)^(1/2)*((
x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1
/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)
^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.145.5 Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

3.145.6 Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**3,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**3, x)
```

3.145.7 Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

3.145.8 Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^3,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

3.146 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

3.146.1 Optimal result

Integrand size = 19, antiderivative size = 213

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx = -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{4a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-2/5*(b*x^(1/3)+a*x)^(3/2)/x^3-12/55*a*(b*x^(1/3)+a*x)^(1/2)/x^2-24/385*a^
2*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+8/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/
3)+4/77*a^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/
3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.146.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.29

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{3}{2},-\frac{11}{4},-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{8/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((a*x^(
2/3))/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x^(8/3))
```

3.146.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1924, 1926, 1926, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{5}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{7/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{15x^3}\right)$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{15x^3}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{15x^3}\right)$$

$\downarrow$ 1931

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{15x^3}\right)$$

$\downarrow$ 1917

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2(\ldots}{}\right.$$

$\downarrow$ 266

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11x^2}\right)-\frac{2(ax\ldots}{}\right.$$

$\downarrow$ 761

$$3\left(\frac{2}{5}a\left(\frac{2}{11}a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\ldots}{}\right.\right.\right.$$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

3.146. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(15*x^3) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(11*x^2) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sq
rt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*
Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3
))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt
[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*b)))/11))/5)
```

3.146.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.146. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.146.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.79

method	result
default	$\frac{\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{14}{3}}}{77}-\frac{262x^{\frac{11}{3}}a^{2}b^{2}}{385}+\frac{16x^{\frac{13}{3}}a^{3}b}{385}-\frac{56a\,b^{3}x^{3}}{55}+}{b^{2}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{14}{3}}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x^{\frac{8}{3}}}-\frac{34a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55x^{2}}-\frac{24a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b\,x^{\frac{4}{3}}}+\frac{8a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{2}x^{\frac{2}{3}}}+\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}\right.}{}}}{77}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/385*(10*a^3*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^
(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
*x^(14/3)-131*x^(11/3)*a^2*b^2+8*x^(13/3)*a^3*b-196*a*b^3*x^3+20*a^4*x^5-7
7*x^(7/3)*b^4)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(14/3)
```

3.146.5 Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

3.146.6 Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**4,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**4, x)
```

3.146.7 Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

3.146.8 Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^4}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^4,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

3.147 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

3.147.1 Optimal result

Integrand size = 19, antiderivative size = 438

$$\begin{aligned}\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx &= -\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\ &\quad - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} \\ &\quad - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} \\ &\quad + \frac{88a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\ &\quad - \frac{44a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-2/7*(b*x^(1/3)+a*x)^(3/2)/x^4-88/1105*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/b^4/
(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-12/119*a*(b*x^(1/3)+a*x)^(
1/2)/x^3-24/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3
)+a*x)^(1/2)/b^2/x^(5/3)-88/3315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*a
^5*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+88/1105*a^(21/4)*x^(1/6)*(cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a
^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)
/(b*x^(1/3)+a*x)^(1/2)-44/1105*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(si
n(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2)
)*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*
x)^(1/2)
```

3.147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.14

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{21}{4},-\frac{3}{2},-\frac{17}{4},-\frac{ax^{2/3}}{b}\right)}{7\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{11/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-21/4, -3/2, -17/4, -((a*x^(
2/3))/b)])/(7*Sqrt[1 + (a*x^(2/3))/b]*x^(11/3))
```

3.147.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1924, 1926, 1926, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{13/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{7}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{10/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{21x^4}\right)$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{21x^4}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{21x^4}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3}}{21x^4}$$

$$\Big\downarrow \text{1931}$$

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}dx$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\int\frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17x^3}$$

$\downarrow$ 1931

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right.\right.\right.-$$

↓ 1938

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+}}{13bx^{7/}}\right.\right.\right.$$

$$\Big\downarrow \text{266}$$

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+}}{13bx^{7}}\right.\right.\right.$$

↓ 834

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\ldots}{13b}\right.\right.\right.$$

3.147. $\int\frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$\downarrow 27$$

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{}}{}\right.}{13b}\right.\right.\right.$$

3.147. $\displaystyle\int\frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$\downarrow$ 761

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d}{\sqrt{a}}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.}{5b}\right.}{9b}\right.}{13b}\right.\right.\right.$$

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

↓ 1510

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

$$\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}} - \frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\right.}{}$$

$$3a \quad \frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\cdots\right.}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$$7a\left(-\frac{\cdots}{5b}\right.$$

$$11a\left(-\frac{\cdots}{9b}\right.$$

$$3\left(\frac{2}{7}a\left(\frac{2}{17}a\left(-\frac{\cdots}{13b}\right.\right.\right.$$

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(21*x^4) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(17*x^3) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) - (11*a*((-2*
Sqrt[b*x^(1/3) + a*x])/(9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5
*b*x) - (3*a*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(
2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/
3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b]
+ Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(
2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcT
an[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*
Sqrt[b*x^(1/3) + a*x])))/(5*b)))/(9*b)))/(13*b)))/17))/7)
```

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.147. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

3.147.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{7x^{\frac{11}{3}}}-\frac{46a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119x^{3}}-\frac{24a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b\,x^{\frac{7}{3}}}+\frac{88a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^{2}x^{\frac{5}{3}}}-\frac{88a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^{3}x}+\frac{88\left(b+a\,x^{\frac{2}{3}}\right)}{1105b^{4}\sqrt{x^{\frac{1}{3}}\left(b+\right.}}$
default	$-\frac{88a^{5}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{1105}+\frac{44a^{5}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-}}{}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/7*b*(b*x^(1/3)+a*x)^(1/2)/x^(11/3)-46/119*a*(b*x^(1/3)+a*x)^(1/2)/x^3-2
4/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3)+a*x)^(1/
2)/b^2/x^(5/3)-88/3315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*(b+a*x^(2/3
))*a^5/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-44/1105*a^5/b^4*(-a*b)^(1/2)*((x^
(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2
))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(
1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/
2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.147.5 Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

3.147.6 Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**5,x)
```

output
```
Integral((a*x + b*x**(1/3))**(3/2)/x**5, x)
```

3.147.7 Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

3.147.8 Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^5}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^5,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

3.148 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

3.148.1 Optimal result

Integrand size = 19, antiderivative size = 301

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx = -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{1768a^6\sqrt{b\sqrt[3]{x}+ax}}{100947b^5x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{884a^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-2/9*(b*x^(1/3)+a*x)^(3/2)/x^5-4/69*a*(b*x^(1/3)+a*x)^(1/2)/x^4-8/1311*a^2
*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+136/19665*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/
x^(8/3)-1768/216315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+1768/168245*a^5*(b*x
^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-1768/100947*a^6*(b*x^(1/3)+a*x)^(1/2)/b^5/x^
(2/3)-884/100947*a^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(
1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3
))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{27}{4},-\frac{3}{2},-\frac{23}{4},-\frac{ax^{2/3}}{b}\right)}{9\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{14/3}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

output
```
(-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-27/4, -3/2, -23/4, -((a*x^(
2/3))/b)])/(9*Sqrt[1 + (a*x^(2/3))/b]*x^(14/3))
```

3.148.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1926, 1926, 1931, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{\left(\sqrt[3]{x}b+ax\right)^{3/2}}{x^{16/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{9}a\int \frac{\sqrt{\sqrt[3]{x}b+ax}}{x^{13/3}}d\sqrt[3]{x}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{27x^5}\right)$$

$$\Big\downarrow \text{1926}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\int \frac{1}{x^{10/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{27x^5}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{27x^5}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\int \frac{1}{x^2\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{27x^5}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{23x^4}\right)\right.$$

$$\Big\downarrow \text{1931}$$

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right.\right.\right.$$

$\downarrow$ 1931

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-$$

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

↓ 1917

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

$$3\Bigg(\frac{2}{9}a\Bigg(\frac{2}{23}a\Bigg(-\frac{17a\Bigg(-\frac{13a\Bigg(-\frac{9a\Bigg(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}\,d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\Bigg)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\Bigg)}{15b}-\frac{2\sqrt{}}{}}{19b}$$

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

$\downarrow$ 266

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+\cdots}}{15b\cdots}\right)}{19b}\cdots\right.\right.\right.$$

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

$\downarrow$ 761

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

$$3\left(\frac{2}{9}a\left(\frac{2}{23}a\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{\cdots}}}{3bx^{2/3}}\cdots}{7b}\cdots}{11b}\cdots}{15b}\cdots}{19b}\cdots$$

3.148. $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

input
```
Int[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

output
```
3*((-2*(b*x^(1/3) + a*x)^(3/2))/(27*x^5) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(23*x^4) + (2*a*((-2*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) - (17*a*((-2
*Sqrt[b*x^(1/3) + a*x])/(15*b*x^(8/3)) - (13*a*((-2*Sqrt[b*x^(1/3) + a*x])
/(11*b*x^2) - (9*a*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*S
qrt[b*x^(1/3) + a*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))
*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/
3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqr
t[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*b)))/(11*b)))/(15*b)))/(19*b)
))/23))/9)
```

3.148.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.148.4 Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\left(6630a^{6}\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{26}{3}}-1768x^{\frac{23}{3}}a^{5}b^{2}+5304x^{\frac{25}{3}}\right.}{1514205b^{5}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{9x^{\frac{14}{3}}}-\frac{58a\sqrt{b\,x^{\frac{1}{3}}+ax}}{207x^{4}}-\frac{8a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311b\,x^{\frac{10}{3}}}+\frac{136a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665b^{2}x^{\frac{8}{3}}}-\frac{1768a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315b^{3}x^{2}}+\frac{1768a^{5}\sqrt{b\,x^{\frac{1}{3}}}}{168245b^{4}x}$

input
```
int((b*x^(1/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/1514205*(6630*a^6*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^
(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*x^(26/3)-1768*x^(23/3)*a^5*b^2+5304*x^(25/3)*a^6*b+952*a^4*b^3*x^7
+216755*x^(17/3)*a^2*b^5-616*x^(19/3)*a^3*b^4+380380*a*b^6*x^5+13260*a^7*x
^9+168245*x^(13/3)*b^7)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(26/3)
```

3.148.5 Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\text{Timed out}$$

input
```
integrate((b*x**(1/3)+a*x)**(3/2)/x**6,x)
```

output
```
Timed out
```

3.148.7 Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

3.148.8 Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^6}\,dx$$

input
```
int((a*x + b*x^(1/3))^(3/2)/x^6,x)
```

output
```
int((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

3.149 $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.149.1 Optimal result

Integrand size = 19, antiderivative size = 304

$$\begin{aligned}\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6}\\&\quad+\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5}-\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}\\&\quad+\frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}-\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2}+\frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a}\\&\quad-\frac{5525b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{29/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
11050/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^7-2210/4807*b^5*x^(2/3)*(b*x^(1/3)
+a*x)^(1/2)/a^6+15470/43263*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-1190/393
3*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^4+350/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(
1/2)/a^3-50/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(1/3)+a*
x)^(1/2)/a-5525/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x
^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(29/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(16575b^7+6630ab^6x^{2/3}-2210a^2b^5x^{4/3}+1190a^3b^4x^2-770a^4b^3x^{8/3}+550a^5b^2x^{10/3}-418a\right.}{43263a^7\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(16575*b^7 + 6630*a*b^6*x^(2/3) - 2210*a^2*b^5*x^
(4/3) + 1190*a^3*b^4*x^2 - 770*a^4*b^3*x^(8/3) + 550*a^5*b^2*x^(10/3) - 41
8*a^6*b*x^4 + 4807*a^7*x^(14/3) - 16575*b^7*Sqrt[1 + (a*x^(2/3))/b]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(43263*a^7*(b + a*x^(2/3)))
```

3.149.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1924, 1930, 1930, 1930, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{14/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\int \frac{x^4}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{27a}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\int\frac{x^{10/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{23a}\right)}{27a}\right)$$

$\downarrow$ 1930

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\int\frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19a}\right)}{23a}\right)}{27a}\right)$$

$\downarrow$ 1930

3.149. $\int\frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}dx$

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int\frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{19a}\right)}{23a}\right)}{27a}\right)$$

$$\downarrow 1930$$

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{27a}-\frac{25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}}{11a}\right.}{15a}\right.}{19a}\right.}{23a}\right.}{27a}\right.$$

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

↓ 1930

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$9b\left(\frac{2x^{2/3}\sqrt{ax+b}}{7a}\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{\quad}{15a}\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{\quad}{19a}\right.$$

$$25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{\quad}{23a}\right.$$

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$\downarrow$ 1930

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$9b \left(\frac{2x^{2/3}\sqrt{ax+b}}{7a} \right.$

$13b \left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a} - \right.$

$17b \left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a} - \right.$

$21b \left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a} - \frac{}{19a} \right.$

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$\downarrow$ 1917

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$9b\left(\frac{2x^{2/3}\sqrt{ax+b}}{7a}\right.$$

$$13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\right.$$

$$17b\left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a}-\right.$$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\right.$$

$$25b\left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a}-\frac{\qquad}{23a}\right.$$

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

↓ 266

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$\left(25b \left(\frac{2x^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{23a} - \frac{21b \left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a} - \frac{17b \left(\frac{2x^{2}\sqrt{ax+b\sqrt[3]{x}}}{15a} - \frac{13b \left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a} - \frac{9b \left(\frac{2x^{2/3}\sqrt{ax+b}}{7a} \right.}{} \right.}{} \right.}{} \right.}{23a} \right. \right.$$

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

↓ 761

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$21b\left(\frac{2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{19a}-\frac{17b\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b}}{7a}\right.}{}\right.}{}\right.}{}\right.$$

3.149. $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

input
```
Int[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^4*Sqrt[b*x^(1/3) + a*x])/(27*a) - (25*b*((2*x^(10/3)*Sqrt[b*x^(1/3
) + a*x])/(23*a) - (21*b*((2*x^(8/3)*Sqrt[b*x^(1/3) + a*x])/(19*a) - (17*b
*((2*x^2*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3)
+ a*x])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*
Sqrt[b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b
 + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*
EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(
1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a)))/(19*a)))/(23*
a)))/(27*a))
```

3.149.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.149.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.64

method	result
default	$-\frac{-1100x^{\frac{11}{3}}a^6b^2+836x^{\frac{13}{3}}a^7b+1540a^5b^3x^3+4420x^{\frac{5}{3}}a^3b^5-2380x^{\frac{7}{3}}a^4b^4-9614x^5a^8+16575b^7\sqrt{-ab}\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-}}{43263\sqrt{x^{\frac{1}{3}}\left(b+a x^{\frac{2}{3}}\right)}a^8}$
derivativedivides	$\frac{2x^4\sqrt{b x^{\frac{1}{3}}+ax}}{9a}-\frac{50b x^{\frac{10}{3}}\sqrt{b x^{\frac{1}{3}}+ax}}{207a^2}+\frac{350b^2x^{\frac{8}{3}}\sqrt{b x^{\frac{1}{3}}+ax}}{1311a^3}-\frac{1190b^3x^2\sqrt{b x^{\frac{1}{3}}+ax}}{3933a^4}+\frac{15470b^4x^{\frac{4}{3}}\sqrt{b x^{\frac{1}{3}}+ax}}{43263a^5}-$

input
```
int(x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/43263*(-1100*x^(11/3)*a^6*b^2+836*x^(13/3)*a^7*b+1540*a^5*b^3*x^3+4420*
x^(5/3)*a^3*b^5-2380*x^(7/3)*a^4*b^4-9614*x^5*a^8+16575*b^7*(-a*b)^(1/2)*(
(a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-13260*a^2*b^6*x-33150*x^(1/
3)*a*b^7)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^8
```

3.149.5 Fricas [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^5 - a*b*x^(13/3) + b^2*x^(11/3))*sqrt(a*x + b*x^(1/3))/(a^
3*x^2 + b^3), x)
```

3.149.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(a*x + b*x**(1/3)), x)
```

3.149.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

3.149.8 Giac [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^4}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x^4/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^4/(a*x + b*x^(1/3))^(1/2), x)
```

3.150 $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.150.1 Optimal result

Integrand size = 19, antiderivative size = 414

$$\begin{aligned}
&\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx \\
&= -\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5}-\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} \\
&\quad+\frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3}-\frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2}+\frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} \\
&\quad+\frac{418b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad-\frac{209b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

output
```
-418/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(11/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x
^(1/3)+a*x)^(1/2)+418/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-2090/4641*
b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^4+570/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)
/a^3-38/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(1/3)+a*x)^(1
/2)/a+418/221*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^
(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/
(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)-209/221*
b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1
/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/
2)+b^(1/2))^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.35

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$
$$= \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(1463b^5\sqrt[3]{x}+418ab^4x-190a^2b^3x^{5/3}+114a^3b^2x^{7/3}-78a^4bx^3+663a^5x^{11/3}-1463b^5\sqrt{1+\right.}{4641a^5\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(1463*b^5*x^(1/3) + 418*a*b^4*x - 190*a^2*b^3*x^(
5/3) + 114*a^3*b^2*x^(7/3) - 78*a^4*b*x^3 + 663*a^5*x^(11/3) - 1463*b^5*Sq
rt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3
))/b)]))/(4641*a^5*(b + a*x^(2/3)))
```

3.150.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1924, 1930, 1930, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{11/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\int \frac{x^3}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{21a}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17a}\right)}{21a}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)}{17a}\right)}{21a}\right)$$

$$\downarrow 1930$$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int\frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{13a}\right)}{17a}\right)}{21a}\right)$$

$$\downarrow 1930$$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{\cdots}}{5a}\right.}{9a}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

3.150. $\int\frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

↓ 1938

3.150. $\int \frac{x^3}{\sqrt{b \sqrt[3]{x}+a x}}\, dx$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\cdots}{5a\sqrt{ax+\cdots}}\right.}{9a}\cdots}{13a}\cdots}{17a}\cdots}{21a}\cdots\right.$$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$\downarrow$ 266

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int}{5a\sqrt{ax+\cdots}}\right.}{9a}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$\downarrow$ 834

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{13a}\cdots\right.}{17a}\cdots\right.}{21a}\cdots$$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$\Big\downarrow 27$$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{13a}\cdots\right.}{17a}\cdots\right.}{21a}\cdots\right.$$

3.150. $\displaystyle\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

↓ 761

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$\left(\quad 19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a} - \frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a} - \frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a} - \frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a} - \frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\right.}{}\right.}{}\right.}{}\right.}{}\right.\right.$$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.149. $\downarrow$ 1510

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

3.150. $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

input
```
Int[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^3*Sqrt[b*x^(1/3) + a*x])/(21*a) - (19*b*((2*x^(7/3)*Sqrt[b*x^(1/3)
 + a*x])/(17*a) - (15*b*((2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*
((2*x*Sqrt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x
])/(5*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4
/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*
Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^
(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (
b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a
]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3
/4)*Sqrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/
(17*a)))/(21*a))
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.150.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.63

method	result
default	$-\frac{-228x^{\frac{8}{3}}a^4b^2+156x^{\frac{10}{3}}a^5b+380a^3b^3x^2+8778b^6\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{4641a^6\sqrt{x}}$ 20
derivativedivides	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a}-\frac{38b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^2}+\frac{570b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^3}-\frac{2090b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^4}+\frac{418b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^5}-$

input
```
int(x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4641/a^6*(-228*x^(8/3)*a^4*b^2+156*x^(10/3)*a^5*b+380*a^3*b^3*x^2+8778*
b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(
1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-4389*b^6*((a*x^(1/3)+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-1326*a^6*x^4-2926*x^(2/3)*a*b^5-836*x^(
4/3)*a^2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

3.150.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^4 - a*b*x^(10/3) + b^2*x^(8/3))*sqrt(a*x + b*x^(1/3))/(a^3
*x^2 + b^3), x)
```

3.150.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a*x + b*x**(1/3)), x)
```

3.150.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

3.150.8 Giac [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^3}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x^3/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^(1/3))^(1/2), x)
```

3.151 $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.151.1 Optimal result

Integrand size = 19, antiderivative size = 216

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4}+\frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3}-\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2}+\frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a}+\frac{39b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{17/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-78/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^4+234/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(
1/2)/a^3-26/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/3)+a*x)
^(1/2)/a+39/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/
4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(17/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.151.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-195b^4-78ab^3x^{2/3}+26a^2b^2x^{4/3}-14a^3bx^2+77a^4x^{8/3}+195b^4\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometr}\right.}{385a^4\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(-195*b^4 - 78*a*b^3*x^(2/3) + 26*a^2*b^2*x^(4/3)
 - 14*a^3*b*x^2 + 77*a^4*x^(8/3) + 195*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^4*(b + a*x^(2/3)))
```

3.151.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1924, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \ 1924$$

$$3\int \frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 1930$$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int \frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)$$

$$\Big\downarrow \ 1930$$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)$$

$\downarrow$ 1930

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right)}{15a}\right)$$

$\downarrow$ 1930

3.151. $\int\frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{7a}\right)}{11a}\right)}{15a}\right)$$

$\downarrow$ 1917

3.151. $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)$$

$\downarrow$ 266

3.151. $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)$$

$\downarrow$ 761

3.151. $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+}{\left(\sqrt{a}x^{2/3}+\sqrt{}\right.}}}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}}\right.}{7a}\right.}{11a}\right.}{15a}\right.$$

input
```
Int[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x^2*Sqrt[b*x^(1/3) + a*x])/(15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3)
 + a*x])/(11*a) - (9*b*((2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2
*Sqrt[b*x^(1/3) + a*x])/(3*a) - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]
*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^
(1/3) + a*x]*Sqrt[b + a*x^(4/3)])))/(7*a)))/(11*a)))/(15*a))
```

3.151. $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.151.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.151. $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.151.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

method	result
default	$-\frac{-52a^3b^2x^{\frac{5}{3}}+28a^4b\,x^{\frac{7}{3}}-195b^4\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+156a^2b^3}{385\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^5}$
derivativedivides	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a}-\frac{26b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^2}+\frac{234b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^3}-\frac{78b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^4}+\frac{39b^4\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$

input
```
int(x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/385*(-52*a^3*b^2*x^(5/3)+28*a^4*b*x^(7/3)-195*b^4*(-a*b)^(1/2)*((a*x^(1
/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+156*a^2*b^3*x-154*a^5*x^3+390*a*b^
4*x^(1/3))/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^5
```

3.151.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^3 - a*b*x^(7/3) + b^2*x^(5/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

3.151.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a*x + b*x**(1/3)), x)
```

3.151.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

3.151.8 Giac [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/3))^(1/2), x)
```

3.152 $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.152.1 Optimal result

Integrand size = 17, antiderivative size = 326

$$\begin{aligned}\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2}+\frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a}\\ &\quad-\frac{14b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}}\\ &\quad+\frac{7b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
14/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(5/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3
)+a*x)^(1/2)-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(1/3)+a*
x)^(1/2)/a-14/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/
4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)+7/5*b^(
9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b
^(1/2))^2)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.152.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-7b^2\sqrt[3]{x}-2abx+5a^2x^{5/3}+7b^2\sqrt{1+\frac{ax^{2/3}}{b}}\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)\right)}{15a^2\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(-7*b^2*x^(1/3) - 2*a*b*x + 5*a^2*x^(5/3) + 7*b^2
*Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(
2/3))/b)]))/(15*a^2*(b + a*x^(2/3)))
```

3.152.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1924, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int \frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)$$

$$\Big\downarrow \text{1938}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$$\Big\downarrow \text{834}$$

3.152. $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}dx$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$\downarrow$ 27

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)$$

$\downarrow$ 761

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}\right.}{5a\sqrt{ax+b\sqrt[3]{x}}}\right.}{9a}\right.$$

$\downarrow$ 1510

3.152. $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}\cdots\right)}{5a\sqrt{ax+b\cdots}}\right)}{9a}\cdots\right)$$

input
```
Int[x/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
3*((2*x*Sqrt[b*x^(1/3) + a*x])/(9*a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a
*x])/(5*a) - (6*b*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^
(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3)
)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) +
 (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt
[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^
(3/4)*Sqrt[b + a*x^(4/3)])))/(5*a*Sqrt[b*x^(1/3) + a*x])))/(9*a))
```

3.152. $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.152.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.152.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3a}-\frac{14b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a^{2}}+\frac{7b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(\cdots\right.}\right.}{\cdots}\right.}{5a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{-42b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+21b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right.}{\sqrt{-ab}}}\cdots}{15a^{3}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$

input
```
int(x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(b*x^(1/3)+a*x)^(1/2)/a-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+7/
5*b^2/a^3*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(
-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/
2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a
*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*Ellipti
cF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.152.5 Fricas [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

3.152.6 Sympy [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(x/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(a*x + b*x**(1/3)), x)
```

3.152.7 Maxima [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

3.152.8 Giac [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(x/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
int(x/(a*x + b*x^(1/3))^(1/2), x)
```

3.153 $\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.153.1 Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
2*(b*x^(1/3)+a*x)^(1/2)/a-b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+
a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^(1/3)+a*x)^(1/2
)
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(b+ax^{2/3}-b\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)\right)}{a\left(b+ax^{2/3}\right)}$$

input
```
Integrate[1/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x]*(b + a*x^(2/3) - b*Sqrt[1 + (a*x^(2/3))/b]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a*(b + a*x^(2/3)))
```

3.153.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1916, 1919, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \ 1916$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}dx}{3a}$$

$$\Big\downarrow \ 1919$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{a}$$

$$\Big\downarrow \ 1917$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{a\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{a\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}$$

input
```
Int[1/Sqrt[b*x^(1/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b
+ a*x^(2/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*E
llipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3
) + a*x]*Sqrt[b + a*x^(4/3)])
```

3.153.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1916
```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Simp[a*((2*n - j - 2)/(b*(n -
2)))   Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
 LtQ[2*(n - 1), j, n]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

3.153.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{-b\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+2ab\,x^{\frac{1}{3}}+2a^{2}x}{\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{2}}$	127
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{a}-\frac{b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$	135

input
```
int(1/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1
/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*Elli
pticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2*a*b*x^(
1/3)+2*a^2*x)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^2
```

3.153.5 Fricas [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^3 + b^3*x), x)
```

3.153.6 Sympy [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**(1/3)), x)
```

3.153.7 Maxima [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

3.153.8 Giac [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\frac{2\,x\,\sqrt{\frac{b}{a\,x^{2/3}}+1}\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ -\frac{b}{a\,x^{2/3}}\right)}{\sqrt{a\,x+b\,x^{1/3}}}$$

input
```
int(1/(a*x + b*x^(1/3))^(1/2),x)
```

output
```
(2*x*(b/(a*x^(2/3)) + 1)^(1/2)*hypergeom([-3/4, 1/2], 1/4, -b/(a*x^(2/3)))
)/(a*x + b*x^(1/3))^(1/2)
```

3.154 $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.154.1 Optimal result

Integrand size = 19, antiderivative size = 294

$$\begin{aligned}\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}}\\
&\quad-\frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad+\frac{3\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
6*(b+a*x^(2/3))*x^(1/3)*a^(1/2)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x
)^(1/2)-6*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-6*a^(1/4)*x^(1/6)*(cos(2*arctan(
a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*
EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(
1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b
*x^(1/3)+a*x)^(1/2)+3*a^(1/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^
(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.154.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^(2/3)
)/b)])/Sqrt[b*x^(1/3) + a*x]
```

3.154.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1924, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{a\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{1938}$$

$$3\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{834}$$

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+}}{b\sqrt[3]{}}\right.$$

$$\Big\downarrow \text{1510}$$

3.154. $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}E}{\sqrt[4]{a}\sqrt{ax^{4/3}+}}\right.}{b\sqrt{ax+b\sqrt[3]{x}}}\right.$$

input
```
Int[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/
6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(
1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x
^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*S
qrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[
(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1
/3) + a*x]))
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.154. $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.154.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{6\left(b+a\,x^{\frac{2}{3}}\right)}{b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}\right.}{b\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{6\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b-3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-}}{\sqrt{-ab}}}}{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)b}$

input
```
int(1/x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-6*(b+a*x^(2/3))/b/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+3/b*(-a*b)^(1/2)*((x^(1/3
)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)
*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.154.5 Fricas [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^4 + b^3*x^2), x)
```

3.154.6 Sympy [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*x + b*x**(1/3))), x)
```

3.154.7 Maxima [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

3.154.8 Giac [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{x\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/3))^(1/2)), x)
```

3.155 $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.155.1 Optimal result

Integrand size = 19, antiderivative size = 163

$$\begin{aligned}\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}\\ &\quad+\frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-6/7*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+10/7*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2
/3)+5/7*a^(7/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.155.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{ax^{2/3}}{b}\right)}{7x\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((a*x^(2/3
))/b)])/(7*x*Sqrt[b*x^(1/3) + a*x])
```

3.155.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1924, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{5a\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{5a\left(-\frac{a\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$\Big\downarrow$ 1917

$$3\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$\Big\downarrow$ 266

$$3\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

$\Big\downarrow$ 761

$$3\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)$$

input
```
Int[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a
*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/
3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[
2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x
]*Sqrt[b + a*x^(4/3)])))/(7*b))
```

3.155. $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.155.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.155. $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.155.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

method	result
default	$\frac{5a\sqrt{-ab}\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}F\left(\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{4}{3}}+4abx+10x^{\frac{5}{3}}a^{2}-6b^{2}x^{\frac{1}{3}}}{7b^{2}\sqrt{x^{\frac{1}{3}}\left(b+a x^{\frac{2}{3}}\right)}\,x^{\frac{4}{3}}}$
derivativedivides	$-\frac{6\sqrt{b x^{\frac{1}{3}}+ax}}{7b x^{\frac{4}{3}}}+\frac{10a\sqrt{b x^{\frac{1}{3}}+ax}}{7b^{2}x^{\frac{2}{3}}}+\frac{5a\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\right.}{7b^{2}\sqrt{b x^{\frac{1}{3}}+ax}}$

input
```
int(1/x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*(5*a*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a
*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)
*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(4
/3)+4*a*b*x+10*x^(5/3)*a^2-6*b^2*x^(1/3))/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2
)/x^(4/3)
```

3.155.5 Fricas [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^5 + b^3*x^3), x)
```

3.155.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^2\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*x + b*x**(1/3))), x)
```

3.155.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

3.155.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{x^2\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/3))^(1/2)), x)
```

3.156 $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.156.1 Optimal result

Integrand size = 19, antiderivative size = 388

$$\begin{aligned}\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}}\\&\quad+\frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}}-\frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}+\frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}}\\&\quad+\frac{154a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\\&\quad-\frac{77a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-154/65*a^(7/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(
1/3)+a*x)^(1/2)-6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*
x)^(1/2)/b^2/x^(5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*a^3*(b
*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+154/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Elli
pticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)
+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^
(1/3)+a*x)^(1/2)-77/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1
/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*
x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.156.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx=-\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{1}{2},-\frac{9}{4},-\frac{ax^{2/3}}{b}\right)}{13x^2\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-13/4, 1/2, -9/4, -((a*x^(2/
3))/b)])/(13*x^2*Sqrt[b*x^(1/3) + a*x])
```

3.156.3 Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

$$\Big\downarrow \ 1924$$

$$3\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{11a\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{11a\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\Big\downarrow \text{1931}$$

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$\downarrow$ 1938

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$$\downarrow 266$$

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)$$

$\downarrow$ 834

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}}{13b}$$

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$\Big\downarrow 27$$

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}\right.$$

3.156. $\int\frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$\downarrow$ 761

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}\,d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax\cdots}}{b\cdots}\right)}{5b}\cdots\right)}{9b}\cdots\right)}{13b}\cdots\right)$$

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

↓ 1510

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}\cdots}{\left(\sqrt{a}x^{2/3}\cdots\right)}}\cdots}{\sqrt[4]{a}\cdots}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}\cdots\right)}{5b}\cdots\right)}{9b}\cdots\right)}{13b}\cdots\right)$$

3.156. $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

input
```
Int[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) - (11*a*((-2*Sqrt[b*x^(1/3) +
 a*x])/(9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-
2*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-
((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*
(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3
))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b
 + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b +
a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1
/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) +
 a*x])))/(5*b)))/(9*b)))/(13*b))
```

3.156.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.156.4 Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{13b\,x^{\frac{7}{3}}}+\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{39b^{2}x^{\frac{5}{3}}}-\frac{154a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{195b^{3}x}+\frac{154\left(b+a\,x^{\frac{2}{3}}\right)a^{3}}{65b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{77a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{\cdots}}{\cdots}$
default	$\frac{-462a^{3}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231a^{3}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-\cdots}}{\sqrt{-ab}}}\cdots}{\cdots}$

input
```
int(1/x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*(b+a*x^(2/3))*a^3/b^4
/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-77/65*a^3/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-
a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^
(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(
-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2)))
```

3.156.5 Fricas [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^6 + b^3*x^4), x)
```

3.156.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a*x + b*x**(1/3))), x)
```

3.156.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

3.156.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^3\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/3))^(1/2)), x)
```

3.157 $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

3.157.1 Optimal result

Integrand size = 19, antiderivative size = 251

$$\begin{aligned}\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}}+\frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}}-\frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2}\\ &\quad+\frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}}-\frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}}\\ &\quad-\frac{663a^{19/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1463b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
-6/19*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+34/95*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x
^(8/3)-442/1045*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+3978/7315*a^3*(b*x^(1/3)
+a*x)^(1/2)/b^4/x^(4/3)-1326/1463*a^4*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-66
3/1463*a^(19/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{19}{4},\frac{1}{2},-\frac{15}{4},-\frac{ax^{2/3}}{b}\right)}{19x^3\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
(-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-19/4, 1/2, -15/4, -((a*x^(2
/3))/b)])/(19*x^3*Sqrt[b*x^(1/3) + a*x])
```

3.157.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1924, 1931, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax+b\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{ 1924}$$

$$3\int \frac{1}{x^{10/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(-\frac{17a\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$$\Big\downarrow \text{ 1931}$$

$$3\left(-\frac{17a\left(-\frac{13a\int \frac{1}{x^2\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$\downarrow$ 1931

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$\downarrow$ 1931

3.157. $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right)$$

$\downarrow$ 1931

3.157. $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}\right.$$

3.157. $\int\frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$\downarrow$ 1917

3.157. $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}\right.$$

3.157. $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$\downarrow 266$$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{19b}-\right.$$

3.157. $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

↓ 761

3.157. $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

$$3\left(-\frac{17a\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b}}{7bx^{4/}}\right)}{11b}\right)}{15b}\right)}{19b}\right)$$

3.157. $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

input
```
Int[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

output
```
3*((-2*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) - (17*a*((-2*Sqrt[b*x^(1/3)
+ a*x])/(15*b*x^(8/3)) - (13*a*((-2*Sqrt[b*x^(1/3) + a*x])/(11*b*x^2) - (9
*a*((-2*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) +
a*x])/(3*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2
/3)]*x^(1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF
[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*
x]*Sqrt[b + a*x^(4/3)])))/(7*b)))/(11*b)))/(15*b)))/(19*b))
```

3.157.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.157.4 Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

method	result
default	$-\frac{3315 a^{4} \sqrt{-a b}\, \sqrt{\frac{a\, x^{\frac{1}{3}}+\sqrt{-a b}}{\sqrt{-a b}}}\, \sqrt{-\frac{2 \left(a\, x^{\frac{1}{3}}-\sqrt{-a b}\right)}{\sqrt{-a b}}}\, \sqrt{-\frac{x^{\frac{1}{3}} a}{\sqrt{-a b}}}\, F\left(\sqrt{\frac{a\, x^{\frac{1}{3}}+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right) x^{\frac{16}{3}}+2652 a^{4} b\, x^{5}+6630 x^{\frac{17}{3}} a^{5}+4}{7315 b^{5} \sqrt{x^{\frac{1}{3}} \left(b+a\, x^{\frac{2}{3}}\right)}\, x^{\frac{16}{3}}}$
derivativedivides	$-\frac{6 \sqrt{b\, x^{\frac{1}{3}}+a x}}{19 b\, x^{\frac{10}{3}}}+\frac{34 a \sqrt{b\, x^{\frac{1}{3}}+a x}}{95 b^{2} x^{\frac{8}{3}}}-\frac{442 a^{2} \sqrt{b\, x^{\frac{1}{3}}+a x}}{1045 b^{3} x^{2}}+\frac{3978 a^{3} \sqrt{b\, x^{\frac{1}{3}}+a x}}{7315 b^{4} x^{\frac{4}{3}}}-\frac{1326 a^{4} \sqrt{b\, x^{\frac{1}{3}}+a x}}{1463 b^{5} x^{\frac{2}{3}}}-663 a^{4} \sqrt{-a}$

input
```
int(1/x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7315*(3315*a^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/
2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*x^(16/3)+2652*a^4*b*x^5+6630*x^(17/3)*a^5+476*x^(11/3)*a^2*b^3-884*x^
(13/3)*a^3*b^2-308*a*b^4*x^3+2310*x^(7/3)*b^5)/b^5/(x^(1/3)*(b+a*x^(2/3)))
^(1/2)/x^(16/3)
```

3.157.5 Fricas [F]

$$\int \frac{1}{x^4 \sqrt{b \sqrt[3]{x}+a x}}\, dx=\int \frac{1}{\sqrt{a x+b x^{\frac{1}{3}}}\, x^{4}}\, dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^7 + b^3*x^5), x)
```

3.157.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a*x + b*x**(1/3))), x)
```

3.157.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

3.157.8 Giac [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(1/3))^(1/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(1/3))^(1/2)), x)
```

3.158 $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.158.1 Optimal result

Integrand size = 19, antiderivative size = 437

$$\begin{aligned}
\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= -\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} \\
&\quad +\frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4}-\frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}+\frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} \\
&\quad +\frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{27/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442a^{27/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

output
```
-3*x^4/a/(b*x^(1/3)+a*x)^(1/2)-4807/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(13/2)
/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4807/663*b^4*x^(1/3)*(b*x
^(1/3)+a*x)^(1/2)/a^6-24035/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+6555/1547
*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-437/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(
1/2)/a^3+23/7*x^3*(b*x^(1/3)+a*x)^(1/2)/a^2+4807/221*b^(21/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x
^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/
a^(27/4)/(b*x^(1/3)+a*x)^(1/2)-4807/442*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(
1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ell
ipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2
)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(27/4)/(b*x
^(1/3)+a*x)^(1/2)
```

3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.30

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2x^{2/3}\left(-33649b^5+4807ab^4x^{2/3}-2185a^2b^3x^{4/3}+1311a^3b^2x^2-897a^4bx^{8/3}+663a^5\right.}{4641a^6\sqrt{b\sqrt[3]{x}+a}}$$

input
```
Integrate[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x^(2/3)*(-33649*b^5 + 4807*a*b^4*x^(2/3) - 2185*a^2*b^3*x^(4/3) + 1311*
a^3*b^2*x^2 - 897*a^4*b*x^(8/3) + 663*a^5*x^(10/3) + 33649*b^5*Sqrt[1 + (a
*x^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(4641*a^
6*Sqrt[b*x^(1/3) + a*x])
```

3.158.3 Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1924, 1928, 1930, 1930, 1930, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\int \frac{x^4}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{14/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1928}$$

$$3\left(\frac{23\int \frac{x^{11/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\int \frac{x^3}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{21a}\right)}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\int \frac{x^{7/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17a}\right)}{21a}\right)}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\int\frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13a}\right)}{17a}\right)}{21a}\right)}{2a}-\frac{x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1930

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int\frac{x}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{9a}\right)}{13a}\right)}{17a}\right)}{21a}\right)}{2a}-\frac{x^4}{a\sqrt{ax+}}\right.$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 1930

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\left(23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)}{13a}\right)}{17a}\right)}{21a}\right)\right.$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.158. $\displaystyle\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 1938

$$\left(23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{\cdots}}{\sqrt{x^{2}\cdots}}}{5a\sqrt{ax+b\sqrt[3]{\cdots}}}\cdots\right.}{9a}\cdots\right.}{13a}\cdots\right.}{17a}\cdots\right.}{21a}\cdots\right.\right.$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 266

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{\cdots}}{\sqrt{ax\cdots}}}{5a\sqrt{ax+b\sqrt[3]{\cdots}}}\right.\cdots}{9a}\right.\cdots}{13a}\right.\cdots}{17a}\right.\cdots}{21a}\right.\cdots}{2a}\right.\cdots$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 834

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int}{\cdots}\right.}{\cdots}\right.}{9\cdots}\right.}{13a}\right.}{17a}\right.}{21a}\right.$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$\downarrow$ 27

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int}{\quad}\right.$$

$$7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{\quad}{\quad}\right.$$

$$11b\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{\quad}{9a}\right.$$

$$15b\left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a}-\frac{\quad}{13a}\right.$$

$$19b\left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a}-\frac{\quad}{17a}\right.$$

$$23\left(\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{21a}-\frac{\quad}{21a}\right.$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 761

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\left(\left(19b \left(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a} - 15b \left(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a} - 11b \left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a} - 7b \left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a} - \frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}}{} \right.}{} \right. \right. \right. \right. \right. \right.$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 1510

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\Bigg(\Bigg(\Bigg(19b \Bigg(\frac{2x^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{17a} - \Bigg(15b \Bigg(\frac{2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{13a} - \Bigg(11b \Bigg(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a} - \Bigg(7b \Bigg(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a} - \frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b} \Bigg(\frac{\sqrt[4]{b}}{\cdots}}{\cdots}$$

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

input
```
Int[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x^4/(a*Sqrt[b*x^(1/3) + a*x])) + (23*((2*x^3*Sqrt[b*x^(1/3) + a*x])/(
21*a) - (19*b*((2*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(17*a) - (15*b*((2*x^(5/3
)*Sqrt[b*x^(1/3) + a*x])/(13*a) - (11*b*((2*x*Sqrt[b*x^(1/3) + a*x])/(9*a)
 - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5*a) - (6*b*Sqrt[b + a*x^(2/3)
]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3)))
 + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sq
rt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^
(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3)
)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(5*a*Sq
rt[b*x^(1/3) + a*x])))/(9*a)))/(13*a)))/(17*a)))/(21*a)))/(2*a))
```

3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.158. $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.158.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}b^5}{a^6\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a^2}-\frac{80b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^3}+\frac{1914b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^4}-\frac{10112b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^5}+28$
default	$5244x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^4b^2-3588x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^5b-8740x^2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^3b^3-201894b^6\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-$

input
```
int(x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*x^(2/3)/a^6*b^5/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/7*x^3*(b*x^(1/3)+a*x)^
(1/2)/a^2-80/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+1914/1547*b^2*x^(5/3)
*(b*x^(1/3)+a*x)^(1/2)/a^4-10112/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+2818
/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^6-4807/442*b^5/a^7*(-a*b)^(1/2)*(
(x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x
)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^
(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.158.5 Fricas [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^6 + 3*a^2*b^2*x^(14/3) - 2*a*b^3*x^4 - (2*a^3*b*x^5 - b^4*
x^3)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

3.158.6 Sympy [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**4/(a*x + b*x**(1/3))**(3/2), x)
```

3.158.7 Maxima [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

3.158.8 Giac [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}} \,d x$$

input
```
int(x^4/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

3.159 $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.159.1 Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}+\frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2}+\frac{663b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-3*x^3/a/(b*x^(1/3)+a*x)^(1/2)-663/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+1989/3
85*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-221/55*b*x^(4/3)*(b*x^(1/3)+a*x)^
(1/2)/a^3+17/5*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+663/154*b^(15/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x
^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/
a^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.159.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{-3315b^4\sqrt[3]{x}-1326ab^3x+442a^2b^2x^{5/3}-238a^3bx^{7/3}+154a^4x^3+3315b^4\sqrt{1+\frac{ax^{2/3}}{b}}}{385a^5\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(-3315*b^4*x^(1/3) - 1326*a*b^3*x + 442*a^2*b^2*x^(5/3) - 238*a^3*b*x^(7/3
) + 154*a^4*x^3 + 3315*b^4*Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2
F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)])/(385*a^5*Sqrt[b*x^(1/3) + a*x])
```

3.159.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1924, 1928, 1930, 1930, 1930, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{11/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1928}$$

$$3\left(\frac{17\int \frac{x^{8/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\int\frac{x^2}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1930

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\int\frac{x^{4/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11a}\right)}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1930

3.159. $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\int\frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7a}\right)}{11a}\right)}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1930

3.159. $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{3a}\right)}{7a}\right)}{11a}\right)}{15a}\right)}{2a}-\frac{x^3}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

3.159. $\int\frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 1917

3.159. $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)}{2a}\right.$$

3.159. $\displaystyle\int\frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\Big\downarrow \; 266$$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{7a}\right)}{11a}\right)}{15a}\right)}{2a}-\right.$$

3.159. $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$\downarrow$ 761

3.159. $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{15a}-\frac{13b\left(\frac{2x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{11a}-\frac{9b\left(\frac{2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{7a}-\frac{5b\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax}}\right.}{7a}\right.}{11a}\right.}{15a}\right.}{2a}\right.$$

3.159. $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

input
```
Int[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x^3/(a*Sqrt[b*x^(1/3) + a*x])) + (17*((2*x^2*Sqrt[b*x^(1/3) + a*x])/(
15*a) - (13*b*((2*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(11*a) - (9*b*((2*x^(2/3)
*Sqrt[b*x^(1/3) + a*x])/(7*a) - (5*b*((2*Sqrt[b*x^(1/3) + a*x])/(3*a) - (b
^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b + a
*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]))
)/(7*a)))/(11*a)))/(15*a)))/(2*a))
```

3.159.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.159.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}b^{4}}{a^{5}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a^{2}}-\frac{56b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^{3}}+\frac{834b^{2}x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^{4}}-\frac{432b^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^{5}}+\frac{663\ldots}{\ldots}$
default	$-\frac{-884\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{5}{3}}a^{3}b^{2}+476\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{7}{3}}a^{4}b-3315\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{\ldots}\right.}{\sqrt{-ab}\ldots}}\ldots}{\ldots}$

input
```
int(x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3*x^(1/3)/a^5*b^4/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/5*x^2*(b*x^(1/3)+a*x)
^(1/2)/a^2-56/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+834/385*b^2*x^(2/3)*(
b*x^(1/3)+a*x)^(1/2)/a^4-432/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+663/154*b^4/
a^6*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^
(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1
/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))
```

3.159.5 Fricas [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^5 + 3*a^2*b^2*x^(11/3) - 2*a*b^3*x^3 - (2*a^3*b*x^4 - b^4*
x^2)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

3.159.6 Sympy [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**3/(a*x + b*x**(1/3))**(3/2), x)
```

3.159.7 Maxima [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

3.159.8 Giac [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

3.160 $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.160.1 Optimal result

Integrand size = 19, antiderivative size = 349

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}$$
$$-\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3}+\frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2}$$
$$-\frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}$$
$$+\frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-3*x^2/a/(b*x^(1/3)+a*x)^(1/2)+77/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x^(
1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-77/15*b*x^(1/3)*(b*x^(1/3)+a*x
)^(1/2)/a^3+11/3*x*(b*x^(1/3)+a*x)^(1/2)/a^2-77/5*b^(9/4)*x^(1/6)*(cos(2*a
rctan(a^(1/4)*x^(1/6)/b^(1/4))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1
/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/
3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(1
5/4)/(b*x^(1/3)+a*x)^(1/2)+77/10*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(
1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(s
in(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2
))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a
*x)^(1/2)
```

3.160.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.27

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2x^{2/3}\left(77b^2-11abx^{2/3}+5a^2x^{4/3}-77b^2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\right.\right.}{15a^3\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(2*x^(2/3)*(77*b^2 - 11*a*b*x^(2/3) + 5*a^2*x^(4/3) - 77*b^2*Sqrt[1 + (a*x
^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(15*a^3*Sq
rt[b*x^(1/3) + a*x])
```

3.160.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1924, 1928, 1930, 1930, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{8/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1928}$$

$$3\left(\frac{11\int \frac{x^{5/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\int \frac{x}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5a}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1938}$$

3.160. $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{3b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 266

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 834

3.160. $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right.$$

$\downarrow$ 27

3.160. $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}-\frac{x^2}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 761

3.160. $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\int\cdots\right)}{5a\sqrt{ax+b\sqrt[3]{x}}}\right)}{9a}\right)}{2a}\cdots\right)$$

$\downarrow$ 1510

3.160. $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{9a}-\frac{7b\left(\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{5a}-\frac{6b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\cdots\right)}{5a\sqrt{ax+b\sqrt[3]{x}}\cdots}\cdots\right)}{9a}\cdots\right)}{2a}\cdots\right.$$

3.160. $\displaystyle\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

input
```
Int[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x^2/(a*Sqrt[b*x^(1/3) + a*x])) + (11*((2*x*Sqrt[b*x^(1/3) + a*x])/(9*
a) - (7*b*((2*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(5*a) - (6*b*Sqrt[b + a*x^(2/
3)]*x^(1/6)*(-((-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3)
)) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] +
Sqrt[a]*x^(2/3))^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(
a^(1/4)*Sqrt[b + a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/
3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(5*a*
Sqrt[b*x^(1/3) + a*x])))/(9*a)))/(2*a))
```

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.160.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{3x^{\frac{2}{3}}b^{2}}{a^{3}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3a^{2}}-\frac{32b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a^{3}}+\frac{77b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{}}{}$
default	$-\frac{-462b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}}{}$

input
```
int(x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3*x^(2/3)/a^3*b^2/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/3*x*(b*x^(1/3)+a*x)^(
1/2)/a^2-32/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+77/10*b^2/a^4*(-a*b)^(1
/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3
)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-
a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.160.5 Fricas [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^4 + 3*a^2*b^2*x^(8/3) - 2*a*b^3*x^2 - (2*a^3*b*x^3 - b^4*x
)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

3.160.6 Sympy [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2/(a*x + b*x**(1/3))**(3/2), x)
```

3.160.7 Maxima [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

3.160.8 Giac [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

3.161 $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.161.1 Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
-3*x/a/(b*x^(1/3)+a*x)^(1/2)+5*(b*x^(1/3)+a*x)^(1/2)/a^2-5/2*b^(3/4)*x^(1/
6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1
/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)
^(1/2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.161.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{\sqrt{b\sqrt[3]{x}+ax}\left(5b+2ax^{2/3}-5b\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)\right)}{a^2\left(b+ax^{2/3}\right)}$$

input
```
Integrate[x/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
(Sqrt[b*x^(1/3) + a*x]*(5*b + 2*a*x^(2/3) - 5*b*Sqrt[1 + (a*x^(2/3))/b]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a^2*(b + a*x^(2/3)))
```

3.161.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1924, 1928, 1930, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{x^{5/3}}{\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1928}$$

$$3\left(\frac{5\int \frac{x^{2/3}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1930}$$

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3a}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 1917

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 266

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{2b\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3a\sqrt{ax+b\sqrt[3]{x}}}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\Big\downarrow$ 761

$$3\left(\frac{5\left(\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}\right)}{2a}-\frac{x}{a\sqrt{ax+b\sqrt[3]{x}}}\right)$$

input
```
Int[x/(b*x^(1/3) + a*x)^(3/2),x]
```

output
```
3*(-(x/(a*Sqrt[b*x^(1/3) + a*x])) + (5*((2*Sqrt[b*x^(1/3) + a*x])/(3*a) -
(b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1/6)*Sqrt[(b +
 a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(
1/6))/b^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b + a*x^(4/3)]
)))/(2*a))
```

3.161. $\int\frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.161.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.161. $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.161.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}}b}{a^{2}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{a^{2}}-\frac{5b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)}{\sqrt{-ab}}}\right.}{2a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{5\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,b-6\sqrt{b\,x^{\frac{1}{3}}+ax}\,x^{\frac{1}{3}}ab-}{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)a^{3}}$

input
```
int(x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)/a^2*b/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2*(b*x^(1/3)+a*x)^(1/2)/a^
2-5/2*b/a^3*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(
1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

3.161.5 Fricas [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

3.161.6 Sympy [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(x/(a*x + b*x**(1/3))**(3/2), x)
```

3.161.7 Maxima [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

3.161.8 Giac [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
int(x/(a*x + b*x^(1/3))^(3/2), x)
```

3.162 $\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.162.1 Optimal result

Integrand size = 15, antiderivative size = 296

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}+\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}-\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
3*x^(2/3)/b/(b*x^(1/3)+a*x)^(1/2)-3*(b+a*x^(2/3))*x^(1/3)/b/a^(1/2)/(x^(1/
3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+3*x^(1/6)*(cos(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Elliptic
E(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(
1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b
*x^(1/3)+a*x)^(1/2)-3/2*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/
4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.162.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{2/3}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[(b*x^(1/3) + a*x)^(-3/2),x]
```

output
```
(2*Sqrt[1 + (a*x^(2/3))/b]*x^(2/3)*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x
^(2/3))/b)])/(b*Sqrt[b*x^(1/3) + a*x])
```

3.162.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {1912, 1924, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}dx}{2b}$$

$$\Big\downarrow \text{1924}$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\int \frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{2b\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \ 266$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \ 834$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \ 27$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \ 761$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

$$\Big\downarrow \ 1510$$

$$\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}E\left(2\text{ arct}\right.}{\sqrt[4]{a}\sqrt{ax^{4/3}+b}}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

input
```
Int[(b*x^(1/3) + a*x)^(-3/2),x]
```

3.162. $\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

output
```
(3*x^(2/3))/(b*Sqrt[b*x^(1/3) + a*x]) - (3*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-(
(-((x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(
Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3)
)^2]*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b
+ a*x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a
*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) +
a*x])
```

3.162.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.162.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}}{b\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\right.}{2ba\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{-3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b+\frac{3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{}}{\sqrt{-ab}}}}{}}{a\,x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)b}$

input
```
int(1/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*x^(2/3)/b/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-3/2/b/a*(-a*b)^(1/2)*((x^(1/3)
+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/
(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*
(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.162.5 Fricas [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^5 + 2*a^3*b^3*x^3 + b^6*x), x)
```

3.162.6 Sympy [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(1/3))**(-3/2), x)
```

3.162.7 Maxima [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(1/3))^(-3/2), x)
```

3.162.8 Giac [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^(1/3))^(-3/2), x)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.14

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2\,x\,\left(\frac{a\,x^{2/3}}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{2};\ \frac{7}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}$$

input
```
int(1/(a*x + b*x^(1/3))^(3/2),x)
```

output
```
(2*x*((a*x^(2/3))/b + 1)^(3/2)*hypergeom([3/4, 3/2], 7/4, -(a*x^(2/3))/b))
/(a*x + b*x^(1/3))^(3/2)
```

3.163 $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.163.1 Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
3/b/x^(1/3)/(b*x^(1/3)+a*x)^(1/2)-5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)-5/2*
a^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2
)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.163.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((a*x^(2/3)
)/b)])/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])
```

3.163.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1924, 1929, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{\sqrt[3]{x}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1929}$$

$$3\left(\frac{5\int \frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{5\left(-\frac{a\int \frac{1}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1917}$$

$$3\left(\frac{5\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{266}$$

$$3\left(\frac{5\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int \frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{761}$$

$$3\left(\frac{5\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{2b}+\frac{1}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

input
```
Int[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) + (5*((-2*Sqrt[b*x^(1/3) + a*x])/(3
*b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(
1/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTa
n[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[
b + a*x^(4/3)])))/(2*b))
```

3.163. $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.163.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.163. $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.163.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}a}{b^2\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{b^2x^{\frac{2}{3}}}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right.}{\sqrt{-ab}}}\right.}{2b^2\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{5x^{\frac{2}{3}}\sqrt{-ab}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+6\sqrt{b\,x^{\frac{1}{3}}+ax}\,xa+}{2b^2x\left(b+a\,x^{\frac{2}{3}}\right)}$

input
```
int(1/x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3*x^(1/3)*a/b^2/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-2*(b*x^(1/3)+a*x)^(1/2)/b
^2/x^(2/3)-5/2/b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2)
)^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(
-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1
/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

3.163.5 Fricas [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^6 + 2*a^3*b^3*x^4 + b^6*x^2), x)
```

3.163.6 Sympy [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(a*x + b*x**(1/3))**(3/2)), x)
```

3.163.7 Maxima [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

3.163.8 Giac [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/3))^(3/2)), x)
```

3.164 $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.164.1 Optimal result

Integrand size = 19, antiderivative size = 383

$$\begin{aligned}\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}+\frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}\\ &-\frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}}+\frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x}-\frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}}\\ &-\frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\\ &+\frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
3/b/x^(4/3)/(b*x^(1/3)+a*x)^(1/2)+77/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(
x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-11/3*(b*x^(1/3)+a*x)^(1/2)/
b^2/x^(5/3)+77/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x-77/5*a^2*(b*x^(1/3)+a*x)^(
1/2)/b^4/x^(1/3)-77/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^
(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)+7
7/10*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a
^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.164.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{3}{2},-\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{3bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((a*x^(2/3
))/b)])/(3*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])
```

3.164.3 Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1924, 1929, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{x^{4/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\downarrow 1929$$

$$3\left(\frac{11\int \frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 1931$$

$$3\left(\frac{11\left(-\frac{7a\int \frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 1931$$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\int \frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 1931$$

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\downarrow$ 1938

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 266$$

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\frac{1}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 834$$

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}\right.$$

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\downarrow 27$$

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{2b}+\right.$$

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.164.

$\downarrow$ 761

3.164. $\displaystyle\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\cdots\right)}{5b}\cdots\right)}{9b}\cdots\right)}{2b}\cdots\right)$$

3.164. $\displaystyle\int\frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 1510

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{11\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\cdots}{\sqrt[4]{a}\sqrt{ax\cdots}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}\cdots\right)}{5b}\cdots\right)}{9b}\cdots\right)}{2b}\cdots\right)$$

3.164. $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

input
```
Int[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(4/3)*Sqrt[b*x^(1/3) + a*x]) + (11*((-2*Sqrt[b*x^(1/3) + a*x])/(
9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-2*Sqrt[b
*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((x^(
1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt[b]
 + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*El
lipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*x^(
4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3
))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x]))
/(5*b)))/(9*b)))/(2*b))
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.164.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{3b^2x^{\frac{5}{3}}}+\frac{32a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15b^3x}-\frac{62\left(b+a\,x^{\frac{2}{3}}\right)a^2}{5b^4\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{3x^{\frac{2}{3}}a^3}{b^4\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{77a^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{\cdots}}{\cdots}$
default	$-\frac{-462a^2b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231a^2b\sqrt{\frac{a\,x^{\frac{1}{3}}+\cdots}{\sqrt{-\cdots}}}\cdots}{\cdots}$

input
```
int(1/x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)+32/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x
-62/5*(b+a*x^(2/3))*a^2/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-3*x^(2/3)*a^3/b^
4/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+77/10*a^2/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a
*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/
a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.164.5 Fricas [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^7 + 2*a^3*b^3*x^5 + b^6*x^3), x)
```

3.164.6 Sympy [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(a*x + b*x**(1/3))**(3/2)), x)
```

3.164.7 Maxima [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

3.164.8 Giac [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}} \,d x$$

input
```
int(1/(x^2*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/3))^(3/2)), x)
```

3.165 $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.165.1 Optimal result

Integrand size = 19, antiderivative size = 246

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}+\frac{663a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

output
```
3/b/x^(7/3)/(b*x^(1/3)+a*x)^(1/2)-17/5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)+2
21/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2-1989/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b
^4/x^(4/3)+663/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)+663/154*a^(15/4)*x
^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/
4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*
2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2)
)^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.165.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},\frac{3}{2},-\frac{11}{4},-\frac{ax^{2/3}}{b}\right)}{5bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-15/4, 3/2, -11/4, -((a*x^(2
/3))/b)])/(5*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])
```

3.165.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1924, 1929, 1931, 1931, 1931, 1931, 1917, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$3\int \frac{1}{x^{7/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1929}$$

$$3\left(\frac{17\int \frac{1}{x^{8/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \text{1931}$$

$$3\left(\frac{17\left(-\frac{13a\int \frac{1}{x^2\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\big\downarrow$ 1931

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\int \frac{1}{x^{4/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$\big\downarrow$ 1931

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\int\frac{1}{x^{2/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \; 1931$$

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\int\frac{1}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{3b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{1}{bx^{7/3}\sqrt{ax+b}}\right.$$

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$\downarrow$ 1917

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{x^{2/3}a+b}\sqrt[6]{x}}d\sqrt[3]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^{2}}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\right.$$

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\downarrow 266$$

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{3b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right)}{11b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{11bx^2}\right)}{15b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{15bx^{8/3}}\right)}{2b}+\frac{\ }{bx^{7}\ldots}\right.$$

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.165.

$\downarrow$ 761

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{17\left(-\frac{13a\left(-\frac{9a\left(-\frac{5a\left(-\frac{a^{3/4}\sqrt[6]{x}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{ax^{2/3}+b}\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+b\sqrt[3]{x}}\sqrt{ax^{4/3}+b}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{3bx^{2/3}}\right)}{7b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}\right.}{11b}\right.}{15b}\right.}{2b}\right.$$

3.165. $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

input
```
Int[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(7/3)*Sqrt[b*x^(1/3) + a*x]) + (17*((-2*Sqrt[b*x^(1/3) + a*x])/(
15*b*x^(8/3)) - (13*a*((-2*Sqrt[b*x^(1/3) + a*x])/(11*b*x^2) - (9*a*((-2*S
qrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) - (5*a*((-2*Sqrt[b*x^(1/3) + a*x])/(3*
b*x^(2/3)) - (a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[b + a*x^(2/3)]*x^(1
/6)*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^(1/3) + a*x]*Sqrt[b
 + a*x^(4/3)])))/(7*b)))/(11*b)))/(15*b)))/(2*b))
```

3.165.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.165.4 Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5b^{2}x^{\frac{8}{3}}}+\frac{56a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55b^{3}x^{2}}-\frac{834a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b^{4}x^{\frac{4}{3}}}+\frac{432a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{5}x^{\frac{2}{3}}}+\frac{3x^{\frac{1}{3}}a^{4}}{b^{5}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{663a^{3}\sqrt{-ab}}{}$
default	$\frac{3315x^{\frac{14}{3}}\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{3}-884x^{\frac{11}{3}}\sqrt{x^{\frac{1}{3}}}}{}$

input
```
int(1/x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)+56/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x
^2-834/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)+432/77*a^3*(b*x^(1/3)+a*x
)^(1/2)/b^5/x^(2/3)+3*x^(1/3)*a^4/b^5/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+663/
154*a^3/b^5*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(
1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

3.165.5 Fricas [F]

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^8 + 2*a^3*b^3*x^6 + b^6*x^4), x)
```

3.165.6 Sympy [F]

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**3*(a*x + b*x**(1/3))**(3/2)), x)
```

3.165.7 Maxima [F]

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

3.165.8 Giac [F]

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/3))^(3/2)), x)
```

3.166 $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.166.1 Optimal result

Integrand size = 19, antiderivative size = 471

$$\begin{aligned}\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221b^7\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}\\
&-\frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}}+\frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3}-\frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}}\\
&+\frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}}-\frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x}+\frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}}\\
&+\frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221b^{27/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&-\frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442b^{27/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

output
```
3/b/x^(10/3)/(b*x^(1/3)+a*x)^(1/2)-4807/221*a^(11/2)*(b+a*x^(2/3))*x^(1/3)
/b^7/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-23/7*(b*x^(1/3)+a*x)^
(1/2)/b^2/x^(11/3)+437/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^3-6555/1547*a^2*(
b*x^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+24035/4641*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/
x^(5/3)-4807/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4807/221*a^5*(b*x^(1/3)+a
*x)^(1/2)/b^7/x^(1/3)+4807/221*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(si
n(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2)
)*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*
x)^(1/2)-4807/442*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))
^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^
(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/
3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)
```

3.166.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{21}{4},\frac{3}{2},-\frac{17}{4},-\frac{ax^{2/3}}{b}\right)}{7bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}$$

input
```
Integrate[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-21/4, 3/2, -17/4, -((a*x^(2
/3))/b)])/(7*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])
```

3.166.3 Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {1924, 1929, 1931, 1931, 1931, 1931, 1931, 1931, 1938, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(ax+b\sqrt[3]{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1924$$

$$3\int \frac{1}{x^{10/3}\left(\sqrt[3]{x}b+ax\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 1929$$

$$3\left(\frac{23\int \frac{1}{x^{11/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \; 1931$$

$$3\left(\frac{23\left(-\frac{19a\int \frac{1}{x^3\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \; 1931$$

$$3\left(\frac{23\left(-\frac{19a\left(-\frac{15a\int \frac{1}{x^{7/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^3}\right)}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\Big\downarrow \; 1931$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{23\left(-\frac{19a\left(-\frac{15a\left(-\frac{11a\int\frac{1}{x^{5/3}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^3}\right)}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}\right)$$

$$\downarrow 1931$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$3\left(\frac{23\left(-\frac{19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\int\frac{1}{x\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^{3}}\right)}{21b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{21bx^{11/3}}\right)}{2b}+\frac{1}{bx^{10/3}\sqrt{ax}}\right.$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 1931

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$
23\left(-\frac{19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\int\frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}b+ax}}d\sqrt[3]{x}}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^{3}}\right)}{21b}-\frac{2\sqrt{ax+b}}{21bx^{11}}
$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 1931

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\int\frac{\sqrt[3]{x}}{\sqrt{\sqrt[3]{x}b+ax}}\,d\sqrt[3]{x}}{b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{17bx^{3}}$$

3.166. $\int\frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.166.

↓ 1938

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\left(\left(19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{\sqrt[6]{x}}{\sqrt{x^{2/3}a+b}}d\sqrt[3]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\right.\right.\right.$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

↓ 266

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\left(\left(23\left(-\frac{19a\left(-\frac{15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\int\frac{x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)}{9b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{9bx^{5/3}}\right)}{13b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}\right)}{17b}-\cdots\right)}{21b}\cdots$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\downarrow \ 834$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{b}\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)$$

$$11a\left(-\frac{7a\left(\cdots\right)}{9b}-\cdots\right)$$

$$15a\left(-\frac{11a\left(\cdots\right)}{13b}\cdots\right)$$

$$19a\left(-\frac{15a\left(\cdots\right)}{17b}\cdots\right)$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\downarrow 27$$

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$\left(\left(\left(\left(19a\left(15a\left(11a\left(7a\left(3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/3}}{\sqrt{ax^{4/3}+b}}d\sqrt[6]{x}}{\sqrt{a}}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\right)-\frac{\ldots}{5b}-\frac{2\sqrt{ax+b\sqrt[3]{x}}}{5bx}\right)-\frac{\ldots}{9b}-\frac{2\sqrt{a\ldots}}{9\ldots}\right.\right.\right.\right.\right.\right.\right.\right.$$

$$-\frac{\ldots}{13b}\ldots\qquad -\frac{\ldots}{17b}\ldots$$

3.166. $\displaystyle\int\frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.166.

↓ 761

3.167. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

$$15a\left(-\frac{11a\left(-\frac{7a\left(-\frac{3a\left(\frac{2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}}-\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{2/\cdots}}{\sqrt{ax^{4/3}+b\cdots}}}{\sqrt{a}}\cdots\right)}{b\sqrt{ax+b\sqrt[3]{x}}}\cdots\right)}{5b}\cdots\right)}{9b}\cdots\right)}{13b}\cdots\right.$$

3.166. $\int\frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.166. $\downarrow$ 1510

3.166. $$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$$

$$2a\sqrt[6]{x}\sqrt{ax^{2/3}+b}\left(\frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)\sqrt{\frac{ax^{4/3}+b}{\left(\sqrt{a}x^{2/3}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{ax^{4/3}+b}} - \frac{\sqrt[4]{b}\left(\sqrt{a}x^{2/3}\right.}{}\right.$$

$$3a \qquad b\sqrt{ax+b\sqrt[3]{x}}$$

$$7a \; - \qquad 5b$$

$$11a \; - \qquad 9b$$

$$15a \; - \qquad 1$$

3.166. $\displaystyle\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

input
```
Int[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

output
```
3*(1/(b*x^(10/3)*Sqrt[b*x^(1/3) + a*x]) + (23*((-2*Sqrt[b*x^(1/3) + a*x])/
(21*b*x^(11/3)) - (19*a*((-2*Sqrt[b*x^(1/3) + a*x])/(17*b*x^3) - (15*a*((-
2*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) - (11*a*((-2*Sqrt[b*x^(1/3) + a*x]
)/(9*b*x^(5/3)) - (7*a*((-2*Sqrt[b*x^(1/3) + a*x])/(5*b*x) - (3*a*((-2*Sqr
t[b*x^(1/3) + a*x])/(b*x^(1/3)) + (2*a*Sqrt[b + a*x^(2/3)]*x^(1/6)*(-((-((
x^(1/6)*Sqrt[b + a*x^(4/3)])/(Sqrt[b] + Sqrt[a]*x^(2/3))) + (b^(1/4)*(Sqrt
[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]
*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b + a*
x^(4/3)]))/Sqrt[a]) + (b^(1/4)*(Sqrt[b] + Sqrt[a]*x^(2/3))*Sqrt[(b + a*x^(
4/3))/(Sqrt[b] + Sqrt[a]*x^(2/3))^2]*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/
b^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[b + a*x^(4/3)])))/(b*Sqrt[b*x^(1/3) + a*x]
)))/(5*b)))/(9*b)))/(13*b)))/(17*b)))/(21*b)))/(2*b))
```

3.166.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.166. $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

3.166.4 Maple [A] (verified)

Time = 6.07 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b^{2}x^{\frac{11}{3}}}+\frac{80a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119b^{3}x^{3}}-\frac{1914a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b^{4}x^{\frac{7}{3}}}+\frac{10112a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^{5}x^{\frac{5}{3}}}-\frac{2818a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^{6}x}+\frac{4144\left(b \right.}{221b^{7}\sqrt{x}}$
default	$-\frac{201894a^{5}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-100947a^{5}b\sqrt{}}{}$

input
```
int(1/x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(11/3)+80/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3
/x^3-1914/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+10112/4641*a^3*(b*x^(
1/3)+a*x)^(1/2)/b^5/x^(5/3)-2818/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4144/
221*(b+a*x^(2/3))/b^7*a^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+3*x^(2/3)*a^6/b^7/
((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-4807/442*a^5/b^7*(-a*b)^(1/2)*((x^(1/3)+1/
a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a
*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2
/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

3.166.5 Fricas [F]

$$\int \frac{1}{x^{4}\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^{4}}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^
(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^9 + 2*a^3*b^3*x^7 + b^6*x^5), x)
```

3.166.6 Sympy [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^4\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**4*(a*x + b*x**(1/3))**(3/2)), x)
```

3.166.7 Maxima [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

3.166.8 Giac [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(b\sqrt[3]{x} + ax\right)^{3/2}}\, dx = \int \frac{1}{x^4\,\left(a\,x + b\,x^{1/3}\right)^{3/2}}\, dx$$

input
```
int(1/(x^4*(a*x + b*x^(1/3))^(3/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(1/3))^(3/2)), x)
```

3.167 $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

3.167.1 Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}\int x^3\sqrt{bx^{2/3}+ax}\,dx &= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x}\\ &-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}\\ &+\frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}-\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}\\ &+\frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}-\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}\\ &+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}\\ &+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}\end{aligned}$$

output
```
-524288/4345965*b^9*(b*x^(2/3)+a*x)^(3/2)/a^10+8388608/152108775*b^12*(b*x
^(2/3)+a*x)^(3/2)/a^13/x-4194304/50702925*b^11*(b*x^(2/3)+a*x)^(3/2)/a^12/
x^(2/3)+1048576/10140585*b^10*(b*x^(2/3)+a*x)^(3/2)/a^11/x^(1/3)+65536/482
885*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^9-360448/2414425*b^7*x^(2/3)*(b*x^
(2/3)+a*x)^(3/2)/a^8+90112/557175*b^6*x*(b*x^(2/3)+a*x)^(3/2)/a^7-45056/26
0015*b^5*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/a^6+2816/15295*b^4*x^(5/3)*(b*x^(2/
3)+a*x)^(3/2)/a^5-1408/7245*b^3*x^2*(b*x^(2/3)+a*x)^(3/2)/a^4+352/1725*b^2
*x^(7/3)*(b*x^(2/3)+a*x)^(3/2)/a^3-16/75*b*x^(8/3)*(b*x^(2/3)+a*x)^(3/2)/a
^2+2/9*x^3*(b*x^(2/3)+a*x)^(3/2)/a
```

3.167.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.50

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(4194304b^{13}-2097152ab^{12}\sqrt[3]{x}+1572864a^2b^{11}x^{2/3}-1310720a^3b^{10}x\right.}{}$$

input
```
Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(4194304*b^13 - 2097152*a*b^12*x^(1/3) + 1572864*
a^2*b^11*x^(2/3) - 1310720*a^3*b^10*x + 1146880*a^4*b^9*x^(4/3) - 1032192*
a^5*b^8*x^(5/3) + 946176*a^6*b^7*x^2 - 878592*a^7*b^6*x^(7/3) + 823680*a^8
*b^5*x^(8/3) - 777920*a^9*b^4*x^3 + 739024*a^10*b^3*x^(10/3) - 705432*a^11
*b^2*x^(11/3) + 676039*a^12*b*x^4 + 16900975*a^13*x^(13/3)))/(152108775*a^
13*x^(1/3))
```

3.167.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{ax+bx^{2/3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\int x^{8/3}\sqrt{x^{2/3}b+ax}dx}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\int x^{7/3}\sqrt{x^{2/3}b+ax}dx}{25a}\right)}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a}-\frac{20b\int x^2\sqrt{x^{2/3}b+ax}dx}{23a}\right)}{25a}\right)}{9a}$$

$$\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a}-\frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\int x^{5/3}\sqrt{x^{2/3}b+ax}dx}{7a}\right)}{23a}\right)}{25a}\right)}{9a}$$

$$\downarrow \text{1922}$$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a}-\frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\int x^{4/3}\sqrt{x^{2/3}b+ax}dx}{19a}\right)}{7a}\right)}{23a}\right)}{25a}\right)}{9a}$$

$$\downarrow \text{1922}$$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-\frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a}-\frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a}-\frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\int x\sqrt{}}{}\right.}{19a}\right.}{7a}\right.}{23a}\right.}{25a}\right.}{9a}$$

$\downarrow$ 1922

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} - \frac{8b\left(\frac{6x^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{25a} - \frac{22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a} - \frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} - \frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a} - \frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \frac{14b\left(\frac{2x\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{7a}\right.}{23a}\right.}{25a}\right.}{\cdots}$$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

↓ 1922

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a} -$$

$$22b\left(\frac{6x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{23a} - \frac{20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} - \frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a} - \frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a} - \frac{14b\left(\frac{2x(}{}\right.}{}\right)}{}\right)}{}\right)}{23a}\right.$$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$\downarrow$ 1922

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-$$

$$20b\left(\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\right.$$

$$6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\right.$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\right.$$

$$14b\left(\frac{2x(\cdots}{\cdots}\right.$$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$\downarrow$ 1908

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-$$

$$14b\left(\frac{2x\left(\right.}{}\right.$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\right.$$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\downarrow \text{1922}$$

3.167. $\int x^3 \sqrt{b x^{2/3} + a x} \, dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-$$

$$14b\left(\frac{2x\left(\right.}{}\right.$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{}{}\right.$$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$\downarrow$ 1922

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-$$

$14b \quad \frac{2x(}{}$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

↓ 1920

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}-$$

$$14b \quad \frac{2x(}{}$$

3.167. $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

input
```
Int[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a) - (8*b*((6*x^(8/3)*(b*x^(2/3) + a*x)
^(3/2))/(25*a) - (22*b*((6*x^(7/3)*(b*x^(2/3) + a*x)^(3/2))/(23*a) - (20*b
*((2*x^2*(b*x^(2/3) + a*x)^(3/2))/(7*a) - (6*b*((6*x^(5/3)*(b*x^(2/3) + a*
x)^(3/2))/(19*a) - (16*b*((6*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(17*a) - (14
*b*((2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a) - (4*b*((6*x^(2/3)*(b*x^(2/3) + a*
x)^(3/2))/(13*a) - (10*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(11*a) - (8*
b*((2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(
7*a*x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2
/3) + a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)))/(11*a)))/(13*a)))/(5*a))
)/(17*a)))/(19*a)))/(7*a)))/(23*a)))/(25*a)))/(9*a)
```

3.167.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.167.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\left(b+a\,x^{\frac{1}{3}}\right)\left(16900975a^{12}x^{4}-16224936a^{11}b\,x^{\frac{11}{3}}+15519504a^{10}b^{2}x^{\frac{10}{3}}-14780480a^{9}x^{3}b^{3}+14002560a^{8}b^{4}x^{\frac{8}{3}}\right.}{}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\left(b+a\,x^{\frac{1}{3}}\right)\left(16224936a^{11}b\,x^{\frac{11}{3}}-15519504a^{10}b^{2}x^{\frac{10}{3}}-14002560a^{8}b^{4}x^{\frac{8}{3}}+13178880a^{7}b^{5}x^{\frac{7}{3}}+11354112a^{5}\right.}{}$

input
```
int(x^3*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/152108775*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(16900975*a^12*x^4-1622493
6*a^11*b*x^(11/3)+15519504*a^10*b^2*x^(10/3)-14780480*a^9*x^3*b^3+14002560
*a^8*b^4*x^(8/3)-13178880*a^7*b^5*x^(7/3)+12300288*a^6*b^6*x^2-11354112*a^
5*b^7*x^(5/3)+10321920*a^4*b^8*x^(4/3)-9175040*a^3*b^9*x+7864320*a^2*b^10*
x^(2/3)-6291456*a*b^11*x^(1/3)+4194304*b^12)/x^(1/3)/a^13
```

3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(277) = 554.

Time = 179.80 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.49

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/304217550*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(
64*a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 -
 393725113600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6
 - 114688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*
(48816*a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^
10 - 7340032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b
^8 + 12288*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(437436
16*a^12 + 89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 42432*(72
17086464*a^15 + 4969216*a^12 + 165*a^9)*b^4 + 7524608*(20570112*a^15 - 210
1*a^12)*b^3 + 2821728*(7815168*a^15 + 181*a^12)*b^2 + 2028117*(2072576*a^1
5 - 3*a^12)*b)*x - 4*(16900975*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196
608*a^13*b^4 - 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b
- 27*a^13)*x^5 + 739024*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^1
0*b^7 - 114688*a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27
*a^10)*b^3)*x^4 - 878592*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^
7*b^10 - 114688*a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a
^7)*b^6)*x^3 + 1146880*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*
b^13 - 114688*a^4*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a
^4)*b^9)*x^2 - 2097152*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 -
 114688*a*b^15 - 2304*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x...
```

3.167.6 Sympy [F]

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int x^3\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

input
```
integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*x + b*x**(2/3)), x)
```

3.167.7 Maxima [F]

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int \sqrt{ax+bx^{\frac{2}{3}}}x^3\,dx$$

input
```
integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))*x^3, x)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.07

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=-\frac{8388608\,b^{\frac{27}{2}}}{152108775\,a^{13}} + \frac{2\left(\frac{27\left(676039\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}-8817900\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b+53117350\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^2-195695500\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^3+492116625\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^4-892371\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-8388608/152108775*b^(27/2)/a^13 + 2/152108775*(27*(676039*(a*x^(1/3) + b)
^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/
2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^
(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3)
+ b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(
1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x
^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sq
rt(a*x^(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 1825
3053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478
056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4
- 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2
)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)
^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3)
 + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(
1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int x^3\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

input
```
int(x^3*(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^3*(a*x + b*x^(2/3))^(1/2), x)
```

3.168 $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

3.168.1 Optimal result

Integrand size = 19, antiderivative size = 283

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=\frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}}-\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}$$

output
```
8192/46189*b^6*(b*x^(2/3)+a*x)^(3/2)/a^7-131072/1616615*b^9*(b*x^(2/3)+a*x
)^(3/2)/a^10/x+196608/1616615*b^8*(b*x^(2/3)+a*x)^(3/2)/a^9/x^(2/3)-49152/
323323*b^7*(b*x^(2/3)+a*x)^(3/2)/a^8/x^(1/3)-9216/46189*b^5*x^(1/3)*(b*x^(
2/3)+a*x)^(3/2)/a^6+4608/20995*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^5-384/1
615*b^3*x*(b*x^(2/3)+a*x)^(3/2)/a^4+576/2261*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(
3/2)/a^3-36/133*b*x^(5/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/7*x^2*(b*x^(2/3)+a*x
)^(3/2)/a
```

3.168.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.47

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(-65536b^9+98304ab^8\sqrt[3]{x}-122880a^2b^7x^{2/3}+143360a^3b^6x-161\right.}{}$$

input
```
Integrate[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2)*(-65536*b^9 + 98304*a*b^8*x^(1/3) - 122880*a^2*
b^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 177408*a^5*b^4*x
^(5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a^8*b*x^(8/3
) + 230945*a^9*x^3))/(1616615*a^10*x)
```

3.168.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax+bx^{2/3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\int x^{5/3}\sqrt{x^{2/3}b+ax}dx}{7a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\int x^{4/3}\sqrt{x^{2/3}b+ax}dx}{19a}\right)}{7a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\int x\sqrt{x^{2/3}b+ax}dx}{17a}\right)}{19a}\right)}{7a}$$

$\downarrow$ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\int x^{2/3}\sqrt{x^{2/3}b+ax}dx}{5a}\right)}{17a}\right)}{19a}\right)}{7a}$$

$\downarrow$ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\int\sqrt[3]{x}\sqrt{x^{2/3}b+ax}dx}{13a}\right)}{5a}\right)}{17a}\right)}{19a}\right)}{7a}$$

$\downarrow$ 1922

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-\frac{6b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{19a}-\frac{16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\int\sqrt{x^{2/\cdots}}\cdots}{11\cdots}\cdots\right.}{13a}\cdots\right.}{5a}\cdots\right.}{17a}\cdots\right.}{19a}\cdots\right.}{7a}\cdots$$

$\downarrow$ 1908

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax\cdots\right.}{\cdots}\right.}{\cdots}\right)}{13\cdots}\right)}{5a}\right)}{17a}\right.$$

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$\downarrow$ 1922

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-$$

$$16b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{17a}-\frac{14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{5a\cdots}\right.}{17a\cdots}\right.$$

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$\downarrow$ 1922

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}-$$

$8b\left(\frac{2\left(ax+\right.}{}\right.$

$10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\right.$

$4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\right.$

$14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\right.$

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$$\Big\downarrow \text{1920}$$

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a} -$$

$$8b\left(\frac{2\left(ax\right.}{}\right.$$

$$10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a} - \right.$$

$$4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a} - \right.$$

$$14b\left(\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a} - \right.$$

3.168. $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

input
```
Int[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^2*(b*x^(2/3) + a*x)^(3/2))/(7*a) - (6*b*((6*x^(5/3)*(b*x^(2/3) + a*x)
^(3/2))/(19*a) - (16*b*((6*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(17*a) - (14*b
*((2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a) - (4*b*((6*x^(2/3)*(b*x^(2/3) + a*x)
^(3/2))/(13*a) - (10*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(11*a) - (8*b*
((2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(7*
a*x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2/3
) + a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)))/(11*a)))/(13*a)))/(5*a)))/
(17*a)))/(19*a)))/(7*a)
```

3.168.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.168.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(230945a^{9}x^{3}-218790a^{8}b\,x^{\frac{8}{3}}+205920a^{7}b^{2}x^{\frac{7}{3}}-192192a^{6}b^{3}x^{2}+177408a^{5}b^{4}x^{\frac{5}{3}}-161280a^{4}b^{5}x^{\frac{4}{3}}\right.}{1616615x^{\frac{1}{3}}a^{10}}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(218790a^{8}b\,x^{\frac{8}{3}}-205920a^{7}b^{2}x^{\frac{7}{3}}-177408a^{5}b^{4}x^{\frac{5}{3}}+161280a^{4}b^{5}x^{\frac{4}{3}}-230945a^{9}x^{3}+122880a^{2}b^{7}\right.}{1616615x^{\frac{1}{3}}a^{10}}$

input
```
int(x^2*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/1616615*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(230945*a^9*x^3-218790*a^8*b
*x^(8/3)+205920*a^7*b^2*x^(7/3)-192192*a^6*b^3*x^2+177408*a^5*b^4*x^(5/3)-
161280*a^4*b^5*x^(4/3)+143360*a^3*b^6*x-122880*a^2*b^7*x^(2/3)+98304*a*b^8
*x^(1/3)-65536*b^9)/x^(1/3)/a^10
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(211) = 422.

Time = 150.03 (sec) , antiderivative size = 1031, normalized size of antiderivative = 3.64

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/3233230*((3298534883328*b^16 + 687194767360*b^15 + 3221225472*(64*a^3 -
3)*b^13 - 64424509440*b^14 - 16777216*(11264*a^3 - 53)*b^12 + 5380094720*a
^12 - 6291456*(5504*a^3 + 1)*b^11 + 196608*(3194880*a^6 - 114688*a^3 - 3)*
b^10 + 7340032*(18816*a^6 + 103*a^3)*b^9 - 786432*(48816*a^6 + 23*a^3)*b^8
 - 12288*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^7 - 114688*(1349120*a^9 +
3439*a^6)*b^6 + 3913728*(5600*a^9 + 3*a^6)*b^5 - 2112*(2027683840*a^12 + 1
958400*a^9 + 63*a^6)*b^4 - 36608*(59351040*a^12 - 8101*a^9)*b^3 - 549120*(
566272*a^12 + 17*a^9)*b^2 - 109395*(516096*a^12 - a^9)*b)*x + 4*(230945*(1
6777216*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 1146
88*a^10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 + 13728*(16777216*
a^7*b^9 + 6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5
 + 864*a^7*b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 - 17920*(16777216*a^4*b^1
2 + 6291456*a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 8
64*a^4*b^7 - (262144*a^7 + 27*a^4)*b^6)*x^2 + 32768*(16777216*a*b^15 + 629
1456*a*b^14 + 196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (
262144*a^4 + 27*a)*b^9)*x - (1099511627776*b^16 + 412316860416*b^15 + 1288
4901888*b^14 - 7516192768*b^13 - 150994944*b^12 - 65536*(262144*a^3 + 27)*
b^10 + 56623104*b^11 - 12155*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*
a^9*b^5 - 114688*a^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*
a^9)*b)*x^3 + 14784*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b...
```

3.168.6 Sympy [F]

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=\int x^2\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

input
```
integrate(x**2*(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*x + b*x**(2/3)), x)
```

3.168.7 Maxima [F]

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=\int \sqrt{ax+bx^{\frac{2}{3}}}x^2\,dx$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))*x^2, x)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.10

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=\frac{131072\,b^{\frac{21}{2}}}{1616615\,a^{10}}$$
$$+\frac{2\left(\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}-122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b+554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^2-1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^3+2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^4-3233230\left(ax^{\frac{1}{3}}+\right.}{a^9}\right.}{}$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
131072/1616615*b^(21/2)/a^10 + 2/1616615*(21*(12155*(a*x^(1/3) + b)^(19/2)
 - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 1
492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3
233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 166
2804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945
*sqrt(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510
*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(
a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108
*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 1662804
0*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*
(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^9)/a
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \int x^2\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

input
```
int(x^2*(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^2*(a*x + b*x^(2/3))^(1/2), x)
```

3.169 $\int x\sqrt{bx^{2/3}+ax}\,dx$

3.169.1 Optimal result

Integrand size = 17, antiderivative size = 195

$$\int x\sqrt{bx^{2/3}+ax}\,dx=-\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}$$

output
```
-128/429*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4+2048/15015*b^6*(b*x^(2/3)+a*x)^(3/2
)/a^7/x-1024/5005*b^5*(b*x^(2/3)+a*x)^(3/2)/a^6/x^(2/3)+256/1001*b^4*(b*x^
(2/3)+a*x)^(3/2)/a^5/x^(1/3)+48/143*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^3-
24/65*b*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/5*x*(b*x^(2/3)+a*x)^(3/2)/a
```

3.169.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.49

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(1024b^6-1536ab^5\sqrt[3]{x}+1920a^2b^4x^{2/3}-2240a^3b^3x+2520a^4b^2x^{4/3}\right.}{15015a^7x}$$

input
```
Integrate[x*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2)*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1920*a^2*b^4*x
^(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^(5/3) + 3003
*a^6*x^2))/(15015*a^7*x)
```

3.169.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1922, 1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax+bx^{2/3}}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\int x^{2/3}\sqrt{x^{2/3}b+ax}dx}{5a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\int \sqrt[3]{x}\sqrt{x^{2/3}b+ax}dx}{13a}\right)}{5a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\int \sqrt{x^{2/3}b+ax}dx}{11a}\right)}{13a}\right)}{5a}$$

$$\Big\downarrow \text{1908}$$

$$\frac{\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\int\frac{\sqrt{x^{2/3}b+ax}}{\sqrt[3]{x}}dx}{3a}\right)}{11a}\right)}{13a}\right)}{5a}}{}$$

$\downarrow$ 1922

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\int\frac{\sqrt{x^{2/3}b+ax}}{x^{2/3}}dx}{7a}\right)}{3a}\right)}{11a}\right)}{13a}\right)}{5a}$$

$\downarrow$ 1922

3.169. $\int x\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a} - \frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a} - \frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a} - \frac{8b\left(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a} - \frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}} - \frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}} - \frac{2b\int\frac{\sqrt{x^{2/3}b+ax}}{x}\,dx}{5a}\right)}{7a}\right)}{3a}\right)}{11a}\right)}{13a}\right)}{5a}$$

$\downarrow$ 1920

3.169. $\int x\sqrt{bx^{2/3}+ax}\,dx$

$$\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}-\frac{4b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{13a}-\frac{10b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{11a}-\frac{8b\left(\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{5a^2x}\right)}{7a}\right)}{3a}\right.}{11a}\right.}{13a}\right.}{5a}$$

input
```
Int[x*Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a) - (4*b*((6*x^(2/3)*(b*x^(2/3) + a*x)^(
3/2))/(13*a) - (10*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(11*a) - (8*b*((
2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(7*a*
x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2/3)
+ a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)))/(11*a)))/(13*a)))/(5*a)
```

3.169. $\int x\sqrt{bx^{2/3}+ax}\,dx$

3.169.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.169.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.46

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{6}x^{2}-2772a^{5}b\,x^{\frac{5}{3}}+2520a^{4}b^{2}x^{\frac{4}{3}}-2240a^{3}b^{3}x+1920a^{2}x^{\frac{2}{3}}b^{4}-1536a\,b^{5}x^{\frac{1}{3}}+1024b^{6}\right)}{15015x^{\frac{1}{3}}a^{7}}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(2772a^{5}b\,x^{\frac{5}{3}}-2520a^{4}b^{2}x^{\frac{4}{3}}-1920a^{2}x^{\frac{2}{3}}b^{4}-3003a^{6}x^{2}+1536a\,b^{5}x^{\frac{1}{3}}+2240a^{3}b^{3}x-1024b^{6}\right)}{15015x^{\frac{1}{3}}a^{7}}$

input
```
int(x*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15015*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(3003*a^6*x^2-2772*a^5*b*x^(5/
3)+2520*a^4*b^2*x^(4/3)-2240*a^3*b^3*x+1920*a^2*x^(2/3)*b^4-1536*a*b^5*x^(
1/3)+1024*b^6)/x^(1/3)/a^7
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(145) = 290$.

Time = 147.95 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.93

$$\int x\sqrt{bx^{2/3}+ax}\,dx =$$

$$-\frac{(51539607552\,b^{13}+10737418240\,b^{12}+50331648\,(64\,a^3-3)b^{10}-1006632960\,b^{11}-262144\,(11264\,a^3-53}{}$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/30030*((51539607552*b^13 + 10737418240*b^12 + 50331648*(64*a^3 - 3)*b^1
0 - 1006632960*b^11 - 262144*(11264*a^3 - 53)*b^9 - 69957888*a^9 - 98304*(
5504*a^3 + 1)*b^8 + 3072*(3194880*a^6 - 114688*a^3 - 3)*b^7 + 114688*(1881
6*a^6 + 103*a^3)*b^6 - 12288*(48816*a^6 + 23*a^3)*b^5 + 192*(302776320*a^9
 + 495872*a^6 + 15*a^3)*b^4 + 1792*(16588800*a^9 - 3439*a^6)*b^3 + 26208*(
163840*a^9 + 7*a^6)*b^2 + 693*(1024000*a^9 - 3*a^6)*b)*x - 4*(3003*(167772
16*a^7*b^6 + 6291456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b
^3 - 2304*a^7*b^2 + 864*a^7*b - 27*a^7)*x^3 + 280*(16777216*a^4*b^9 + 6291
456*a^4*b^8 + 196608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4
 - (262144*a^7 + 27*a^4)*b^3)*x^2 - 512*(16777216*a*b^12 + 6291456*a*b^11
+ 196608*a*b^10 - 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27
*a)*b^6)*x + (17179869184*b^13 + 6442450944*b^12 + 201326592*b^11 - 117440
512*b^10 - 2359296*b^9 - 1024*(262144*a^3 + 27)*b^7 + 884736*b^8 + 231*(16
777216*a^6*b^7 + 6291456*a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*
a^6*b^3 + 864*a^6*b^2 - (262144*a^9 + 27*a^6)*b)*x^2 - 320*(16777216*a^3*b
^10 + 6291456*a^3*b^9 + 196608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 8
64*a^3*b^5 - (262144*a^6 + 27*a^3)*b^4)*x)*x^(2/3) - 12*(21*(16777216*a^5*
b^8 + 6291456*a^5*b^7 + 196608*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 8
64*a^5*b^3 - (262144*a^8 + 27*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291
456*a^2*b^10 + 196608*a^2*b^9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2...
```

3.169.6 Sympy [F]

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\int x\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

input
```
integrate(x*(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(a*x + b*x**(2/3)), x)
```

3.169.7 Maxima [F]

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\int \sqrt{ax+bx^{\frac{2}{3}}}x\,dx$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))*x, x)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.17

$$\int x\sqrt{bx^{2/3}+ax}\,dx=-\frac{2048\,b^{\frac{15}{2}}}{15015\,a^7}+\frac{2\left(\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^2-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^3+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^4-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^5+3003\sqrt{ax^{\frac{1}{3}}-\cdots}}{a^6}\cdots}{\cdots}$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-2048/15015*b^(15/2)/a^7 + 2/15015*(15*(231*(a*x^(1/3) + b)^(13/2) - 1638*
(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3
) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(
3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) + b)^(1
5/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 -
25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*
(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*
x^(1/3) + b)*b^7)/a^6)/a
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\int x\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

input
```
int(x*(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x*(a*x + b*x^(2/3))^(1/2), x)
```

3.170 $\int \sqrt{bx^{2/3}+ax}\,dx$

3.170.1 Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}$$

output
```
2/3*(b*x^(2/3)+a*x)^(3/2)/a-32/105*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4/x+16/35*b
^2*(b*x^(2/3)+a*x)^(3/2)/a^3/x^(2/3)-4/7*b*(b*x^(2/3)+a*x)^(3/2)/a^2/x^(1/
3)
```

3.170.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{2\sqrt{bx^{2/3}+ax}\left(-16b^4+8ab^3\sqrt[3]{x}-6a^2b^2x^{2/3}+5a^3bx+35a^4x^{4/3}\right)}{105a^4\sqrt[3]{x}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(-16*b^4 + 8*a*b^3*x^(1/3) - 6*a^2*b^2*x^(2/3) +
5*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^4*x^(1/3))
```

3.170.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+bx^{2/3}}dx$$

$$\Big\downarrow \ 1908$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\int \frac{\sqrt{x^{2/3}b+ax}}{\sqrt[3]{x}}dx}{3a}$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\int \frac{\sqrt{x^{2/3}b+ax}}{x^{2/3}}dx}{7a}\right)}{3a}$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}}-\frac{2b\int \frac{\sqrt{x^{2/3}b+ax}}{x}dx}{5a}\right)}{7a}\right)}{3a}$$

$$\Big\downarrow \ 1920$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{7a\sqrt[3]{x}}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{3/2}}{5ax^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{5a^2x}\right)}{7a}\right)}{3a}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (2*b*((6*(b*x^(2/3) + a*x)^(3/2))/(7*a
*x^(1/3)) - (4*b*((-4*b*(b*x^(2/3) + a*x)^(3/2))/(5*a^2*x) + (6*(b*x^(2/3)
 + a*x)^(3/2))/(5*a*x^(2/3))))/(7*a)))/(3*a)
```

3.170.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.170.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^{3}x-30a^{2}b\,x^{\frac{2}{3}}+24a\,b^{2}x^{\frac{1}{3}}-16b^{3}\right)}{105x^{\frac{1}{3}}a^{4}}$	57
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(30a^{2}b\,x^{\frac{2}{3}}-24a\,b^{2}x^{\frac{1}{3}}-35a^{3}x+16b^{3}\right)}{105x^{\frac{1}{3}}a^{4}}$	57

input
```
int((b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(35*a^3*x-30*a^2*b*x^(2/3)+24*a*
b^2*x^(1/3)-16*b^3)/x^(1/3)/a^4
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(81) = 162$.

Time = 162.93 (sec) , antiderivative size = 501, normalized size of antiderivative = 4.60

$$\int \sqrt{bx^{2/3} + ax}\, dx = \frac{(805306368\, b^{10} + 167772160\, b^{9} + 786432\, (64\, a^{3} - 3)b^{7} - 15728640\, b^{8} - 4096\, (11264\, a^{3}}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/210*((805306368*b^10 + 167772160*b^9 + 786432*(64*a^3 - 3)*b^7 - 1572864
0*b^8 - 4096*(11264*a^3 - 53)*b^6 + 815360*a^6 - 1536*(5504*a^3 + 1)*b^5 -
 48*(15728640*a^6 + 114688*a^3 + 3)*b^4 - 1792*(221184*a^6 - 103*a^3)*b^3
- 192*(307200*a^6 + 23*a^3)*b^2 - 15*(499712*a^6 - 3*a^3)*b)*x + 4*(35*(16
777216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^
4*b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 - 6*(16777216*a^2*b^8 + 629
1456*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^
3 - (262144*a^5 + 27*a^2)*b^2)*x^(4/3) + 8*(16777216*a*b^9 + 6291456*a*b^8
 + 196608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27
*a)*b^3)*x - (268435456*b^10 + 100663296*b^9 + 3145728*b^8 - 1835008*b^7 -
 36864*b^6 - 16*(262144*a^3 + 27)*b^4 + 13824*b^5 - 5*(16777216*a^3*b^7 +
6291456*a^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3
*b^2 - (262144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777
216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*b
^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x)
```

3.170.6 Sympy [F]

$$\int \sqrt{bx^{2/3} + ax}\, dx = \int \sqrt{ax + bx^{\frac{2}{3}}}\, dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3)), x)
```

3.170.7 Maxima [F]

$$\int \sqrt{bx^{2/3}+ax}\,dx=\int \sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3)), x)
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{32\,b^{\frac{9}{2}}}{105\,a^4}+\frac{2\left(\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}b^3\right)b}{a^3}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2-420\left(ax^{\frac{1}{3}}\right.}{a^3}}{105\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
32/105*b^(9/2)/a^4 + 2/105*(9*(5*(a*x^(1/3) + b)^(7/2) - 21*(a*x^(1/3) + b
)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3) + b)*b^3)*b/a
^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/2)*b + 378*(a*x^(1
/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*sqrt(a*x^(1/3) +
b)*b^4)/a^3)/a
```

3.170.9 Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{3\,x\,\sqrt{a\,x+b\,x^{2/3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},4;\ 5;\ -\frac{a\,x^{1/3}}{b}\right)}{4\,\sqrt{\frac{a\,x^{1/3}}{b}+1}}$$

input
```
int((a*x + b*x^(2/3))^(1/2),x)
```

output
```
(3*x*(a*x + b*x^(2/3))^(1/2)*hypergeom([-1/2, 4], 5, -(a*x^(1/3))/b))/(4*(
(a*x^(1/3))/b + 1)^(1/2))
```

3.170. $\int \sqrt{bx^{2/3} + ax}\, dx$

3.171 $\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx$

3.171.1 Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

output
```
2*(b*x^(2/3)+a*x)^(3/2)/a/x
```

3.171.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

3.171.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x}\,dx$$

$$\Big\downarrow \; 1920$$

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

3.171.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.171.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}\,a}$	27
default	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}\,a}$	27

input
```
int((b*x^(2/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)*(b+a*x^(1/3))/a
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(19) = 38$.

Time = 170.84 (sec) , antiderivative size = 224, normalized size of antiderivative = 9.74

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = -\frac{(50331648\,b^7 + 10485760\,b^6 + 49152\,(1024\,a^3-3)b^4 - 983040\,b^5 + 256\,(73728\,a^3+53)b^3 - 23296\,a^3 + 96}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
-1/2*((50331648*b^7 + 10485760*b^6 + 49152*(1024*a^3 - 3)*b^4 - 983040*b^5
 + 256*(73728*a^3 + 53)*b^3 - 23296*a^3 + 96*(16384*a^3 - 1)*b^2 - 9*(8192
*a^3 + 1)*b)*x - 4*((16777216*a*b^6 + 6291456*a*b^5 + 196608*a*b^4 - 26214
4*a^4 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x + (16777216*b^7 + 62
91456*b^6 + 196608*b^5 - 114688*b^4 - 2304*b^3 - (262144*a^3 + 27)*b + 864
*b^2)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a*b^6 + 6291456*a*b^5 + 1
96608*a*b^4 - 262144*a^4 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x)
```

3.171.6 Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x, x)
```

3.171.7 Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx=\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x, x)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx=\frac{2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}}{a}-\frac{2\,b^{\frac{3}{2}}}{a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx=\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x, x)
```

3.172 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$

3.172.1 Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{2x} - \frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}} + \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

output
```
3/4*a^2*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3/2*(b*x^(2
/3)+a*x)^(1/2)/x-3/4*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(2/3)
```

3.172.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = -\frac{3\left(2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{4bx} + \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

output
```
(-3*(2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(4*b*x) + (3*a^2*ArcTanh[(Sqr
t[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

3.172.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1926, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^2}dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{1}{4}a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{1}{4}a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{1}{4}a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}a\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(2*x) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/
3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/4
```

3.172.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.172.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{3\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{2} x^{\frac{2}{3}}+\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}\right)}{4x\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}}$	79
default	$\frac{3\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{2} x^{\frac{2}{3}}-\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{3}{2}}-\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}\right)}{4x\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}}$	80

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4*(b*x^(2/3)+a*x)^(1/2)*((b+a*x^(1/3))^(3/2)*b^(3/2)-arctanh((b+a*x^(1/
3))^(1/2)/b^(1/2))*b*a^2*x^(2/3)+(b+a*x^(1/3))^(1/2)*b^(5/2))/x/(b+a*x^(1/
3))^(1/2)/b^(5/2)
```

3.172.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.172.6 Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**2, x)
```

3.172.7 Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^2, x)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = -\frac{3\left(\frac{a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3+\sqrt{ax^{\frac{1}{3}}+b}a^3b}{a^2bx^{\frac{2}{3}}}\right)}{4\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3)
+ b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^2}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^2,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^2, x)
```

3.173 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

3.173.1 Optimal result

Integrand size = 19, antiderivative size = 178

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}+\frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}}-\frac{21a^5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}$$

output
```
-21/128*a^5*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/5*(b*
x^(2/3)+a*x)^(1/2)/x^2-3/40*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/80*a^2*(b*
x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/128
*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

3.173.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx=\frac{\sqrt{bx^{2/3}+ax}\left(-384b^4-48ab^3\sqrt[3]{x}+56a^2b^2x^{2/3}-70a^3bx+105a^4x^{4/3}\right)}{640b^4x^2}-\frac{21a^5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-384*b^4 - 48*a*b^3*x^(1/3) + 56*a^2*b^2*x^(2/3) -
 70*a^3*b*x + 105*a^4*x^(4/3)))/(640*b^4*x^2) - (21*a^5*ArcTanh[(Sqrt[b]*x
^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

3.173.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1926, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^3}dx$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{1}{10}a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{1}{10}a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \; 1931$$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$$\Big\downarrow \; 1935$$

3.173. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

$\downarrow$ 219

3.173. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

$$\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x
^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^
(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*
a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b
)))/(8*b)))/10
```

3.173. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

3.173.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.173.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\left(-105b^{\frac{9}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}+490b^{\frac{11}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}-896b^{\frac{13}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}+105\,\text{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b^{4}a^{5}x^{\frac{5}{3}}+79\right.}{640x^{2}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{17}{2}}}$
default	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\left(-105b^{\frac{9}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}+490b^{\frac{11}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}-896b^{\frac{13}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}+105\,\text{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b^{4}a^{5}x^{\frac{5}{3}}+79\right.}{640x^{2}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{17}{2}}}$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(b*x^(2/3)+a*x)^(1/2)*(-105*b^(9/2)*(b+a*x^(1/3))^(9/2)+490*b^(11/2
)*(b+a*x^(1/3))^(7/2)-896*b^(13/2)*(b+a*x^(1/3))^(5/2)+105*arctanh((b+a*x^
(1/3))^(1/2)/b^(1/2))*b^4*a^5*x^(5/3)+790*b^(15/2)*(b+a*x^(1/3))^(3/2)+105
*b^(17/2)*(b+a*x^(1/3))^(1/2))/x^2/(b+a*x^(1/3))^(1/2)/b^(17/2)
```

3.173.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.173.6 Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**3, x)
```

3.173.7 Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^3, x)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \frac{\frac{105\,a^6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6 - 490\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b + 896\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^2 - 790\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6b^3 - 10}{a^5b^4x^{\frac{5}{3}}}}{640\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/640*(105*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*
(a*x^(1/3) + b)^(9/2)*a^6 - 490*(a*x^(1/3) + b)^(7/2)*a^6*b + 896*(a*x^(1/
3) + b)^(5/2)*a^6*b^2 - 790*(a*x^(1/3) + b)^(3/2)*a^6*b^3 - 105*sqrt(a*x^(
1/3) + b)*a^6*b^4)/(a^5*b^4*x^(5/3)))/a
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^3}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^3,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^3, x)
```

3.174 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

3.174.1 Optimal result

Integrand size = 19, antiderivative size = 266

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}} - \frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}} + \frac{429a^6\sqrt{bx^{2/3}+ax}}{8192b^6x} - \frac{1287a^7\sqrt{bx^{2/3}+ax}}{16384b^7x^{2/3}} + \frac{1287a^8\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}$$

output
```
1287/16384*a^8*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/8
*(b*x^(2/3)+a*x)^(1/2)/x^3-3/112*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/448*
a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143/4480*a^3*(b*x^(2/3)+a*x)^(1/2)/b
^3/x^2+1287/35840*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5/3)-429/10240*a^5*(b*x
^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+429/8192*a^6*(b*x^(2/3)+a*x)^(1/2)/b^6/x-128
7/16384*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

3.174.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-215040b^7-15360ab^6\sqrt[3]{x}+16640a^2b^5x^{2/3}-18304a^3b^4x+20592a^4b^3x\right.}{573440b^7x^3} + \frac{1287a^8\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-215040*b^7 - 15360*a*b^6*x^(1/3) + 16640*a^2*b^5*
x^(2/3) - 18304*a^3*b^4*x + 20592*a^4*b^3*x^(4/3) - 24024*a^5*b^2*x^(5/3)
+ 30030*a^6*b*x^2 - 45045*a^7*x^(7/3)))/(573440*b^7*x^3) + (1287*a^8*ArcTa
nh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(16384*b^(15/2))
```

3.174.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^4}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{16}a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{16}a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$\downarrow$ 1931

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int\frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$\downarrow$ 1931

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$\downarrow$ 1931

3.174. $\int\frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+}}{7bx^{8/}}\right.$$

$$\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

$\downarrow$ 1931

3.174. $\int\frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

$$\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}\right.$$

3.174. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

$\downarrow$ 1931

3.174. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

$$13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+\cdots}}{5bx\cdots}\cdots}{12b}\cdots$$

3.174. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

3.174. $\downarrow$ 1935

3.174. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

$$\Bigg(\quad\Bigg(\quad\Bigg(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\ldots$$

3.174. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

↓ 219

3.174. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

$$\left(\left(\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{}}{}\right.\right.\right.\right.$$

3.174. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x
^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[
b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3
)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3)
 + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*Arc
Tanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(
8*b)))/(10*b)))/(12*b)))/(14*b)))/16
```

3.174.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.174.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.63

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(45045 b^{\frac{15}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{15}{2}}-345345 b^{\frac{17}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{13}{2}}+1150149 b^{\frac{19}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}-2167737 b^{\frac{21}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}\right.}{5}$
default	$\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-45045 b^{\frac{15}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{15}{2}}+345345 b^{\frac{17}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{13}{2}}-1150149 b^{\frac{19}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}+2167737 b^{\frac{21}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}\right.}{57}$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/573440*(b*x^(2/3)+a*x)^(1/2)*(45045*b^(15/2)*(b+a*x^(1/3))^(15/2)-34534
5*b^(17/2)*(b+a*x^(1/3))^(13/2)+1150149*b^(19/2)*(b+a*x^(1/3))^(11/2)-2167
737*b^(21/2)*(b+a*x^(1/3))^(9/2)+2518087*b^(23/2)*(b+a*x^(1/3))^(7/2)-1831
739*b^(25/2)*(b+a*x^(1/3))^(5/2)+801535*b^(27/2)*(b+a*x^(1/3))^(3/2)-45045
*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^8*x^(8/3)+45045*b^(29/2)*(b+a*
x^(1/3))^(1/2))/x^3/(b+a*x^(1/3))^(1/2)/b^(29/2)
```

3.174.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.174.6 Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**4, x)
```

3.174.7 Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^4, x)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx =$$
$$-\frac{\frac{45045\,a^9 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7} + \frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9 - 345345\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^9 b + 1150149\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^9 b^2 - 2167737\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^9 b^3 + 2518087}{a^8 b^7 x^{\frac{8}{3}}}}{573440\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/573440*(45045*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
 (45045*(a*x^(1/3) + b)^(15/2)*a^9 - 345345*(a*x^(1/3) + b)^(13/2)*a^9*b +
 1150149*(a*x^(1/3) + b)^(11/2)*a^9*b^2 - 2167737*(a*x^(1/3) + b)^(9/2)*a^
9*b^3 + 2518087*(a*x^(1/3) + b)^(7/2)*a^9*b^4 - 1831739*(a*x^(1/3) + b)^(5
/2)*a^9*b^5 + 801535*(a*x^(1/3) + b)^(3/2)*a^9*b^6 + 45045*sqrt(a*x^(1/3)
+ b)*a^9*b^7)/(a^8*b^7*x^(8/3)))/a
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^4}\,dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^4,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^4, x)
```

3.175 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

3.175.1 Optimal result

Integrand size = 19, antiderivative size = 354

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}} - \frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}} + \frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2} - \frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}} + \frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}} - \frac{4199a^9\sqrt{bx^{2/3}+ax}}{131072b^9x} + \frac{12597a^{10}\sqrt{bx^{2/3}+ax}}{262144b^{10}x^{2/3}} - \frac{12597a^{11}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}$$

output

```
-12597/262144*a^11*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)
-3/11*(b*x^(2/3)+a*x)^(1/2)/x^4-3/220*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+1
9/1320*a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)-323/21120*a^3*(b*x^(2/3)+a*x
)^(1/2)/b^3/x^3+323/19712*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(8/3)-4199/23654
4*a^5*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+4199/215040*a^6*(b*x^(2/3)+a*x)^(1
/2)/b^6/x^2-12597/573440*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/3)+4199/163840
*a^8*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-4199/131072*a^9*(b*x^(2/3)+a*x)^(1/
2)/b^9/x+12597/262144*a^10*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

3.175.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-82575360b^{10}-4128768ab^9\sqrt[3]{x}+4358144a^2b^8x^{2/3}-4630528a^3b^7x+4\right.}{} - \frac{12597a^{11}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}$$

input
```
Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-82575360*b^10 - 4128768*a*b^9*x^(1/3) + 4358144*a
^2*b^8*x^(2/3) - 4630528*a^3*b^7*x + 4961280*a^4*b^6*x^(4/3) - 5374720*a^5
*b^5*x^(5/3) + 5912192*a^6*b^4*x^2 - 6651216*a^7*b^3*x^(7/3) + 7759752*a^8
*b^2*x^(8/3) - 9699690*a^9*b*x^3 + 14549535*a^10*x^(10/3)))/(302776320*b^1
0*x^4) - (12597*a^11*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(26
2144*b^(21/2))
```

3.175.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^{2/3}}}{x^5}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{22}a\int \frac{1}{x^4\sqrt{x^{2/3}b+ax}}dx - \frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{22}a\left(-\frac{19a\int \frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b} - \frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right) - \frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\int \frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax}}{10b} \ldots\right.$$

$$\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

$\downarrow$ 1931

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$$\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int\frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}\right.$$

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

↓ 1931

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$$\left(\left(19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}\,dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-3\sqrt{}\right)}{18b}\right.\right.\right.$$

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

↓ 1931

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$$\left(\left(\left(17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}\,dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\cdots\right.}{16b}\right.\right.\right.\right.$$

3.175. $\int\frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

3.175. $\displaystyle\int \frac{\sqrt{b x^{2/3}+a x}}{x^5}\,dx$

$$15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\cdots}{12b}\cdots}{14b}\cdots\right.$$

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$\downarrow$ 1931

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$$\left(\quad\left(\quad\left(\quad\left(\quad\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{\cdots}}{4\cdots}\cdots}{10b}\cdots}{12b}\cdots\right.\right.\right.\right.\right.\right.$$

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

↓ 1935

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\cdots}{8b}\cdots}{10b}\cdots$$

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$\downarrow$ 219

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

$$\left(\left(\left(\left(\left(\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}- \cdots}{10b} \cdots$$

3.175. $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

input
```
Int[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(10*b
*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(2/3) + a*x]/(b*x^(10/3)) - (17*a*((-3*S
qrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x
^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[
b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3
)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3)
 + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*Arc
Tanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(
8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b)))/(20*b)))/22
```

3.175.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.175.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.59

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}} b^{\frac{21}{2}}+155195040\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}} b^{\frac{23}{2}}-749786037\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{25}{2}}+2163862272\left(\right.\right.}{}$
default	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}} b^{\frac{21}{2}}+155195040\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}} b^{\frac{23}{2}}-749786037\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{25}{2}}+2163862272\left(\right.\right.}{}$

input
```
int((b*x^(2/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/302776320*(b*x^(2/3)+a*x)^(1/2)*(-14549535*(b+a*x^(1/3))^(21/2)*b^(21/2
)+155195040*(b+a*x^(1/3))^(19/2)*b^(23/2)-749786037*(b+a*x^(1/3))^(17/2)*b
^(25/2)+2163862272*(b+a*x^(1/3))^(15/2)*b^(27/2)-4139920070*(b+a*x^(1/3))^
(13/2)*b^(29/2)+5503713280*(b+a*x^(1/3))^(11/2)*b^(31/2)-5174056250*(b+a*x
^(1/3))^(9/2)*b^(33/2)+3424523520*(b+a*x^(1/3))^(7/2)*b^(35/2)-1551313995*
(b+a*x^(1/3))^(5/2)*b^(37/2)+14549535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))
*b^10*a^11*x^(11/3)+450357600*(b+a*x^(1/3))^(3/2)*b^(39/2)+14549535*(b+a*x
^(1/3))^(1/2)*b^(41/2))/x^4/(b+a*x^(1/3))^(1/2)/b^(41/2)
```

3.175.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

3.175.6 Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx=\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a*x + b*x**(2/3))/x**5, x)
```

3.175.7 Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx=\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^(2/3))/x^5, x)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx=\frac{\frac{14549535\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}-155195040\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b+749786037\left(ax^{\frac{1}{3}}+b\right)}{}}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/302776320*(14549535*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*
b^10) + (14549535*(a*x^(1/3) + b)^(21/2)*a^12 - 155195040*(a*x^(1/3) + b)^
(19/2)*a^12*b + 749786037*(a*x^(1/3) + b)^(17/2)*a^12*b^2 - 2163862272*(a*
x^(1/3) + b)^(15/2)*a^12*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*a^12*b^4
- 5503713280*(a*x^(1/3) + b)^(11/2)*a^12*b^5 + 5174056250*(a*x^(1/3) + b)^
(9/2)*a^12*b^6 - 3424523520*(a*x^(1/3) + b)^(7/2)*a^12*b^7 + 1551313995*(a
*x^(1/3) + b)^(5/2)*a^12*b^8 - 450357600*(a*x^(1/3) + b)^(3/2)*a^12*b^9 -
14549535*sqrt(a*x^(1/3) + b)*a^12*b^10)/(a^11*b^10*x^(11/3)))/a
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3} + ax}}{x^5} \, dx = \int \frac{\sqrt{a\,x + b\,x^{2/3}}}{x^5} \, dx$$

input
```
int((a*x + b*x^(2/3))^(1/2)/x^5,x)
```

output
```
int((a*x + b*x^(2/3))^(1/2)/x^5, x)
```

3.176 $\int x^2 \left(bx^{2/3}+ax\right)^{3/2} \, dx$

3.176.1 Optimal result

Integrand size = 19, antiderivative size = 343

$$\begin{aligned}\int x^2 \left(bx^{2/3}+ax\right)^{3/2} \, dx &= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}} \\ &+\frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} \\ &-\frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}}-\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} \\ &+\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} \\ &+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}\end{aligned}$$

output
```
45056/557175*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7-1048576/152108775*b^11*(b*x^(2/
3)+a*x)^(5/2)/a^12/x^(5/3)+524288/30421755*b^10*(b*x^(2/3)+a*x)^(5/2)/a^11
/x^(4/3)-131072/4345965*b^9*(b*x^(2/3)+a*x)^(5/2)/a^10/x+65536/1448655*b^8
*(b*x^(2/3)+a*x)^(5/2)/a^9/x^(2/3)-90112/1448655*b^7*(b*x^(2/3)+a*x)^(5/2)
/a^8/x^(1/3)-11264/111435*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(5/2)/a^6+5632/45885
*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^5-352/2415*b^3*x*(b*x^(2/3)+a*x)^(5/2
)/a^4+176/1035*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(5/2)/a^3-44/225*b*x^(5/3)*(b*x
^(2/3)+a*x)^(5/2)/a^2+2/9*x^2*(b*x^(2/3)+a*x)^(5/2)/a
```

3.176.2 Mathematica [A] (verified)

Time = 6.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.49

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(-524288b^{11}+1310720ab^{10}\sqrt[3]{x}-2293760a^2b^9x^{2/3}+3440640a^3b\right.}{}$$

input
```
Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-524288*b^11 + 1310720*a*b^10*
x^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^
(4/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^
(7/3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*
x^(10/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x)
```

3.176.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax+bx^{2/3}\right)^{3/2}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\int x^{5/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{27a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\int x^{4/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{5a}\right)}{27a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\int x\left(x^{2/3}b+ax\right)^{3/2}dx}{23a}\right)}{5a}\right)}{27a}$$

$\Big\downarrow$ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\int x^{2/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{21a}\right)}{23a}\right)}{5a}\right)}{27a}$$

$\Big\downarrow$ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\int\sqrt[3]{x}\left(x^{2/3}b+ax\right)^{3/2}dx}{19a}\right)}{21a}\right)}{23a}\right)}{5a}\right)}{27a}$$

$\Big\downarrow$ 1922

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-\frac{22b\left(\frac{6x^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{25a}-\frac{4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\int\left(x^{2}\cdots\right.}{\cdots}\cdots\right.}{19a}\right.}{21a}\right.}{23a}\right.}{5a}\right.}{27a}$$

$\downarrow$ 1908

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a} -$$

$$12b\left(\frac{2\left(\cdots\right.}{\cdots}\right.$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} - \cdots\right.$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a} - \cdots\right.$$

$$18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a} - \frac{\cdots}{21a}\right.$$

$$4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a} - \frac{\cdots}{23a}\right.$$

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$

↓ 1922

3.176. $\int x^2 \left(bx^{2/3} + ax\right)^{3/2} \, dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-$$

$$12b\ \frac{2(}{}$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\right.$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\right.$$

$$18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\right.$$

$$4b\left(\frac{6x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{23a}-\frac{}{23a}\right.$$

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}dx$

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$

↓ 1922

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-$$

$$12b\left(\frac{2(\cdots}{\cdots}\right.$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\right.$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\right.$$

$$18b\left(\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\right.$$

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$

↓ 1922

3.176. $\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\, dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-$$

$$12b\ \frac{2(\ldots$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\ldots\right.$$

$$16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\ldots\right.$$

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$

↓ 1922

3.176. $\int x^2 \left(bx^{2/3} + ax\right)^{3/2} \, dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}-$$

$$12b\left(\frac{2(\ldots}{\ldots}\right.$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\ldots\right.$$

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$\downarrow$ 1920

3.176. $\int x^2 \left(bx^{2/3} + ax\right)^{3/2} \, dx$

$$\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a} -$$

$$12b \quad \frac{2\left(\right.}{}$$

$$14b \quad \frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} -$$

3.176. $\int x^2\left(bx^{2/3}+ax\right)^{3/2}\, dx$

input
```
Int[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a) - (22*b*((6*x^(5/3)*(b*x^(2/3) + a*x
)^(5/2))/(25*a) - (4*b*((6*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(23*a) - (18*b
*((2*x*(b*x^(2/3) + a*x)^(5/2))/(7*a) - (16*b*((6*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(19*a) - (14*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(17*a) - (12*
b*((2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (2*b*((6*(b*x^(2/3) + a*x)^(5/2))/(
13*a*x^(1/3)) - (8*b*((6*(b*x^(2/3) + a*x)^(5/2))/(11*a*x^(2/3)) - (6*b*((
2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2))
/(35*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)))/(1
1*a)))/(13*a)))/(3*a)))/(17*a)))/(19*a)))/(21*a)))/(23*a)))/(5*a)))/(27*a)
```

3.176.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.176.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(16900975a^{11}x^{\frac{11}{3}}-14872858a^{10}b\,x^{\frac{10}{3}}+12932920b^{2}a^{9}x^{3}-11085360a^{8}b^{3}x^{\frac{8}{3}}+9335040a^{7}b^{4}x^{\frac{7}{3}}\right.}{1521087}$
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(16900975a^{11}x^{\frac{11}{3}}-14872858a^{10}b\,x^{\frac{10}{3}}+12932920b^{2}a^{9}x^{3}-11085360a^{8}b^{3}x^{\frac{8}{3}}+9335040a^{7}b^{4}x^{\frac{7}{3}}\right.}{1521087}$

input
```
int(x^2*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/152108775*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(16900975*a^11*x^(11/3)-14
872858*a^10*b*x^(10/3)+12932920*b^2*a^9*x^3-11085360*a^8*b^3*x^(8/3)+93350
40*a^7*b^4*x^(7/3)-7687680*a^6*b^5*x^2+6150144*a^5*b^6*x^(5/3)-4730880*a^4
*b^7*x^(4/3)+3440640*a^3*b^8*x-2293760*a^2*b^9*x^(2/3)+1310720*a*b^10*x^(1
/3)-524288*b^11)/x/a^12
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(255) = 510.

Time = 199.50 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.77

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/152108775*((6597069766656*b^19 + 1374389534720*b^18 + 6442450944*(64*a^3
 - 3)*b^16 - 128849018880*b^17 - 33554432*(11264*a^3 - 53)*b^15 + 98431278
400*a^15 - 12582912*(5504*a^3 + 1)*b^14 + 393216*(3194880*a^6 - 114688*a^3
 - 3)*b^13 + 14680064*(18816*a^6 + 103*a^3)*b^12 - 1572864*(48816*a^6 + 23
*a^3)*b^11 - 24576*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10 - 229376*(134
9120*a^9 + 3439*a^6)*b^9 + 7827456*(5600*a^9 + 3*a^6)*b^8 - 384*(620420562
944*a^12 + 21542400*a^9 + 693*a^6)*b^7 - 6656*(7444688384*a^12 - 89111*a^9
)*b^6 + 19968*(232361024*a^12 - 935*a^9)*b^5 - 1326*(173210075136*a^15 - 5
33564416*a^12 - 165*a^9)*b^4 - 1881152*(45121536*a^15 + 34547*a^12)*b^3 -
352716*(19243008*a^15 - 1339*a^12)*b^2 + 2028117*(237568*a^15 + 21*a^12)*b
)*x + (16900975*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^4 -
262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x^5
- 92378*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*b^7 - 114688*a
^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a^10)*b^3)*x^4
+ 109824*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7*b^10 - 114688*
a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^7)*b^6)*x^3 - 1
43360*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b^13 - 114688*a^4
*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^4)*b^9)*x^2 + 26
2144*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 - 114688*a*b^15 - 2
304*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x - 4*(219902325555...
```

3.176.6 Sympy [F]

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x^2\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2*(a*x + b*x**(2/3))**(3/2), x)
```

3.176.7 Maxima [F]

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx = \int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)
```

3.176.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(255) = 510$.

Time = 0.31 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.24

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
2/16900975*b*(524288*b^(25/2)/a^12 + (25*(88179*(a*x^(1/3) + b)^(23/2) - 1
062347*(a*x^(1/3) + b)^(21/2)*b + 5870865*(a*x^(1/3) + b)^(19/2)*b^2 - 196
84665*(a*x^(1/3) + b)^(17/2)*b^3 + 44618574*(a*x^(1/3) + b)^(15/2)*b^4 - 7
2076158*(a*x^(1/3) + b)^(13/2)*b^5 + 85180914*(a*x^(1/3) + b)^(11/2)*b^6 -
 74364290*(a*x^(1/3) + b)^(9/2)*b^7 + 47805615*(a*x^(1/3) + b)^(7/2)*b^8 -
 22309287*(a*x^(1/3) + b)^(5/2)*b^9 + 7436429*(a*x^(1/3) + b)^(3/2)*b^10 -
 2028117*sqrt(a*x^(1/3) + b)*b^11)*b/a^11 + 3*(676039*(a*x^(1/3) + b)^(25/
2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)*b^
2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17/2
)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)^
(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3)
+ b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3
) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*
x^(1/3) + b)*b^12)/a^11)/a) - 2/152108775*a*(4194304*b^(27/2)/a^13 - (27*(
676039*(a*x^(1/3) + b)^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 5311735
0*(a*x^(1/3) + b)^(21/2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 4921
16625*(a*x^(1/3) + b)^(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 +
1201269300*(a*x^(1/3) + b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*
b^7 + 929553625*(a*x^(1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2
)*b^9 + 223092870*(a*x^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)...
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x^2\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

input
```
int(x^2*(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^2*(a*x + b*x^(2/3))^(3/2), x)
```

3.177 $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

3.177.1 Optimal result

Integrand size = 17, antiderivative size = 255

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}$$

output
```
-256/1615*b^3*(b*x^(2/3)+a*x)^(5/2)/a^4+65536/4849845*b^8*(b*x^(2/3)+a*x)^
(5/2)/a^9/x^(5/3)-32768/969969*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8/x^(4/3)+8192/
138567*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7/x-4096/46189*b^5*(b*x^(2/3)+a*x)^(5/2
)/a^6/x^(2/3)+512/4199*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(1/3)+64/323*b^2*x^
(1/3)*(b*x^(2/3)+a*x)^(5/2)/a^3-32/133*b*x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^2
+2/7*x*(b*x^(2/3)+a*x)^(5/2)/a
```

3.177.2 Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(32768b^8-81920ab^7\sqrt[3]{x}+143360a^2b^6x^{2/3}-215040a^3b^5x+2956\right.}{4849845a^9x}$$

input
```
Integrate[x*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(32768*b^8 - 81920*a*b^7*x^(1/3
) + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 3
84384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835
*a^8*x^(8/3)))/(4849845*a^9*x)
```

3.177.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1922, 1922, 1922, 1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^{2/3}\right)^{3/2}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\int x^{2/3}\left(x^{2/3}b+ax\right)^{3/2}dx}{21a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\int\sqrt[3]{x}\left(x^{2/3}b+ax\right)^{3/2}dx}{19a}\right)}{21a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\int\left(x^{2/3}b+ax\right)^{3/2}dx}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1908

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{\sqrt[3]{x}}dx}{3a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1922

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{2/3}}dx}{13a}\right)}{3a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1922

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x}\cdots}{11a}\cdots\right)}{13a}\cdots\right)}{3a}\cdots\right)}{17a}\cdots\right)}{19a}\cdots\right)}{21a}\cdots$$

$\downarrow$ 1922

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-\frac{16b\left(\frac{6x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{19a}-\frac{14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}-\frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}\cdots\right)}{\cdots}\right)}{13a}\right)}{3a}\right)}{17a}\right)}{19a}\right)}{\cdots}$$

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

↓ 1922

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}-$$

$$6b\ \frac{2\left(ax+bx^{2/3}\right)^{5/}}{3ax}$$

$$8b\ \frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-$$

$$2b\ \frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-$$

$$12b\ \frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}- \qquad 3a$$

$$14b\ \frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a}- \qquad 17a$$

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$\downarrow$ 1920

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a} -$$

$$14b\left(\frac{6\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{17a} - \frac{12b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a} - \frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}} - \frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}} - \frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/}}{3ax}\right.}{}\right.}{}\right.}{3a}\right.}{17a}\right.$$

3.177. $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

input
```
Int[x*(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*x*(b*x^(2/3) + a*x)^(5/2))/(7*a) - (16*b*((6*x^(2/3)*(b*x^(2/3) + a*x)^
(5/2))/(19*a) - (14*b*((6*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(17*a) - (12*b*
((2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (2*b*((6*(b*x^(2/3) + a*x)^(5/2))/(13
*a*x^(1/3)) - (8*b*((6*(b*x^(2/3) + a*x)^(5/2))/(11*a*x^(2/3)) - (6*b*((2*
(b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2))/(
35*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)))/(11*
a)))/(13*a)))/(3*a)))/(17*a)))/(19*a)))/(21*a)
```

3.177.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.177.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.44

method	result
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(692835a^{8}x^{\frac{8}{3}}-583440a^{7}b\,x^{\frac{7}{3}}+480480a^{6}x^{2}b^{2}-384384a^{5}b^{3}x^{\frac{5}{3}}+295680x^{\frac{4}{3}}a^{4}b^{4}-215040a^{3}b^{5}x\right.}{4849845x\,a^{9}}$
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(692835a^{8}x^{\frac{8}{3}}-583440a^{7}b\,x^{\frac{7}{3}}+480480a^{6}x^{2}b^{2}-384384a^{5}b^{3}x^{\frac{5}{3}}+295680x^{\frac{4}{3}}a^{4}b^{4}-215040a^{3}b^{5}x\right.}{4849845x\,a^{9}}$

input
```
int(x*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/4849845*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(692835*a^8*x^(8/3)-583440*a
^7*b*x^(7/3)+480480*a^6*x^2*b^2-384384*a^5*b^3*x^(5/3)+295680*x^(4/3)*a^4*
b^4-215040*a^3*b^5*x+143360*a^2*b^6*x^(2/3)-81920*x^(1/3)*a*b^7+32768*b^8)
/x/a^9
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(189) = 378.

Time = 144.93 (sec) , antiderivative size = 1031, normalized size of antiderivative = 4.04

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/4849845*((824633720832*b^16 + 171798691840*b^15 + 805306368*(64*a^3 - 3
)*b^13 - 16106127360*b^14 - 4194304*(11264*a^3 - 53)*b^12 - 8070142080*a^1
2 - 1572864*(5504*a^3 + 1)*b^11 + 49152*(3194880*a^6 - 114688*a^3 - 3)*b^1
0 + 1835008*(18816*a^6 + 103*a^3)*b^9 - 196608*(48816*a^6 + 23*a^3)*b^8 +
3072*(6575923200*a^9 + 495872*a^6 + 15*a^3)*b^7 + 28672*(146455680*a^9 - 3
439*a^6)*b^6 - 419328*(934400*a^9 - 7*a^6)*b^5 + 1584*(12166103040*a^12 -
38275840*a^9 - 21*a^6)*b^4 + 164736*(43008000*a^12 + 33737*a^9)*b^3 + 5148
0*(10838016*a^12 - 799*a^9)*b^2 - 109395*(401408*a^12 + 33*a^9)*b)*x - 2*(
692835*(16777216*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^
13 - 114688*a^10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 - 6864*(1
6777216*a^7*b^9 + 6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304
*a^7*b^5 + 864*a^7*b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 + 8960*(16777216*
a^4*b^12 + 6291456*a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*
b^8 + 864*a^4*b^7 - (262144*a^7 + 27*a^4)*b^6)*x^2 - 16384*(16777216*a*b^1
5 + 6291456*a*b^14 + 196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b
^10 - (262144*a^4 + 27*a)*b^9)*x + 2*(274877906944*b^16 + 103079215104*b^1
5 + 3221225472*b^14 - 1879048192*b^13 - 37748736*b^12 - 16384*(262144*a^3
+ 27)*b^10 + 14155776*b^11 + 401115*(16777216*a^9*b^7 + 6291456*a^9*b^6 +
196608*a^9*b^5 - 114688*a^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^1
2 + 27*a^9)*b)*x^3 + 3696*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608...
```

3.177.6 Sympy [F]

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x*(a*x + b*x**(2/3))**(3/2), x)
```

3.177.7 Maxima [F]

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)*x, x)
```

3.177.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(189) = 378$.

Time = 0.32 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.36

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=$$

$$-\frac{2}{692835}\,b\left(\frac{32768\,b^{\frac{19}{2}}}{a^9}-\frac{\frac{19\left(6435\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}-58344\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b+235620\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^2-556920\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^3+850850\left(ax^{\frac{1}{3}}+b\right)}{a^8}}{}\right.$$

$$+\frac{2}{1616615}\,a\left(\frac{65536\,b^{\frac{21}{2}}}{a^{10}}+\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}-122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b+554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^2-1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^3+2645370\left(ax\right.}{}\right.$$

input
```
integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2/692835*b*(32768*b^(19/2)/a^9 - (19*(6435*(a*x^(1/3) + b)^(17/2) - 58344
*(a*x^(1/3) + b)^(15/2)*b + 235620*(a*x^(1/3) + b)^(13/2)*b^2 - 556920*(a*
x^(1/3) + b)^(11/2)*b^3 + 850850*(a*x^(1/3) + b)^(9/2)*b^4 - 875160*(a*x^(
1/3) + b)^(7/2)*b^5 + 612612*(a*x^(1/3) + b)^(5/2)*b^6 - 291720*(a*x^(1/3)
 + b)^(3/2)*b^7 + 109395*sqrt(a*x^(1/3) + b)*b^8)*b/a^8 + 9*(12155*(a*x^(1
/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)
^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)
^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^
(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/
2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)/a^8)/a) + 2/1616615*a*(65536*b^(2
1/2)/a^10 + (21*(12155*(a*x^(1/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17
/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)
*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*
b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^
7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)*b/a
^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510*(a*x^(1/3) + b)^(19/2)*b + 2
567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(a*x^(1/3) + b)^(15/2)*b^3 + 1
5668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108*(a*x^(1/3) + b)^(11/2)*b^5 +
 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 16628040*(a*x^(1/3) + b)^(7/2)*b^7 +
 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(a*x^(1/3) + b)^(3/2)*b^9 ...
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}\,dx$$

input
```
int(x*(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x*(a*x + b*x^(2/3))^(3/2), x)
```

3.178 $\int \left(bx^{2/3}+ax\right)^{3/2} dx$

3.178.1 Optimal result

Integrand size = 15, antiderivative size = 169

$$\int \left(bx^{2/3}+ax\right)^{3/2} dx=\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}$$

output
```
2/5*(b*x^(2/3)+a*x)^(5/2)/a-512/15015*b^5*(b*x^(2/3)+a*x)^(5/2)/a^6/x^(5/3
)+256/3003*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(4/3)-64/429*b^3*(b*x^(2/3)+a*x
)^(5/2)/a^4/x+32/143*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(2/3)-4/13*b*(b*x^(2/
3)+a*x)^(5/2)/a^2/x^(1/3)
```

3.178.2 Mathematica [A] (verified)

Time = 6.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int \left(bx^{2/3}+ax\right)^{3/2} dx=\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(-256b^5+640ab^4\sqrt[3]{x}-1120a^2b^3x^{2/3}+1680a^3b^2x-2310a^4bx^{4/3}\right.}{15015a^6x}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-256*b^5 + 640*a*b^4*x^(1/3) -
 1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(
5/3)))/(15015*a^6*x)
```

3.178.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^{2/3}\right)^{3/2}dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{\sqrt[3]{x}}dx}{3a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{2/3}}dx}{13a}\right)}{3a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{x}dx}{11a}\right)}{13a}\right)}{3a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{4/3}}dx}{9a}\right)}{11a}\right)}{13a}\right)}{3a}$$

$\downarrow$ 1922

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{2b\int\frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{5/3}}dx}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{3a}$$

$\downarrow$ 1920

3.178. $\int\left(bx^{2/3}+ax\right)^{3/2}\,dx$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}-\frac{2b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{13a\sqrt[3]{x}}-\frac{8b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{11ax^{2/3}}-\frac{6b\left(\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{12b\left(ax+bx^{2/3}\right)^{5/2}}{35a^2x^{5/3}}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{3a}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (2*b*((6*(b*x^(2/3) + a*x)^(5/2))/(13*
a*x^(1/3)) - (8*b*((6*(b*x^(2/3) + a*x)^(5/2))/(11*a*x^(2/3)) - (6*b*((2*(
b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2))/(3
5*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)))/(11*a
)))/(13*a)))/(3*a)
```

3.178.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.178.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.47

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{5}x^{\frac{5}{3}}-2310a^{4}b\,x^{\frac{4}{3}}+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640a\,b^{4}x^{\frac{1}{3}}-256b^{5}\right)}{15015x\,a^{6}}$	79
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{5}x^{\frac{5}{3}}-2310a^{4}b\,x^{\frac{4}{3}}+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640a\,b^{4}x^{\frac{1}{3}}-256b^{5}\right)}{15015x\,a^{6}}$	79

input
```
int((b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15015*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(3003*a^5*x^(5/3)-2310*a^4*b*x
^(4/3)+1680*a^3*b^2*x-1120*a^2*b^3*x^(2/3)+640*a*b^4*x^(1/3)-256*b^5)/x/a^
6
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(125) = 250.

Time = 153.11 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.54

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx = \frac{2\left(4\left(805306368\,b^{13}+167772160\,b^{12}+786432\left(64\,a^{3}-3\right)b^{10}-15728640\,b^{11}-4096\left(11264\,a^{3}\right.\right.\right.}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(4*(805306368*b^13 + 167772160*b^12 + 786432*(64*a^3 - 3)*b^10 - 1
5728640*b^11 - 4096*(11264*a^3 - 53)*b^9 + 4372368*a^9 - 1536*(5504*a^3 +
1)*b^8 - 48*(242810880*a^6 + 114688*a^3 + 3)*b^7 - 1792*(1353984*a^6 - 103
*a^3)*b^6 + 192*(1152384*a^6 - 23*a^3)*b^5 - 3*(3633315840*a^9 - 12027392*
a^6 - 15*a^3)*b^4 - 112*(35389440*a^9 + 29281*a^6)*b^3 - 819*(368640*a^9 -
 31*a^6)*b^2 + 693*(40960*a^9 + 3*a^6)*b)*x + (3003*(16777216*a^7*b^6 + 62
91456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2304*a^7*b
^2 + 864*a^7*b - 27*a^7)*x^3 - 70*(16777216*a^4*b^9 + 6291456*a^4*b^8 + 19
6608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (262144*a^7 +
 27*a^4)*b^3)*x^2 + 128*(16777216*a*b^12 + 6291456*a*b^11 + 196608*a*b^10
- 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b^6)*x - 16*
(268435456*b^13 + 100663296*b^12 + 3145728*b^11 - 1835008*b^10 - 36864*b^9
 - 16*(262144*a^3 + 27)*b^7 + 13824*b^8 - 231*(16777216*a^6*b^7 + 6291456*
a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6*b^3 + 864*a^6*b^2 - (
262144*a^9 + 27*a^6)*b)*x^2 - 5*(16777216*a^3*b^10 + 6291456*a^3*b^9 + 196
608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 864*a^3*b^5 - (262144*a^6 +
27*a^3)*b^4)*x)*x^(2/3) + 3*(21*(16777216*a^5*b^8 + 6291456*a^5*b^7 + 1966
08*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 864*a^5*b^3 - (262144*a^8 + 2
7*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291456*a^2*b^10 + 196608*a^2*b^
9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2*b^6 - (262144*a^5 + 27*a^2)...
```

3.178.6 Sympy [F]

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2), x)
```

3.178.7 Maxima [F]

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2), x)
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(125) = 250$.

Time = 0.31 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.57

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2}{3003}\,b\left(\frac{256\,b^{\frac{13}{2}}}{a^6}+\frac{13\left(63\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b+990\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^2-1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^3+1155\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^4-\cdots}{a^5}\cdots\right.$$
$$\left.-\frac{2}{15015}\,a\left(\frac{1024\,b^{\frac{15}{2}}}{a^7}-\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^2-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^3+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^4-6006\left(a\cdots\right.\right.}{a^6}\cdots\right.$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
2/3003*b*(256*b^(13/2)/a^6 + (13*(63*(a*x^(1/3) + b)^(11/2) - 385*(a*x^(1/
3) + b)^(9/2)*b + 990*(a*x^(1/3) + b)^(7/2)*b^2 - 1386*(a*x^(1/3) + b)^(5/
2)*b^3 + 1155*(a*x^(1/3) + b)^(3/2)*b^4 - 693*sqrt(a*x^(1/3) + b)*b^5)*b/a
^5 + 3*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*
(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/
3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) +
 b)*b^6)/a^5)/a) - 2/15015*a*(1024*b^(15/2)/a^7 - (15*(231*(a*x^(1/3) + b)
^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 -
 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a
*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*
x^(1/3) + b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b
)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/
2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6
 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^6)/a)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.24

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{x\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},6;\ 7;\ -\frac{a\,x^{1/3}}{b}\right)}{2\,\left(\frac{a\,x^{1/3}}{b}+1\right)^{3/2}}$$

input
```
int((a*x + b*x^(2/3))^(3/2),x)
```

output
```
(x*(a*x + b*x^(2/3))^(3/2)*hypergeom([-3/2, 6], 7, -(a*x^(1/3))/b))/(2*((a
*x^(1/3))/b + 1)^(3/2))
```

3.179 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx$

3.179.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}$$

output
```
16/105*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(5/3)-8/21*b*(b*x^(2/3)+a*x)^(5/2)/
a^2/x^(4/3)+2/3*(b*x^(2/3)+a*x)^(5/2)/a/x
```

3.179.2 Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\frac{2\left(b+a\sqrt[3]{x}\right)\left(8b^2-20ab\sqrt[3]{x}+35a^2x^{2/3}\right)\left(bx^{2/3}+ax\right)^{3/2}}{105a^3x}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

output
```
(2*(b + a*x^(1/3))*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*(b*x^(2/3) +
a*x)^(3/2))/(105*a^3*x)
```

3.179.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x}\,dx$$

$$\Big\downarrow \; 1922$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{4/3}}dx}{9a}$$

$$\Big\downarrow \; 1922$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{2b\int \frac{\left(x^{2/3}b+ax\right)^{3/2}}{x^{5/3}}dx}{7a}\right)}{9a}$$

$$\Big\downarrow \; 1920$$

$$\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}-\frac{4b\left(\frac{6\left(ax+bx^{2/3}\right)^{5/2}}{7ax^{4/3}}-\frac{12b\left(ax+bx^{2/3}\right)^{5/2}}{35a^2x^{5/3}}\right)}{9a}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x) - (4*b*((-12*b*(b*x^(2/3) + a*x)^(5/2)
)/(35*a^2*x^(5/3)) + (6*(b*x^(2/3) + a*x)^(5/2))/(7*a*x^(4/3))))/(9*a)
```

3.179.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.179.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^{2}x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^{2}\right)}{105x\,a^{3}}$	48
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^{2}x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^{2}\right)}{105x\,a^{3}}$	48

input
```
int((b*x^(2/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(35*a^2*x^(2/3)-20*a*b*x^(1/3)+8
*b^2)/x/a^3
```

3.179.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(62) = 124.

Time = 175.32 (sec) , antiderivative size = 501, normalized size of antiderivative = 5.96

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx =$$

$$-\frac{(201326592\,b^{10}+41943040\,b^{9}+196608\,(6784\,a^{3}-3)b^{7}-3932160\,b^{8}+1024\,(257536\,a^{3}+53)b^{6}-407680}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
-1/105*((201326592*b^10 + 41943040*b^9 + 196608*(6784*a^3 - 3)*b^7 - 39321
60*b^8 + 1024*(257536*a^3 + 53)*b^6 - 407680*a^6 - 384*(72704*a^3 + 1)*b^5
 + 12*(94371840*a^6 - 437248*a^3 - 3)*b^4 + 896*(442368*a^6 + 449*a^3)*b^3
 + 24*(1105920*a^6 - 151*a^3)*b^2 - 15*(253952*a^6 + 15*a^3)*b)*x - 2*(35*
(16777216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688
*a^4*b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 3*(16777216*a^2*b^8 +
6291456*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2
*b^3 - (262144*a^5 + 27*a^2)*b^2)*x^(4/3) - 4*(16777216*a*b^9 + 6291456*a*
b^8 + 196608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 +
 27*a)*b^3)*x + 2*(67108864*b^10 + 25165824*b^9 + 786432*b^8 - 458752*b^7
- 9216*b^6 - 4*(262144*a^3 + 27)*b^4 + 3456*b^5 + 25*(16777216*a^3*b^7 + 6
291456*a^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3*
b^2 - (262144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((167772
16*a^3*b^6 + 6291456*a^3*b^5 + 196608*a^3*b^4 - 262144*a^6 - 114688*a^3*b^
3 - 2304*a^3*b^2 + 864*a^3*b - 27*a^3)*x)
```

3.179.6 Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x, x)
```

3.179.7 Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x, x)
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(62) = 124$.

Time = 0.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.15

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx =$$

$$-\frac{2}{35}b\left(\frac{8b^{\frac{7}{2}}}{a^3}-\frac{\frac{7\left(3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}-10\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b+15\sqrt{ax^{\frac{1}{3}}+b}b^2\right)b}{a^2}+\frac{3\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}b\right.}{a^2}}{a}\right.$$

$$+\frac{2}{105}a\left(\frac{16b^{\frac{9}{2}}}{a^4}+\frac{\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}b^3\right)b}{a^3}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+\right.}{a^3}}{a}\right.$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
-2/35*b*(8*b^(7/2)/a^3 - (7*(3*(a*x^(1/3) + b)^(5/2) - 10*(a*x^(1/3) + b)^
(3/2)*b + 15*sqrt(a*x^(1/3) + b)*b^2)*b/a^2 + 3*(5*(a*x^(1/3) + b)^(7/2) -
 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(
1/3) + b)*b^3)/a^2)/a) + 2/105*a*(16*b^(9/2)/a^4 + (9*(5*(a*x^(1/3) + b)^(
7/2) - 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt
(a*x^(1/3) + b)*b^3)*b/a^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) +
b)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3
 + 315*sqrt(a*x^(1/3) + b)*b^4)/a^3)/a)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x, x)
```

3.180 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$

3.180.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)$$

output
```
2*(b*x^(2/3)+a*x)^(3/2)/x-6*b^(3/2)*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x
)^(1/2))+6*b*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)
```

3.180.2 Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\frac{2\sqrt{bx^{2/3}+ax}\left(\sqrt{b+a\sqrt[3]{x}}\left(4b+a\sqrt[3]{x}\right)-3b^{3/2}\text{arctanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)\right)}{\sqrt{b+a\sqrt[3]{x}}\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2
)*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

3.180.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1927, 1927, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$b\int \frac{\sqrt{x^{2/3}b+ax}}{x^{4/3}}dx+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

$$\Big\downarrow \text{1927}$$

$$b\left(b\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx+\frac{6\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}\right)+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

$$\Big\downarrow \text{1935}$$

$$b\left(\frac{6\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}-6b\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}\right)+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{6\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}-6\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)\right)+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

output
```
(2*(b*x^(2/3) + a*x)^(3/2))/x + b*((6*Sqrt[b*x^(2/3) + a*x])/x^(1/3) - 6*S
qrt[b]*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])
```

3.180.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.180.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(-3 b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)+\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}+3 b \sqrt{b+a\,x^{\frac{1}{3}}}\right)}{x \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}}$	67
default	$-\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(3 b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)-\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}-3 b \sqrt{b+a\,x^{\frac{1}{3}}}\right)}{x \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}}$	69

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(2/3)+a*x)^(3/2)*(-3*b^(3/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))+(
b+a*x^(1/3))^(3/2)+3*b*(b+a*x^(1/3))^(1/2))/x/(b+a*x^(1/3))^(3/2)
```

3.180.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.180.6 Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)
```

3.180.7 Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \frac{6\,b^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} + 6\sqrt{ax^{\frac{1}{3}}+b}\,b - \frac{2\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}b^{\frac{3}{2}}\right)}{\sqrt{-b}}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3
/2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt
(-b)*b^(3/2))/sqrt(-b)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^2,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

3.181 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$

3.181.1 Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=-\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}$$

output
```
-(b*x^(2/3)+a*x)^(3/2)/x^2+3/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)
^(1/2))/b^(3/2)-3/4*a*(b*x^(2/3)+a*x)^(1/2)/x-3/8*a^2*(b*x^(2/3)+a*x)^(1/2
)/b/x^(2/3)
```

3.181.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\frac{6a^3\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},4,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^4\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

output
```
(6*a^3*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 4, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^4*x^(1/3))
```

3.181.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1926, 1926, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^2}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{1935}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{2x}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

output
```
-((b*x^(2/3) + a*x)^(3/2)/x^2) + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*x) + (a
*((-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/
Sqrt[b*x^(2/3) + a*x]])/b^(3/2)))/4))/2
```

3.181.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.181.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(3 \,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{3} x-3 b^{\frac{3}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}-8 b^{\frac{5}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}+3 b^{\frac{7}{2}} \sqrt{b+a\,x^{\frac{1}{3}}}\right)}{8 x^{2} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{5}{2}}}$	93
default	$\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(3 \,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{3} x-3 b^{\frac{3}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}-8 b^{\frac{5}{2}} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}+3 b^{\frac{7}{2}} \sqrt{b+a\,x^{\frac{1}{3}}}\right)}{8 x^{2} \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{5}{2}}}$	93

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(b*x^(2/3)+a*x)^(3/2)*(3*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*a^3*x-
3*b^(3/2)*(b+a*x^(1/3))^(5/2)-8*b^(5/2)*(b+a*x^(1/3))^(3/2)+3*b^(7/2)*(b+a
*x^(1/3))^(1/2))/x^2/(b+a*x^(1/3))^(3/2)/b^(5/2)
```

3.181.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.181.6 Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)
```

3.181.7 Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = -\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^4+8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^4b-3\sqrt{ax^{\frac{1}{3}}+b}a^4b^2}{a^3bx}}{8\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/8*(3*a^4*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1
/3) + b)^(5/2)*a^4 + 8*(a*x^(1/3) + b)^(3/2)*a^4*b - 3*sqrt(a*x^(1/3) + b)
*a^4*b^2)/(a^3*b*x))/a
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^3,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

3.182 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

3.182.1 Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}-\frac{21a^6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{9/2}}$$

output
```
-1/2*(b*x^(2/3)+a*x)^(3/2)/x^3-21/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/
3)+a*x)^(1/2))/b^(9/2)-3/20*a*(b*x^(2/3)+a*x)^(1/2)/x^2-3/160*a^2*(b*x^(2/
3)+a*x)^(1/2)/b/x^(5/3)+7/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/256*
a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/
3)
```

3.182.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.30

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{6a^6\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},7,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^7\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

output
```
(-6*a^6*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7,
7/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))
```

3.182.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1926, 1926, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{4}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^3}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}\right)-$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$\downarrow$ 1931

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-3\sqrt{}\right)$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$\downarrow$ 1935

3.182. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right. - \right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

$$\downarrow 219$$

3.182. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

$$\frac{1}{4}a\left(\frac{1}{10}a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)-\cdots\right)$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

output
```
-1/2*(b*x^(2/3) + a*x)^(3/2)/x^3 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2)
+ (a*((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) +
a*x]/(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*S
qrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/10))/4
```

3.182. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

3.182.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.182.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(105\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{9}{2}}-595\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}b^{\frac{11}{2}}+1386\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}b^{\frac{13}{2}}-1686\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}b^{\frac{15}{2}}-105\,\text{arctanh}\left(\right.\right.}{2560x^{3}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$
default	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(105\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{9}{2}}-595\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}b^{\frac{11}{2}}+1386\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}b^{\frac{13}{2}}-1686\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}b^{\frac{15}{2}}-105\,\text{arctanh}\left(\right.\right.}{2560x^{3}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2560*(b*x^(2/3)+a*x)^(3/2)*(105*(b+a*x^(1/3))^(11/2)*b^(9/2)-595*(b+a*x^
(1/3))^(9/2)*b^(11/2)+1386*(b+a*x^(1/3))^(7/2)*b^(13/2)-1686*(b+a*x^(1/3))
^(5/2)*b^(15/2)-105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^4*a^6*x^2-595*(
b+a*x^(1/3))^(3/2)*b^(17/2)+105*(b+a*x^(1/3))^(1/2)*b^(19/2))/x^3/(b+a*x^(
1/3))^(3/2)/b^(17/2)
```

3.182.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.182.6 Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**4, x)
```

3.182.7 Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \frac{\frac{105\,a^7 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^7 - 595\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^7b + 1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^7b^2 - 1686\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a}{a^6b^4x^2}}{2560\,a}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105
*(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^
(1/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(
1/3) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2))/
a
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^4,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

3.183 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

3.183.1 Optimal result

Integrand size = 19, antiderivative size = 291

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} - \frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}} + \frac{143a^7\sqrt{bx^{2/3}+ax}}{16384b^6x} - \frac{429a^8\sqrt{bx^{2/3}+ax}}{32768b^7x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} + \frac{429a^9\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{15/2}}$$

output
```
-1/3*(b*x^(2/3)+a*x)^(3/2)/x^4+429/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^
(2/3)+a*x)^(1/2))/b^(15/2)-1/16*a*(b*x^(2/3)+a*x)^(1/2)/x^3-1/224*a^2*(b*x
^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-
143/26880*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+429/71680*a^5*(b*x^(2/3)+a*x)^
(1/2)/b^4/x^(5/3)-143/20480*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+143/1638
4*a^7*(b*x^(2/3)+a*x)^(1/2)/b^6/x-429/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^7/
x^(2/3)
```

3.183.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = \frac{6a^9\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},10,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{10}\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

output
```
(6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10,
7/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))
```

3.183.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1926, 1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{6}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^4}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^{2}\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{ 1931}$$

3.183. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right.\right.-\ldots$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

3.183. $\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{a\ldots}}{\ldots}\right.\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^{4}}$$

$$\Big\downarrow \text{ 1931}$$

3.183. $\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{5}}\,dx$

$$\frac{1}{6}a\left(\frac{1}{16}a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}\,dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+\cdots}}{2bx\cdots}\cdots}{14b}\cdots\right.\right.$$

3.183. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

$\downarrow$ 1931

3.183. $\int \frac{(bx^{2/3}+ax)^{3/2}}{x^5}\, dx$

$$13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\cdots}{\cdots}\right)}{12b}\cdots\right.$$

3.183. $\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

↓ 1935

3.183. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/}}}{4bx^{5/3}}}{10b}\right.$$

3.183. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

$\downarrow$ 219

3.183. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}\right.$$

3.183. $\int \frac{(bx^{2/3}+ax)^{3/2}}{x^5}\,dx$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

output
```
-1/3*(b*x^(2/3) + a*x)^(3/2)/x^4 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3)
+ (a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3
) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*
((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/
(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b
*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/
3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/1
6))/6
```

3.183.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.183.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(45045 b^{\frac{31}{2}} \sqrt{b+a x^{\frac{1}{3}}}-390390 b^{\frac{29}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{3}{2}}-2633274 b^{\frac{27}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{5}{2}}+4349826 b^{\frac{25}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{7}{2}}-4685\right.$
default	$\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(45045 b^{\frac{31}{2}} \sqrt{b+a x^{\frac{1}{3}}}-390390 b^{\frac{29}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{3}{2}}-2633274 b^{\frac{27}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{5}{2}}+4349826 b^{\frac{25}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{7}{2}}-4685\right.$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3440640*(b*x^(2/3)+a*x)^(3/2)*(45045*b^(31/2)*(b+a*x^(1/3))^(1/2)-390390
*b^(29/2)*(b+a*x^(1/3))^(3/2)-2633274*b^(27/2)*(b+a*x^(1/3))^(5/2)+4349826
*b^(25/2)*(b+a*x^(1/3))^(7/2)-4685824*b^(23/2)*(b+a*x^(1/3))^(9/2)+3317886
*b^(21/2)*(b+a*x^(1/3))^(11/2)-1495494*b^(19/2)*(b+a*x^(1/3))^(13/2)+39039
0*b^(17/2)*(b+a*x^(1/3))^(15/2)-45045*b^(15/2)*(b+a*x^(1/3))^(17/2)+45045*
arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^9*x^3)/x^4/(b+a*x^(1/3))^(3/2)/
b^(29/2)
```

3.183.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

3.183.6 Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)
```

output
```
Integral((a*x + b*x**(2/3))**(3/2)/x**5, x)
```

3.183.7 Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.67

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=$$
$$-\frac{\frac{45045\,a^{10}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{10}-390390\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{10}b+1495494\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{10}b^2-3317886\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{10}b^3+4\cdots}{\cdots}}{3440640\,a\cdots}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-1/3440640*(45045*a^10*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7)
 + (45045*(a*x^(1/3) + b)^(17/2)*a^10 - 390390*(a*x^(1/3) + b)^(15/2)*a^10
*b + 1495494*(a*x^(1/3) + b)^(13/2)*a^10*b^2 - 3317886*(a*x^(1/3) + b)^(11
/2)*a^10*b^3 + 4685824*(a*x^(1/3) + b)^(9/2)*a^10*b^4 - 4349826*(a*x^(1/3)
 + b)^(7/2)*a^10*b^5 + 2633274*(a*x^(1/3) + b)^(5/2)*a^10*b^6 + 390390*(a*
x^(1/3) + b)^(3/2)*a^10*b^7 - 45045*sqrt(a*x^(1/3) + b)*a^10*b^8)/(a^9*b^7
*x^3))/a
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^5}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^5,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

3.184 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

3.184.1 Optimal result

Integrand size = 19, antiderivative size = 379

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}}-\frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}}+\frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2}-\frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}}+\frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}}-\frac{4199a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^9x}+\frac{12597a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{10}x^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{12597a^{12}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{21/2}}$$

output
```
-1/4*(b*x^(2/3)+a*x)^(3/2)/x^5-12597/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/
(b*x^(2/3)+a*x)^(1/2))/b^(21/2)-3/88*a*(b*x^(2/3)+a*x)^(1/2)/x^4-3/1760*a^
2*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/10560*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/
x^(10/3)-323/168960*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/157696*a^5*(b*x^
(2/3)+a*x)^(1/2)/b^4/x^(8/3)-4199/1892352*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^
(7/3)+4199/1720320*a^7*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-12597/4587520*a^8*(b*
x^(2/3)+a*x)^(1/2)/b^7/x^(5/3)+4199/1310720*a^9*(b*x^(2/3)+a*x)^(1/2)/b^8/
x^(4/3)-4199/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^9/x+12597/2097152*a^11*(
b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

3.184.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = -\frac{6a^{12}\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},13,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{13}\sqrt[3]{x}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

output
```
(-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13
, 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))
```

3.184.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {1926, 1926, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{8}a\int \frac{\sqrt{x^{2/3}b+ax}}{x^5}dx-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{8}a\left(\frac{1}{22}a\int \frac{1}{x^4\sqrt{x^{2/3}b+ax}}dx-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\int\frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$\downarrow$ 1931

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\int\frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$\downarrow$ 1931

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\int\frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\right)-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$\downarrow$ 1931

3.184. $\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int\frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)-\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$\downarrow$ 1931

3.184. $\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\cdots}{\cdots}\right.\right.$$

$$\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}$$

$\big\downarrow$ 1931

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$\frac{1}{8}a\left(\frac{1}{22}a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int\frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\sqrt{\;}\cdots}{20b}\cdots\right.\right.$$

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

↓ 1931

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$\Bigg(\Bigg(\Bigg(19a\Bigg(-\frac{17a\Bigg(-\frac{15a\Bigg(-\frac{13a\Bigg(-\frac{11a\bigg(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}\,dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\bigg)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\Bigg)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\cdots\Bigg)}{16b}\cdots}{18b}\cdots$$

3.184. $\int \frac{(bx^{2/3}+ax)^{3/2}}{x^6}\,dx$

↓ 1931

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$\left(\left(\left(\left(17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx}}{2bx^{7/}}}{14b}}{16b}\right.\right.\right.\right.$$

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

↓ 1931

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax}}{5}}{12b}}{14b}\right.$$

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

↓ 1931

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$\left(\left(\left(\left(\left(\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\cdots\right)}{8b}\cdots\right)}{10b}\cdots\right)}{12b}\cdots\right.\right.\right.\right.\right.\right.$$

3.184. $\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$\downarrow$ 1935

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+\cdots}}{bx^{\cdots}}\cdots}{8b}\cdots}{10b}\cdots$$

3.184. $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

↓ 219

3.184. $\int \frac{(bx^{2/3}+ax)^{3/2}}{x^6}\,dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx}}{bx^{4/3}}}{8b}}{10b}\right.$$

3.184. $\int \frac{(bx^{2/3}+ax)^{3/2}}{x^6}\,dx$

input
```
Int[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

output
```
-1/4*(b*x^(2/3) + a*x)^(3/2)/x^5 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4)
 + (a*((-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(
2/3) + a*x]/(b*x^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (
15*a*((-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3
) + a*x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*
((-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/
(b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b
*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/
3) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(
16*b)))/(18*b)))/(20*b)))/22))/8
```

3.184.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.184.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(145495535b^{\frac{21}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{23}{2}}-169744575b^{\frac{23}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}}+904981077b^{\frac{25}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}}-2913648309b^{\frac{27}{2}}\right.}{}$
default	$-\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(-145495535b^{\frac{21}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{23}{2}}+169744575b^{\frac{23}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}}-904981077b^{\frac{25}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}}+291364830\right.}{}$

input
```
int((b*x^(2/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2422210560*(b*x^(2/3)+a*x)^(3/2)*(14549535*b^(21/2)*(b+a*x^(1/3))^(23/2)
-169744575*b^(23/2)*(b+a*x^(1/3))^(21/2)+904981077*b^(25/2)*(b+a*x^(1/3))^
(19/2)-2913648309*b^(27/2)*(b+a*x^(1/3))^(17/2)+6303782342*b^(29/2)*(b+a*x
^(1/3))^(15/2)-9643633350*b^(31/2)*(b+a*x^(1/3))^(13/2)+10677769530*b^(33/
2)*(b+a*x^(1/3))^(11/2)-8598579770*b^(35/2)*(b+a*x^(1/3))^(9/2)+4975837515
*b^(37/2)*(b+a*x^(1/3))^(7/2)-2001671595*b^(39/2)*(b+a*x^(1/3))^(5/2)-1454
9535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*a^12*x^4-169744575*b^(41/2)
*(b+a*x^(1/3))^(3/2)+14549535*b^(43/2)*(b+a*x^(1/3))^(1/2))/x^5/(b+a*x^(1/
3))^(3/2)/b^(41/2)
```

3.184.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
Timed out
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)
```

output
```
Timed out
```

3.184.7 Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.65

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \frac{\frac{14549535\,a^{13}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{13} - 169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{13}b + 904981077\left(ax^{\frac{1}{3}}+\right.}{}$$

input
```
integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)
*b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)
^(21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a
*x^(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4
 - 9643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b
)^(11/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515
*(a*x^(1/3) + b)^(7/2)*a^13*b^8 - 2001671595*(a*x^(1/3) + b)^(5/2)*a^13*b^
9 - 169744575*(a*x^(1/3) + b)^(3/2)*a^13*b^10 + 14549535*sqrt(a*x^(1/3) +
b)*a^13*b^11)/(a^12*b^10*x^4))/a
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^6}\,dx$$

input
```
int((a*x + b*x^(2/3))^(3/2)/x^6,x)
```

output
```
int((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

3.185 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

3.185.1 Optimal result

Integrand size = 19, antiderivative size = 401

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a}$$

output
```
8388608/11700675*b^12*(b*x^(2/3)+a*x)^(1/2)/a^13-16777216/11700675*b^13*(b
*x^(2/3)+a*x)^(1/2)/a^14/x^(1/3)-2097152/3900225*b^11*x^(1/3)*(b*x^(2/3)+a
*x)^(1/2)/a^12+1048576/2340135*b^10*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^11-131
072/334305*b^9*x*(b*x^(2/3)+a*x)^(1/2)/a^10+65536/185725*b^8*x^(4/3)*(b*x^
(2/3)+a*x)^(1/2)/a^9-180224/557175*b^7*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+1
171456/3900225*b^6*x^2*(b*x^(2/3)+a*x)^(1/2)/a^7-73216/260015*b^5*x^(7/3)*
(b*x^(2/3)+a*x)^(1/2)/a^6+36608/137655*b^4*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a
^5-9152/36225*b^3*x^3*(b*x^(2/3)+a*x)^(1/2)/a^4+416/1725*b^2*x^(10/3)*(b*x
^(2/3)+a*x)^(1/2)/a^3-52/225*b*x^(11/3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/9*x^4*
(b*x^(2/3)+a*x)^(1/2)/a
```

3.185.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(-8388608b^{13}+4194304ab^{12}\sqrt[3]{x}-3145728a^2b^{11}x^{2/3}+2621440a^3b^{10}x-\right.}{}$$

input
```
Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728
*a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384
*a^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360
*a^8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 14108
64*a^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700
675*a^14*x^(1/3))
```

3.185.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\int \frac{x^{11/3}}{\sqrt{x^{2/3}b+ax}}dx}{27a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a}-\frac{24b\int \frac{x^{10/3}}{\sqrt{x^{2/3}b+ax}}dx}{25a}\right)}{27a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a}-\frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a}-\frac{22b\int\frac{x^3}{\sqrt{x^{2/3}b+ax}}dx}{23a}\right)}{25a}\right)}{27a}$$

$\downarrow$ 1922

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-\frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a}-\frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a}-\frac{22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\int\frac{x^{8/3}}{\sqrt{x^{2/3}b+ax}}dx}{21a}\right)}{23a}\right)}{25a}\right)}{27a}$$

$\downarrow$ 1922

3.185. $\int\frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} - \frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a} - \frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a} - \frac{22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{18b\int \frac{x^{7/3}}{\sqrt{x^{2/3}b+ax}}dx}{19a}\right)}{21a}\right)}{23a}\right)}{25a}\right)}{27a}$$

$\downarrow$ 1922

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} - \frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a} - \frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a} - \frac{22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \frac{16b\int\frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{17a}\right)}{19a}\right.}{21a}\right.}{23a}\right.}{25a}\right.}{27a}$$

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1922

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} - \frac{26b\left(\frac{6x^{11/3}\sqrt{ax+bx^{2/3}}}{25a} - \frac{24b\left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a} - \frac{22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \cdots\right)}{17\cdots}\right)}{19a}\right)}{21a}\right)}{23a}\right)}{25a}\right)}{\cdots}$$

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$\downarrow$ 1922

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} - \left(24b \left(\frac{6x^{10/3}\sqrt{ax+bx^{2/3}}}{23a} - \frac{\left(22b \left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \frac{\left(20b \left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{\left(18b \left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \frac{\left(16b \left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \cdots \right.\right.}{\cdots} \right.\right.}{\cdots} \right.\right.}{21a} \right.\right.}{23a} \right.\right.$$

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\, dx$

↓ 1922

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} -\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \right.$$

$$22b\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} - \right.$$

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

3.185. $\downarrow$ 1922

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} -\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \right.$$

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1922

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\, dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}-$$

$$\left(\quad\left(\quad\left(\quad\left(\quad\left(\quad\left(\quad 16b\,\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\right.\right.\right.\right.\right.\right.$$

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$\downarrow$ 1922

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} -\right.$$

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1908

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} - \Bigg(\Bigg(\Bigg(\Bigg($$

1

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

3.185.

$\downarrow$ 1920

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a} -$$

1

3.185. $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

input
```
Int[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^4*Sqrt[b*x^(2/3) + a*x])/(9*a) - (26*b*((6*x^(11/3)*Sqrt[b*x^(2/3) +
a*x])/(25*a) - (24*b*((6*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(23*a) - (22*b*((
2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a) - (20*b*((6*x^(8/3)*Sqrt[b*x^(2/3) + a*
x])/(19*a) - (18*b*((6*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(17*a) - (16*b*((2*x
^2*Sqrt[b*x^(2/3) + a*x])/(5*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x])
/(13*a) - (12*b*((6*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*Sq
rt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a)
 - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3)
 + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(
9*a)))/(11*a)))/(13*a)))/(15*a)))/(17*a)))/(19*a)))/(21*a)))/(23*a)))/(25*
a)))/(27*a)
```

3.185.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.185.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.42

method	result
derivativedivides	$2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^{4}+1410864a^{11}b^{2}x^{\frac{11}{3}}-1478048a^{10}b^{3}x^{\frac{10}{3}}+1555840a^{9}b^{4}x^{3}-1647360a^{8}\right.$
default	$2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^{4}+1410864a^{11}b^{2}x^{\frac{11}{3}}-1478048a^{10}b^{3}x^{\frac{10}{3}}+1555840a^{9}b^{4}x^{3}-1647360a^{8}\right.$

input
```
int(x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/11700675*x^(1/3)*(b+a*x^(1/3))*(1300075*a^13*x^(13/3)-1352078*a^12*b*x^4
+1410864*a^11*b^2*x^(11/3)-1478048*a^10*b^3*x^(10/3)+1555840*a^9*b^4*x^3-1
647360*a^8*b^5*x^(8/3)+1757184*a^7*b^6*x^(7/3)-1892352*a^6*b^7*x^2+2064384
*a^5*b^8*x^(5/3)-2293760*a^4*b^9*x^(4/3)+2621440*a^3*b^10*x-3145728*a^2*b^
11*x^(2/3)+4194304*a*b^12*x^(1/3)-8388608*b^13)/(b*x^(2/3)+a*x)^(1/2)/a^14
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. 2(299) = 598.

Time = 148.43 (sec) , antiderivative size = 1294, normalized size of antiderivative = 3.23

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/11700675*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(64
*a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 + 1
5143273600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6 -
114688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*(48
816*a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10
- 7340032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b^8
+ 12288*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(43743616*
a^12 + 89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 3264*(360854
3232*a^15 + 64599808*a^12 + 2145*a^9)*b^4 + 578816*(13049856*a^15 - 27313*
a^12)*b^3 + 217056*(6211584*a^15 + 2353*a^12)*b^2 - 156009*(2547712*a^15 +
 39*a^12)*b)*x + 2*(1300075*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608
*a^13*b^4 - 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 2
7*a^13)*x^5 - 1478048*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*
b^7 - 114688*a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a
^10)*b^3)*x^4 + 1757184*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7
*b^10 - 114688*a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^
7)*b^6)*x^3 - 2293760*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b
^13 - 114688*a^4*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^
4)*b^9)*x^2 + 4194304*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 -
114688*a*b^15 - 2304*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x ...
```

3.185.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(a*x + b*x**(2/3)), x)
```

3.185.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(a*x + b*x^(2/3)), x)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{16777216\,b^{\frac{27}{2}}}{11700675\,a^{14}} + \frac{2\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}} - 18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b + 119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^2 - 478056150\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}\right.}{}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/
2) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*
b^2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(1
9/2)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) +
 b)^(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(
1/3) + b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(
a*x^(1/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 1521087
75*(a*x^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^14
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^4}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input

```
int(x^4/(a*x + b*x^(2/3))^(1/2),x)
```

output

```
int(x^4/(a*x + b*x^(2/3))^(1/2), x)
```

3.186 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

3.186.1 Optimal result

Integrand size = 19, antiderivative size = 313

$$\begin{aligned}\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = &-\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}}+\frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}}\\ &+\frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8}+\frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7}\\ &-\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6}+\frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5}-\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4}\\ &+\frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}-\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}\end{aligned}$$

output
```
-262144/323323*b^9*(b*x^(2/3)+a*x)^(1/2)/a^10+524288/323323*b^10*(b*x^(2/3
)+a*x)^(1/2)/a^11/x^(1/3)+196608/323323*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/
a^9-163840/323323*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+20480/46189*b^6*x*
(b*x^(2/3)+a*x)^(1/2)/a^7-18432/46189*b^5*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^
6+1536/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-768/2261*b^3*x^2*(b*x^(2
/3)+a*x)^(1/2)/a^4+720/2261*b^2*x^(7/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-40/133*b
*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(2/3)+a*x)^(1/2)/a
```

3.186.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(262144b^{10}-131072ab^9\sqrt[3]{x}+98304a^2b^8x^{2/3}-81920a^3b^7x+71680a^4b^6\right.}{}$$

input
```
Integrate[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b
^8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/
3) + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4
8620*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

3.186.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\int \frac{x^{8/3}}{\sqrt{x^{2/3}b+ax}}dx}{21a}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\int \frac{x^{7/3}}{\sqrt{x^{2/3}b+ax}}dx}{19a}\right)}{21a}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\int\frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{17a}\right)}{19a}\right)}{21a}$$

$\Big\downarrow$ 1922

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int\frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\Big\downarrow$ 1922

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int\frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1922

3.186. $\int\frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\int\frac{x}{\sqrt{x^{2/3}b+ax}}dx}{11a}\right)}{13a}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$$

$\downarrow$ 1922

3.186. $\int\frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a} - \frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a} - \frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \frac{8b}{\cdots}\right.}{11a}\right.}{13a}\right.}{15a}\right.}{17a}\right.}{19a}\right.$$

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1922

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$8b$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \frac{\cdots}{13a}\right.$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \frac{\cdots}{15a}\right.$$

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1922

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$8b$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$$

$$16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \right.$$

15

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1908

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}} \, dx$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \right.$$

$$8b$$

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

$\downarrow$ 1920

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a} -$$

$$8b$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$$

3.186. $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

input
```
Int[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a) - (20*b*((6*x^(8/3)*Sqrt[b*x^(2/3) + a
*x])/(19*a) - (18*b*((6*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(17*a) - (16*b*((2*
x^2*Sqrt[b*x^(2/3) + a*x])/(5*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x]
)/(13*a) - (12*b*((6*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*S
qrt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a
) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3
) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/
(9*a)))/(11*a)))/(13*a)))/(15*a)))/(17*a)))/(19*a)))/(21*a)
```

3.186.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.186.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136x^{2}a^{6}b^{4}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}x^{\frac{4}{3}}b\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{11}}$
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136x^{2}a^{6}b^{4}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}x^{\frac{4}{3}}b\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{11}}$

input
```
int(x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/323323*x^(1/3)*(b+a*x^(1/3))*(46189*a^10*x^(10/3)-48620*a^9*b*x^3+51480*
a^8*b^2*x^(8/3)-54912*a^7*b^3*x^(7/3)+59136*x^2*a^6*b^4-64512*a^5*b^5*x^(5
/3)+71680*a^4*x^(4/3)*b^6-81920*a^3*b^7*x+98304*a^2*b^8*x^(2/3)-131072*a*b
^9*x^(1/3)+262144*b^10)/(b*x^(2/3)+a*x)^(1/2)/a^11
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(233) = 466.

Time = 176.92 (sec) , antiderivative size = 1031, normalized size of antiderivative = 3.29

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/323323*((3298534883328*b^16 + 687194767360*b^15 + 3221225472*(64*a^3 -
3)*b^13 - 64424509440*b^14 - 16777216*(11264*a^3 - 53)*b^12 - 269004736*a^
12 - 6291456*(5504*a^3 + 1)*b^11 + 196608*(3194880*a^6 - 114688*a^3 - 3)*b
^10 + 7340032*(18816*a^6 + 103*a^3)*b^9 - 786432*(48816*a^6 + 23*a^3)*b^8
- 12288*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^7 - 114688*(1349120*a^9 + 3
439*a^6)*b^6 + 3913728*(5600*a^9 + 3*a^6)*b^5 - 2112*(101384192*a^12 + 195
8400*a^9 + 63*a^6)*b^4 - 36608*(3784704*a^12 - 8101*a^9)*b^3 - 109824*(226
688*a^12 + 85*a^9)*b^2 + 7293*(974848*a^12 + 15*a^9)*b)*x - (46189*(167772
16*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 114688*a^
10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 - 54912*(16777216*a^7*b
^9 + 6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5 + 86
4*a^7*b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 + 71680*(16777216*a^4*b^12 + 6
291456*a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 864*a^
4*b^7 - (262144*a^7 + 27*a^4)*b^6)*x^2 - 131072*(16777216*a*b^15 + 6291456
*a*b^14 + 196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (2621
44*a^4 + 27*a)*b^9)*x + 4*(1099511627776*b^16 + 412316860416*b^15 + 128849
01888*b^14 - 7516192768*b^13 - 150994944*b^12 - 65536*(262144*a^3 + 27)*b^
10 + 56623104*b^11 - 12155*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*a^
9*b^5 - 114688*a^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*a^
9)*b)*x^3 + 14784*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b^8...
```

3.186.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a*x + b*x**(2/3)), x)
```

3.186.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^3}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a*x + b*x^(2/3)), x)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx=-\frac{524288\,b^{\frac{21}{2}}}{323323\,a^{11}} + \frac{2\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}-510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b+2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^2-7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^3+156\cdots\right.}{\cdots}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 51
0510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 77597
52*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222
1108*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 166
28040*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233
230*(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^3}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(x^3/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^3/(a*x + b*x^(2/3))^(1/2), x)
```

3.187 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

3.187.1 Optimal result

Integrand size = 19, antiderivative size = 225

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a}$$

output
```
2048/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^7-4096/2145*b^7*(b*x^(2/3)+a*x)^(1/2
)/a^8/x^(1/3)-512/715*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+256/429*b^4*x^
(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-224/429*b^3*x*(b*x^(2/3)+a*x)^(1/2)/a^4+33
6/715*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-28/65*b*x^(5/3)*(b*x^(2/3)+a*x
)^(1/2)/a^2+2/5*x^2*(b*x^(2/3)+a*x)^(1/2)/a
```

3.187.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.49

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(-2048b^7 + 1024ab^6\sqrt[3]{x} - 768a^2b^5x^{2/3} + 640a^3b^4x - 560a^4b^3x^{4/3} + 504a\right.}{2145a^8\sqrt[3]{x}}$$

input
```
Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(
2/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6
*b*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))
```

3.187.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int \frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int \frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\int \frac{x}{\sqrt{x^{2/3}b+ax}}dx}{11a}\right)}{13a}\right)}{15a}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int \frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}\right)}{11a}\right)}{13a}\right)}{15a}$$

$\downarrow$ 1922

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int \frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}dx}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{15a}$$

$\downarrow$ 1922

3.187. $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int\frac{1}{\sqrt{x^{2/3}b+ax}}dx}{5a}\right)}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{15a}$$

$\downarrow$ 1908

3.187. $\int\frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a} - \frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a} - \frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a} - \frac{2b\int \frac{}{\sqrt[3]{3}}}{}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{}$$

3.187. $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

$\downarrow$ 1920

3.187. $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

$$\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a} - \frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a} - \frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a} - \frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a} - \frac{4b\sqrt{ax+}}{a^2\sqrt[3]{}}\right.}{5a}\right.}{7a}\right.}{9a}\right.}{11a}\right.}{13a}\right.}{}$$

3.187. $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

input
```
Int[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a
*x])/(13*a) - (12*b*((6*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*
x*Sqrt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(
7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(
2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)
))/(9*a)))/(11*a)))/(13*a)))/(15*a)
```

3.187.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.187.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{7}x^{\frac{7}{3}}-462a^{6}b\,x^{2}+504a^{5}b^{2}x^{\frac{5}{3}}-560a^{4}b^{3}x^{\frac{4}{3}}+640a^{3}b^{4}x-768a^{2}b^{5}x^{\frac{2}{3}}+1024a\,b^{6}x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{8}}$	101
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{7}x^{\frac{7}{3}}-462a^{6}b\,x^{2}+504a^{5}b^{2}x^{\frac{5}{3}}-560a^{4}b^{3}x^{\frac{4}{3}}+640a^{3}b^{4}x-768a^{2}b^{5}x^{\frac{2}{3}}+1024a\,b^{6}x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{8}}$	101

input
```
int(x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/2145*x^(1/3)*(b+a*x^(1/3))*(429*a^7*x^(7/3)-462*a^6*b*x^2+504*a^5*b^2*x^
(5/3)-560*a^4*b^3*x^(4/3)+640*a^3*b^4*x-768*a^2*b^5*x^(2/3)+1024*a*b^6*x^(
1/3)-2048*b^7)/(b*x^(2/3)+a*x)^(1/2)/a^8
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(167) = 334$.

Time = 180.27 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.41

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{(51539607552\,b^{13} + 10737418240\,b^{12} + 50331648\,(64\,a^3-3)b^{10} - 1006632960\,b^{11} - 26}{}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2145*((51539607552*b^13 + 10737418240*b^12 + 50331648*(64*a^3 - 3)*b^10
- 1006632960*b^11 - 262144*(11264*a^3 - 53)*b^9 + 4996992*a^9 - 98304*(550
4*a^3 + 1)*b^8 + 3072*(3194880*a^6 - 114688*a^3 - 3)*b^7 + 114688*(18816*a
^6 + 103*a^3)*b^6 - 12288*(48816*a^6 + 23*a^3)*b^5 + 192*(21626880*a^9 + 4
95872*a^6 + 15*a^3)*b^4 + 256*(10690560*a^9 - 24073*a^6)*b^3 + 3744*(13312
0*a^9 + 49*a^6)*b^2 - 297*(450560*a^9 + 7*a^6)*b)*x + 2*(429*(16777216*a^7
*b^6 + 6291456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2
304*a^7*b^2 + 864*a^7*b - 27*a^7)*x^3 - 560*(16777216*a^4*b^9 + 6291456*a^
4*b^8 + 196608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (26
2144*a^7 + 27*a^4)*b^3)*x^2 + 1024*(16777216*a*b^12 + 6291456*a*b^11 + 196
608*a*b^10 - 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b
^6)*x - 2*(17179869184*b^13 + 6442450944*b^12 + 201326592*b^11 - 117440512
*b^10 - 2359296*b^9 - 1024*(262144*a^3 + 27)*b^7 + 884736*b^8 + 231*(16777
216*a^6*b^7 + 6291456*a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6
*b^3 + 864*a^6*b^2 - (262144*a^9 + 27*a^6)*b)*x^2 - 320*(16777216*a^3*b^10
 + 6291456*a^3*b^9 + 196608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 864*
a^3*b^5 - (262144*a^6 + 27*a^3)*b^4)*x)*x^(2/3) + 24*(21*(16777216*a^5*b^8
 + 6291456*a^5*b^7 + 196608*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 864*
a^5*b^3 - (262144*a^8 + 27*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291456
*a^2*b^10 + 196608*a^2*b^9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2*b^...
```

3.187.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a*x + b*x**(2/3)), x)
```

3.187.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a*x + b*x^(2/3)), x)
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{4096\,b^{\frac{15}{2}}}{2145\,a^8} + \frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} - 3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b + 12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^2 - 25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^3 + 32175\left(ax^{\frac{1}{3}}+\right.}{2145\,a^8}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1
/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) +
b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/
2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a
^8
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^2}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(x^2/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^(2/3))^(1/2), x)
```

3.188 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$

3.188.1 Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4}+\frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3}-\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2}+\frac{2x\sqrt{bx^{2/3}+ax}}{3a}$$

output
```
-128/105*b^3*(b*x^(2/3)+a*x)^(1/2)/a^4+256/105*b^4*(b*x^(2/3)+a*x)^(1/2)/a
^5/x^(1/3)+32/35*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-16/21*b*x^(2/3)*(b*
x^(2/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(2/3)+a*x)^(1/2)/a
```

3.188.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(128b^4-64ab^3\sqrt[3]{x}+48a^2b^2x^{2/3}-40a^3bx+35a^4x^{4/3}\right)}{105a^5\sqrt[3]{x}}$$

input
```
Integrate[x/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3)
- 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

3.188.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int \frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int \frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}dx}{7a}\right)}{9a}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int \frac{1}{\sqrt{x^{2/3}b+ax}}dx}{5a}\right)}{7a}\right)}{9a}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int \frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}\right)}{5a}\right)}{7a}\right)}{9a}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}\right)}{5a}\right)}{7a}\right)}{9a}$$

input
```
Int[x/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*x*Sqrt[b*x^(2/3) + a*x])/(3*a) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x]
)/(7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*
x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7
*a)))/(9*a)
```

3.188.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.188.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^4x^{\frac{4}{3}}-40a^3bx+48a^2b^2x^{\frac{2}{3}}-64a\,x^{\frac{1}{3}}b^3+128b^4\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^5}$	68
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^4x^{\frac{4}{3}}-40a^3bx+48a^2b^2x^{\frac{2}{3}}-64a\,x^{\frac{1}{3}}b^3+128b^4\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^5}$	68

input
```
int(x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*x^(1/3)*(b+a*x^(1/3))*(35*a^4*x^(4/3)-40*a^3*b*x+48*a^2*b^2*x^(2/3)-
64*a*x^(1/3)*b^3+128*b^4)/(b*x^(2/3)+a*x)^(1/2)/a^5
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(101) = 202.

Time = 170.47 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.66

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx =$$

$$-\frac{2\left(2\,(805306368\,b^{10}+167772160\,b^9+786432\,(64\,a^3-3)b^7-15728640\,b^8-4096\,(11264\,a^3-53)b^6-101\right.}{}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(2*(805306368*b^10 + 167772160*b^9 + 786432*(64*a^3 - 3)*b^7 - 1572
8640*b^8 - 4096*(11264*a^3 - 53)*b^6 - 101920*a^6 - 1536*(5504*a^3 + 1)*b^
5 - 48*(1966080*a^6 + 114688*a^3 + 3)*b^4 - 1792*(36864*a^6 - 103*a^3)*b^3
 - 192*(65280*a^6 + 23*a^3)*b^2 + 15*(188416*a^6 + 3*a^3)*b)*x - (35*(1677
7216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*
b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 48*(16777216*a^2*b^8 + 6291
456*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^3
 - (262144*a^5 + 27*a^2)*b^2)*x^(4/3) - 64*(16777216*a*b^9 + 6291456*a*b^8
 + 196608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27
*a)*b^3)*x + 8*(268435456*b^10 + 100663296*b^9 + 3145728*b^8 - 1835008*b^7
 - 36864*b^6 - 16*(262144*a^3 + 27)*b^4 + 13824*b^5 - 5*(16777216*a^3*b^7
+ 6291456*a^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a
^3*b^2 - (262144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((167
77216*a^5*b^6 + 6291456*a^5*b^5 + 196608*a^5*b^4 - 262144*a^8 - 114688*a^5
*b^3 - 2304*a^5*b^2 + 864*a^5*b - 27*a^5)*x)
```

3.188.6 Sympy [F]

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(a*x + b*x**(2/3)), x)
```

3.188.7 Maxima [F]

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a*x + b*x^(2/3)), x)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{256\,b^{\frac{9}{2}}}{105\,a^5} + \frac{2\left(35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} - 180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b + 378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2 - 420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^3 + 315\sqrt{ax^{\frac{1}{3}}+b}\,b^4\right)}{105\,a^5}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) +
b)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3
 + 315*sqrt(a*x^(1/3) + b)*b^4)/a^5
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(x/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
int(x/(a*x + b*x^(2/3))^(1/2), x)
```

3.189 $\int \frac{1}{\sqrt{bx^{2/3}+ax}} dx$

3.189.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}} dx = \frac{2\sqrt{bx^{2/3}+ax}}{a} - \frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

output
```
2*(b*x^(2/3)+a*x)^(1/2)/a-4*b*(b*x^(2/3)+a*x)^(1/2)/a^2/x^(1/3)
```

3.189.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}} dx = \frac{2\left(-2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

input
```
Integrate[1/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

3.189.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int \frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

input
```
Int[1/Sqrt[b*x^(2/3) + a*x],x]
```

output
```
(2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

3.189.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.189.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(a\,x^{\frac{1}{3}}-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{2}}$	36
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(a\,x^{\frac{1}{3}}-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{2}}$	36

input
```
int(1/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/3)*(b+a*x^(1/3))*(a*x^(1/3)-2*b)/(b*x^(2/3)+a*x)^(1/2)/a^2
```

3.189.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(37) = 74$.

Time = 138.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.06

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{(50331648\,b^7 + 10485760\,b^6 + 49152\,(512\,a^3 - 3)b^4 - 983040\,b^5 + 256\,(24576\,a^3 + 53)b}{}$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
((50331648*b^7 + 10485760*b^6 + 49152*(512*a^3 - 3)*b^4 - 983040*b^5 + 256
*(24576*a^3 + 53)*b^3 + 11648*a^3 - 96*(2048*a^3 + 1)*b^2 - 3*(155648*a^3
+ 3)*b)*x + 2*((16777216*a*b^6 + 6291456*a*b^5 + 196608*a*b^4 - 262144*a^4
 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x - 2*(16777216*b^7 + 62914
56*b^6 + 196608*b^5 - 114688*b^4 - 2304*b^3 - (262144*a^3 + 27)*b + 864*b^
2)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^2*b^6 + 6291456*a^2*b^5 +
196608*a^2*b^4 - 262144*a^5 - 114688*a^2*b^3 - 2304*a^2*b^2 + 864*a^2*b -
27*a^2)*x)
```

3.189.6 Sympy [F]

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x + b*x**(2/3)), x)
```

3.189.7 Maxima [F]

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x + b*x^(2/3)), x)
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx=\frac{4\,b^{\frac{3}{2}}}{a^2}+\frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}-3\sqrt{ax^{\frac{1}{3}}+b}b\right)}{a^2}$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2
```

3.189.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{3\,x\,\sqrt{\frac{a\,x^{1/3}}{b}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{a\,x^{1/3}}{b}\right)}{2\,\sqrt{a\,x+b\,x^{2/3}}}$$

input
```
int(1/(a*x + b*x^(2/3))^(1/2),x)
```

output
```
(3*x*((a*x^(1/3))/b + 1)^(1/2)*hypergeom([1/2, 2], 3, -(a*x^(1/3))/b))/(2*
(a*x + b*x^(2/3))^(1/2))
```

3.190 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$

3.190.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

output
```
3*a*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3*(b*x^(2/3)+a*
x)^(1/2)/b/x^(2/3)
```

3.190.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sq
rt[b*x^(2/3) + a*x]])/b^(3/2)
```

3.190.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+bx^{2/3}}}dx$$
$$\Big\downarrow \text{1931}$$
$$-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$
$$\Big\downarrow \text{1935}$$
$$\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$
$$\Big\downarrow \text{219}$$
$$\frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

input
```
Int[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sq
rt[b*x^(2/3) + a*x]])/b^(3/2)
```

3.190.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.190.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{3\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+a x}\,b^{\frac{5}{2}}}$	61
default	$\frac{3\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}-\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{3}{2}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+a x}\,b^{\frac{5}{2}}}$	61

input
```
int(1/x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3*(b+a*x^(1/3))^(1/2)*((b+a*x^(1/3))^(1/2)*b^(3/2)-arctanh((b+a*x^(1/3))^
(1/2)/b^(1/2))*b*a*x^(1/3))/(b*x^(2/3)+a*x)^(1/2)/b^(5/2)
```

3.190.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.190.6 Sympy [F]

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)
```

3.190.7 Maxima [F]

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x}\,dx$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\left(\frac{a^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\sqrt{ax^{\frac{1}{3}}+b}a}{bx^{\frac{1}{3}}}\right)}{a}$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-3*(a^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3)
 + b)*a/(b*x^(1/3)))/a
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(2/3))^(1/2)), x)
```

3.191 $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$

3.191.1 Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}}+\frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}}-\frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x}+\frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}}-\frac{105a^4\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

output
```
-105/64*a^4*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/4*(b*
x^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/8*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-35/32
*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x+105/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2
/3)
```

3.191.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx=\frac{\sqrt{bx^{2/3}+ax}\left(-48b^3+56ab^2\sqrt[3]{x}-70a^2bx^{2/3}+105a^3x\right)}{64b^4x^{5/3}}-\frac{105a^4\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

input
```
Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-48*b^3 + 56*a*b^2*x^(1/3) - 70*a^2*b*x^(2/3) + 10
5*a^3*x))/(64*b^4*x^(5/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(
2/3) + a*x]])/(64*b^(9/2))
```

3.191.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$\downarrow$ 1935

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

$\downarrow$ 219

$$-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

3.191. $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}dx$

input
```
Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(
b*x^(4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*
x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3
) + a*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)
```

3.191.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.191.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(48\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{9}{2}}-56b^{\frac{7}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a\,x^{\frac{1}{3}}+70b^{\frac{5}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{2}x^{\frac{2}{3}}-105b^{\frac{3}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{3}x+105\operatorname{arctanh}\left(\frac{\sqrt{b+}}{\sqrt{}}\right.\right.}{64x\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$
default	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(105x^{\frac{7}{3}}\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{4}b+70x^{\frac{5}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}a^{2}-56x^{\frac{4}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a+48\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{9}{2}}x-105x^{2}\right.}{64x^{2}\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$

input
```
int(1/x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(b+a*x^(1/3))^(1/2)*(48*(b+a*x^(1/3))^(1/2)*b^(9/2)-56*b^(7/2)*(b+a*
x^(1/3))^(1/2)*a*x^(1/3)+70*b^(5/2)*(b+a*x^(1/3))^(1/2)*a^2*x^(2/3)-105*b^
(3/2)*(b+a*x^(1/3))^(1/2)*a^3*x+105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a
^4*b*x^(4/3))/x/(b*x^(2/3)+a*x)^(1/2)/b^(11/2)
```

3.191.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.191.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{x^2\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)
```

3.191.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \frac{\frac{105\,a^5\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^5-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^5b+511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^5b^2-279\sqrt{ax^{\frac{1}{3}}+b}a^5b^3}{a^4b^4x^{\frac{4}{3}}}}{64\,a}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(
a*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3
) + b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3)))
/a
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^2\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(2/3))^(1/2)), x)
```

3.192 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

3.192.1 Optimal result

Integrand size = 19, antiderivative size = 241

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}+\frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}}-\frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2}+\frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}}-\frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{512b^6x}-\frac{1287a^6\sqrt{bx^{2/3}+ax}}{1024b^7x^{2/3}}+\frac{1287a^7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}$$

output
```
1287/1024*a^7*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/7*
(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/28*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-
143/280*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+1287/2240*a^3*(b*x^(2/3)+a*x)^(1
/2)/b^4/x^(5/3)-429/640*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+429/512*a^5*
(b*x^(2/3)+a*x)^(1/2)/b^6/x-1287/1024*a^6*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3
)
```

3.192.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-15360b^6+16640ab^5\sqrt[3]{x}-18304a^2b^4x^{2/3}+20592a^3b^3x-24024a^4b^2\right.}{35840b^7x^{8/3}} + \frac{1287a^7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}$$

input
```
Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-15360*b^6 + 16640*a*b^5*x^(1/3) - 18304*a^2*b^4*x
^(2/3) + 20592*a^3*b^3*x - 24024*a^4*b^2*x^(4/3) + 30030*a^5*b*x^(5/3) - 4
5045*a^6*x^2))/(35840*b^7*x^(8/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(1/3))/S
qrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

3.192.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$\downarrow$ 1931

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$\downarrow$ 1931

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$\downarrow$ 1931

3.192. $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

$$-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

$$\Big\downarrow \; 1931$$

3.192. $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

$$\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}\right.\right.$$

3.192. $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1935

3.192. $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

$$\left(\quad\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right.\right.\right.$$

3.192. $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

$\downarrow$ 219

3.192. $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

$$\left(\quad 11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/}}}{5bx^2}\right.\right.$$

3.192. $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

input
```
Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*
x]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*S
qrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(
4/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/
3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a
*x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)
```

3.192.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.192.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(45045b^{\frac{3}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{6}x^{2}-45045\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{7}b\,x^{\frac{7}{3}}-30030b^{\frac{5}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{5}x^{\frac{5}{3}}+24024b^{\frac{7}{2}}\sqrt{b+a}}{35840x^{2}\sqrt{b\,x}}$
default	$\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(45045x^{\frac{13}{3}}\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{7}b+30030x^{\frac{11}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}a^{5}-24024x^{\frac{10}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a^{4}-18304x^{\frac{8}{3}}\sqrt{b+}}{35840x^{4}\sqrt{b\,x}}$

input
```
int(1/x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35840/x^2*(b+a*x^(1/3))^(1/2)*(45045*b^(3/2)*(b+a*x^(1/3))^(1/2)*a^6*x^
2-45045*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^7*b*x^(7/3)-30030*b^(5/2)*(
b+a*x^(1/3))^(1/2)*a^5*x^(5/3)+24024*b^(7/2)*(b+a*x^(1/3))^(1/2)*a^4*x^(4/
3)-20592*b^(9/2)*(b+a*x^(1/3))^(1/2)*a^3*x+18304*b^(11/2)*(b+a*x^(1/3))^(1
/2)*a^2*x^(2/3)-16640*b^(13/2)*(b+a*x^(1/3))^(1/2)*a*x^(1/3)+15360*(b+a*x^
(1/3))^(1/2)*b^(15/2))/(b*x^(2/3)+a*x)^(1/2)/b^(17/2)
```

3.192.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.192.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)
```

3.192.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx =$$

$$-\frac{\frac{45045\,a^8\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7} + \frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^8-300300\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^8b+849849\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^8b^2-1317888\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^8b^3+1200199\left(\cdots\right.}{a^7b^7x^{\frac{7}{3}}}}{35840\,a}$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/35840*(45045*a^8*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
(45045*(a*x^(1/3) + b)^(13/2)*a^8 - 300300*(a*x^(1/3) + b)^(11/2)*a^8*b +
849849*(a*x^(1/3) + b)^(9/2)*a^8*b^2 - 1317888*(a*x^(1/3) + b)^(7/2)*a^8*b
^3 + 1200199*(a*x^(1/3) + b)^(5/2)*a^8*b^4 - 631540*(a*x^(1/3) + b)^(3/2)*
a^8*b^5 + 169995*sqrt(a*x^(1/3) + b)*a^8*b^6)/(a^7*b^7*x^(7/3)))/a
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(2/3))^(1/2)), x)
```

3.193 $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

3.193.1 Optimal result

Integrand size = 19, antiderivative size = 329

$$\begin{aligned}\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = &-\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}\\ &-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}}\\ &+\frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2}-\frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}}+\frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}}\\ &-\frac{46189a^8\sqrt{bx^{2/3}+ax}}{65536b^9x}+\frac{138567a^9\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{2/3}}-\frac{138567a^{10}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}\end{aligned}$$

output
```
-138567/131072*a^10*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2
)-3/10*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/60*a*(b*x^(2/3)+a*x)^(1/2)/b^2/
x^(10/3)-323/960*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/896*a^3*(b*x^(2/3)+
a*x)^(1/2)/b^4/x^(8/3)-4199/10752*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+46
189/107520*a^5*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-138567/286720*a^6*(b*x^(2/3)+
a*x)^(1/2)/b^7/x^(5/3)+46189/81920*a^7*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-4
6189/65536*a^8*(b*x^(2/3)+a*x)^(1/2)/b^9/x+138567/131072*a^9*(b*x^(2/3)+a*
x)^(1/2)/b^10/x^(2/3)
```

3.193.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-4128768b^9+4358144ab^8\sqrt[3]{x}-4630528a^2b^7x^{2/3}+4961280a^3b^6x-5\right.}{}$$
$$-\frac{138567a^{10}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}$$

input
```
Integrate[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(Sqrt[b*x^(2/3) + a*x]*(-4128768*b^9 + 4358144*a*b^8*x^(1/3) - 4630528*a^2
*b^7*x^(2/3) + 4961280*a^3*b^6*x - 5374720*a^4*b^5*x^(4/3) + 5912192*a^5*b
^4*x^(5/3) - 6651216*a^6*b^3*x^2 + 7759752*a^7*b^2*x^(7/3) - 9699690*a^8*b
*x^(8/3) + 14549535*a^9*x^3))/(13762560*b^10*x^(11/3)) - (138567*a^10*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(21/2))
```

3.193.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{19a\int \frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{19a\left(-\frac{17a\int \frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$\downarrow$ 1931

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$\downarrow$ 1931

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$\downarrow$ 1931

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$$-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

$$\downarrow 1931$$

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$$19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/\ldots}}}{8bx^3}\ldots\right)}{18b}\ldots$$

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

3.193. $\displaystyle \downarrow$ 1931

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$$17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}\,dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}\right.$$

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1931

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$$\left(\vphantom{\int}\left(\vphantom{\int}\left(\vphantom{\int} 15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}\,dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{a\cdots}}{2\cdots}\right)}{14b}\cdots\right.\right.\right.\right.$$

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

↓ 1931

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$$\left(\left(\left(\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\cdots\right)}{10b}\cdots\right)}{12b}\cdots\right.\right.\right.\right.\right.$$

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$\downarrow$ 1935

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax}}{4b}\cdots}{10b}\right.$$

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$\downarrow$ 219

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+b\cdots}}{4bx^{5\cdots}}\cdots}{10b}\cdots\right.$$

3.193. $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

input
```
Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

output
```
(-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(2/3) +
a*x]/(b*x^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((
-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x
]/(b*x^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sq
rt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4
/3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3
) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/b^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))
/(18*b)))/(20*b)
```

3.193.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.193.4 Maple [A] (verified)

Time = 10.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(4128768\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{21}{2}}-4358144b^{\frac{19}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a\,x^{\frac{1}{3}}+4630528b^{\frac{17}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{2}x^{\frac{2}{3}}-4961280b^{\frac{15}{2}}\sqrt{b+a\,x}\right.}{}$
default	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(14549535x^{\frac{19}{3}}\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{10}b+9699690x^{\frac{17}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}a^{8}-7759752x^{\frac{16}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a^{7}-59\right.}{}$

input
```
int(1/x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/13762560*(b+a*x^(1/3))^(1/2)*(4128768*(b+a*x^(1/3))^(1/2)*b^(21/2)-4358
144*b^(19/2)*(b+a*x^(1/3))^(1/2)*a*x^(1/3)+4630528*b^(17/2)*(b+a*x^(1/3))^
(1/2)*a^2*x^(2/3)-4961280*b^(15/2)*(b+a*x^(1/3))^(1/2)*a^3*x+5374720*b^(13
/2)*(b+a*x^(1/3))^(1/2)*a^4*x^(4/3)-5912192*b^(11/2)*(b+a*x^(1/3))^(1/2)*a
^5*x^(5/3)+6651216*b^(9/2)*(b+a*x^(1/3))^(1/2)*a^6*x^2-7759752*b^(7/2)*(b+
a*x^(1/3))^(1/2)*a^7*x^(7/3)+9699690*b^(5/2)*(b+a*x^(1/3))^(1/2)*a^8*x^(8/
3)-14549535*b^(3/2)*(b+a*x^(1/3))^(1/2)*a^9*x^3+14549535*arctanh((b+a*x^(1
/3))^(1/2)/b^(1/2))*a^10*b*x^(10/3))/x^3/(b*x^(2/3)+a*x)^(1/2)/b^(23/2)
```

3.193.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.193.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a*x + b*x**(2/3))), x)
```

3.193.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \frac{\frac{14549535\,a^{11}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{11}-140645505\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{11}b+609140532\left(ax^{\frac{1}{3}}+\right.}{}}{}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b
^10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(
17/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x
^(1/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 -
 2918514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7
/2)*a^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^
(1/3) + b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b
^10*x^(10/3)))/a
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(2/3))^(1/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(2/3))^(1/2)), x)
```

3.194 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.194.1 Optimal result

Integrand size = 19, antiderivative size = 336

$$\begin{aligned}\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = &-\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}}\\&+\frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}}+\frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}}\\&-\frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9}+\frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8}\\&-\frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}-\frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5}\\&+\frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}-\frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3}+\frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}\end{aligned}$$

output
```
-6*x^4/a/(b*x^(2/3)+a*x)^(1/2)-524288/29393*b^9*(b*x^(2/3)+a*x)^(1/2)/a^11
+1048576/29393*b^10*(b*x^(2/3)+a*x)^(1/2)/a^12/x^(1/3)+393216/29393*b^8*x^
(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^10-327680/29393*b^7*x^(2/3)*(b*x^(2/3)+a*x)^
(1/2)/a^9+40960/4199*b^6*x*(b*x^(2/3)+a*x)^(1/2)/a^8-36864/4199*b^5*x^(4/3
)*(b*x^(2/3)+a*x)^(1/2)/a^7+33792/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a
^6-16896/2261*b^3*x^2*(b*x^(2/3)+a*x)^(1/2)/a^5+15840/2261*b^2*x^(7/3)*(b*
x^(2/3)+a*x)^(1/2)/a^4-880/133*b*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+44/7*x^
3*(b*x^(2/3)+a*x)^(1/2)/a^2
```

3.194.2 Mathematica [A] (verified)

Time = 5.59 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.48

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{2\sqrt[3]{x}\left(524288b^{11}+262144ab^{10}\sqrt[3]{x}-65536a^2b^9x^{2/3}+32768a^3b^8x-20480a^4b^7x^{4/3}+\right.}{}$$

input
```
Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) +
32768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^
6*b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3
 - 4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[b*x^(2/3)
+ a*x])
```

3.194.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{22\int \frac{x^3}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\int \frac{x^{8/3}}{\sqrt{x^{2/3}b+ax}}dx}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\int\frac{x^{7/3}}{\sqrt{x^{2/3}b+ax}}dx}{19a}\right)}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1922

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\int\frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{17a}\right)}{19a}\right)}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1922

3.194. $\int\frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int\frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}\right)}{17a}\right)}{19a}\right)}{21a}\right)}{a}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \ 1922$$

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\frac{22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int\frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}\right)}{a}$$

$$\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1922

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$22\left(\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}-\frac{20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{16b\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\cdots}{\cdots}\right.}{13a}\right.}{15a}\right.}{17a}\right.}{19a}\right.}{21a}\right.\cdots$$

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1922

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\, dx$

$$10b$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\right.$$

$$16b\left(\frac{2x^{2}\sqrt{ax+bx^{2/3}}}{5a}-\frac{\quad}{15a}\right.$$

$$18b\left(\frac{6x^{7/3}\sqrt{ax+bx^{2/3}}}{17a}-\frac{\quad}{17a}\right.$$

$$20b\left(\frac{6x^{8/3}\sqrt{ax+bx^{2/3}}}{19a}-\frac{\quad}{19a}\right.$$

3.194. $\displaystyle\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1922

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\Bigg(\quad \Bigg(\quad \Bigg(\quad 16b \left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a} - \frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \frac{10b}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1922

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$10b$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$$

$$16b\left(\frac{2x^{2}\sqrt{ax+bx^{2/3}}}{5a} - \right.$$

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$\downarrow$ 1908

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$10b$

$12b \left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$

$14b \left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1920

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$10b$

$12b \left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a} - \right.$

$14b \left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a} - \right.$

3.194. $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

input
```
Int[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) + (22*((2*x^3*Sqrt[b*x^(2/3) + a*x])/(7
*a) - (20*b*((6*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(19*a) - (18*b*((6*x^(7/3)*
Sqrt[b*x^(2/3) + a*x])/(17*a) - (16*b*((2*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a)
 - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(13*a) - (12*b*((6*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*Sqrt[b*x^(2/3) + a*x])/(3*a) - (8
*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x^(2
/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3
) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15*a)
))/(17*a)))/(19*a)))/(21*a)))/a
```

3.194.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.194.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720b^{2}a^{9}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-20}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720b^{2}a^{9}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-20}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$

input
```
int(x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/29393*x*(b+a*x^(1/3))*(4199*a^11*x^(11/3)-4862*a^10*b*x^(10/3)+5720*b^2*
a^9*x^3-6864*a^8*b^3*x^(8/3)+8448*a^7*b^4*x^(7/3)-10752*a^6*b^5*x^2+14336*
a^5*b^6*x^(5/3)-20480*a^4*b^7*x^(4/3)+32768*a^3*b^8*x-65536*a^2*b^9*x^(2/3
)+262144*a*b^10*x^(1/3)+524288*b^11)/(b*x^(2/3)+a*x)^(3/2)/a^12
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2566 vs. 2(252) = 504.

Time = 142.89 (sec) , antiderivative size = 2566, normalized size of antiderivative = 7.64

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/29393*((6442450944*a^3*b^19 + 5368709120*a^3*b^18 - 2013265920*a^3*b^17
 - 6113744*a^18 + 402653184*(17*a^6 - 3*a^3)*b^16 + 8388608*(464*a^6 + 53*
a^3)*b^15 - 12582912*(246*a^6 + a^3)*b^14 + 1572864*(1036*a^9 - 2560*a^6 -
 3*a^3)*b^13 - 524288*(758*a^9 - 1569*a^6)*b^12 - 393216*(5803*a^9 + 124*a
^6)*b^11 + 98304*(1315*a^12 - 20924*a^9 - 33*a^6)*b^10 - 57344*(2264*a^12
- 3153*a^9)*b^9 - 6144*(83789*a^12 + 2066*a^9)*b^8 - 1536*(46256*a^15 - 15
9272*a^12 - 267*a^9)*b^7 - 128*(264488*a^15 + 382229*a^12)*b^6 + 9984*(155
47*a^15 + 482*a^12)*b^5 - 24*(2376192*a^18 + 4735792*a^15 + 7887*a^12)*b^4
 - 1664*(107856*a^18 - 16759*a^15)*b^3 - 156*(935424*a^18 + 17935*a^15)*b^
2 + 663*(97664*a^18 + 123*a^15)*b)*x^2 + (6442450944*b^22 + 5368709120*b^2
1 + 402653184*(17*a^3 - 3)*b^19 - 2013265920*b^20 + 8388608*(464*a^3 + 53)
*b^18 - 6113744*a^15*b^3 - 12582912*(246*a^3 + 1)*b^17 + 1572864*(1036*a^6
 - 2560*a^3 - 3)*b^16 - 524288*(758*a^6 - 1569*a^3)*b^15 - 393216*(5803*a^
6 + 124*a^3)*b^14 + 98304*(1315*a^9 - 20924*a^6 - 33*a^3)*b^13 - 57344*(22
64*a^9 - 3153*a^6)*b^12 - 6144*(83789*a^9 + 2066*a^6)*b^11 - 1536*(46256*a
^12 - 159272*a^9 - 267*a^6)*b^10 - 128*(264488*a^12 + 382229*a^9)*b^9 + 99
84*(15547*a^12 + 482*a^9)*b^8 - 24*(2376192*a^15 + 4735792*a^12 + 7887*a^9
)*b^7 - 1664*(107856*a^15 - 16759*a^12)*b^6 - 156*(935424*a^15 + 17935*a^1
2)*b^5 + 663*(97664*a^15 + 123*a^12)*b^4)*x - 2*(4199*(4096*a^13*b^9 + 614
4*a^13*b^8 + 768*a^13*b^7 - 4096*a^19 - 144*a^16*b^2 + 216*a^16*b - 27*...
```

3.194.6 Sympy [F]

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**4/(a*x + b*x**(2/3))**(3/2), x)
```

3.194.7 Maxima [F]

$$\int \frac{x^4}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{(bx^{2/3}+ax)^{3/2}}\,dx = -\frac{1048576\,b^{\frac{21}{2}}}{29393\,a^{12}} + \frac{6\,b^{11}}{\sqrt{ax^{\frac{1}{3}}+b}a^{12}}$$
$$+ \frac{2\left(4199\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{240} - 51051\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{240}b + 285285\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{240}b^2 - 969969\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{2}\right.}{}$$

input
```
integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393
*(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b
 + 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)
*a^240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3)
 + b)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010
*(a*x^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8
 - 1616615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^
240*b^10)/a^252
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

input
```
int(x^4/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

3.195 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.195.1 Optimal result

Integrand size = 19, antiderivative size = 248

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8}-\frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5}+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}$$

output
```
-6*x^3/a/(b*x^(2/3)+a*x)^(1/2)+32768/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^8-65
536/2145*b^7*(b*x^(2/3)+a*x)^(1/2)/a^9/x^(1/3)-8192/715*b^5*x^(1/3)*(b*x^(
2/3)+a*x)^(1/2)/a^7+4096/429*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-3584/42
9*b^3*x*(b*x^(2/3)+a*x)^(1/2)/a^5+5376/715*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/
2)/a^4-448/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+32/5*x^2*(b*x^(2/3)+a*x)
^(1/2)/a^2
```

3.195.2 Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.49

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{2\left(-32768b^8\sqrt[3]{x}-16384ab^7x^{2/3}+4096a^2b^6x-2048a^3b^5x^{4/3}+1280a^4b^4x^{5/3}-896a\right.}{2145a^9\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b
^5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3)
- 528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])
```

3.195.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{16\int \frac{x^2}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\int \frac{x^{5/3}}{\sqrt{x^{2/3}b+ax}}dx}{15a}\right)}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\int\frac{x^{4/3}}{\sqrt{x^{2/3}b+ax}}dx}{13a}\right)}{15a}\right)}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1922

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\int\frac{x}{\sqrt{x^{2/3}b+ax}}dx}{11a}\right)}{13a}\right)}{15a}\right)}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1922

$$\frac{16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int\frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}\right)}{11a}\right)}{13a}\right)}{15a}\right)}{a}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

3.195. $\int\frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1922

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}dx}{7a}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{15a}\right)$$

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1922

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$16\left(\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}-\frac{14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int \cdots}{\sqrt{\cdots}}\cdots\right)}{7a}\cdots\right)}{9a}\cdots\right)}{11a}\cdots\right)}{13a}\cdots\right)}{15a}\cdots\right.$$

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1908

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{}\dots}{}\right.}{}\right.$$

$$8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{\dots}{}\right.$$

$$10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{\dots}{9a}\right.$$

$$12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{\dots}{11a}\right.$$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{\dots}{13a}\right.$$

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1920

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$14b\left(\frac{6x^{5/3}\sqrt{ax+bx^{2/3}}}{13a}-\frac{12b\left(\frac{6x^{4/3}\sqrt{ax+bx^{2/3}}}{11a}-\frac{10b\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{7\cdots}\right.}{9a}\right.}{11a}\right.}{13a}\right.$$

3.195. $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

input
```
Int[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (16*((2*x^2*Sqrt[b*x^(2/3) + a*x])/(5
*a) - (14*b*((6*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(13*a) - (12*b*((6*x^(4/3)*
Sqrt[b*x^(2/3) + a*x])/(11*a) - (10*b*((2*x*Sqrt[b*x^(2/3) + a*x])/(3*a) -
 (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a) - (6*b*((6*x^(1/3)*Sqrt[b*x
^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(
2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15
*a)))/a
```

3.195.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.195.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.44

method	result
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{8}x^{\frac{8}{3}}-528a^{7}b\,x^{\frac{7}{3}}+672a^{6}x^{2}b^{2}-896a^{5}b^{3}x^{\frac{5}{3}}+1280x^{\frac{4}{3}}a^{4}b^{4}-2048a^{3}b^{5}x+4096a^{2}b^{6}x^{\frac{2}{3}}-16384x^{\frac{1}{3}}a\,b^{7}-\right.}{2145\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{9}}$
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{8}x^{\frac{8}{3}}-528a^{7}b\,x^{\frac{7}{3}}+672a^{6}x^{2}b^{2}-896a^{5}b^{3}x^{\frac{5}{3}}+1280x^{\frac{4}{3}}a^{4}b^{4}-2048a^{3}b^{5}x+4096a^{2}b^{6}x^{\frac{2}{3}}-16384x^{\frac{1}{3}}a\,b^{7}-\right.}{2145\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{9}}$

input
```
int(x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/2145*x*(b+a*x^(1/3))*(429*a^8*x^(8/3)-528*a^7*b*x^(7/3)+672*a^6*x^2*b^2-
896*a^5*b^3*x^(5/3)+1280*x^(4/3)*a^4*b^4-2048*a^3*b^5*x+4096*a^2*b^6*x^(2/
3)-16384*x^(1/3)*a*b^7-32768*b^8)/(b*x^(2/3)+a*x)^(3/2)/a^9
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2083 vs. 2(186) = 372.

Time = 123.02 (sec) , antiderivative size = 2083, normalized size of antiderivative = 8.40

$$\int \frac{x^3}{(bx^{2/3}+ax)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/2145*((402653184*a^3*b^16 + 335544320*a^3*b^15 - 125829120*a^3*b^14 + 62
4624*a^15 + 25165824*(17*a^6 - 3*a^3)*b^13 + 524288*(464*a^6 + 53*a^3)*b^1
2 - 786432*(246*a^6 + a^3)*b^11 + 98304*(1036*a^9 - 2560*a^6 - 3*a^3)*b^10
 - 32768*(758*a^9 - 1569*a^6)*b^9 - 24576*(5803*a^9 + 124*a^6)*b^8 + 6144*
(600*a^12 - 20924*a^9 - 33*a^6)*b^7 - 1536*(7666*a^12 - 7357*a^9)*b^6 - 76
8*(40107*a^12 + 1033*a^9)*b^5 + 96*(63360*a^15 + 167852*a^12 + 267*a^9)*b^
4 + 32*(613440*a^15 - 105031*a^12)*b^3 + 468*(34560*a^15 + 661*a^12)*b^2 -
 99*(68480*a^15 + 87*a^12)*b)*x^2 + (402653184*b^19 + 335544320*b^18 + 251
65824*(17*a^3 - 3)*b^16 - 125829120*b^17 + 524288*(464*a^3 + 53)*b^15 + 62
4624*a^12*b^3 - 786432*(246*a^3 + 1)*b^14 + 98304*(1036*a^6 - 2560*a^3 - 3
)*b^13 - 32768*(758*a^6 - 1569*a^3)*b^12 - 24576*(5803*a^6 + 124*a^3)*b^11
 + 6144*(600*a^9 - 20924*a^6 - 33*a^3)*b^10 - 1536*(7666*a^9 - 7357*a^6)*b
^9 - 768*(40107*a^9 + 1033*a^6)*b^8 + 96*(63360*a^12 + 167852*a^9 + 267*a^
6)*b^7 + 32*(613440*a^12 - 105031*a^9)*b^6 + 468*(34560*a^12 + 661*a^9)*b^
5 - 99*(68480*a^12 + 87*a^9)*b^4)*x + 2*(429*(4096*a^10*b^9 + 6144*a^10*b^
8 + 768*a^10*b^7 - 4096*a^16 - 144*a^13*b^2 + 216*a^13*b - 27*a^13 + 256*(
16*a^13 - 7*a^10)*b^6 + 48*(128*a^13 - 3*a^10)*b^5 + 24*(32*a^13 + 9*a^10)
*b^4 - (5888*a^13 + 27*a^10)*b^3)*x^4 - 2096*(4096*a^7*b^12 + 6144*a^7*b^1
1 + 768*a^7*b^10 - 144*a^10*b^5 + 216*a^10*b^4 + 256*(16*a^10 - 7*a^7)*b^9
 + 48*(128*a^10 - 3*a^7)*b^8 + 24*(32*a^10 + 9*a^7)*b^7 - (5888*a^10 + ...
```

3.195.6 Sympy [F]

$$\int \frac{x^3}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)
```

3.195.7 Maxima [F]

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{65536\,b^{\frac{15}{2}}}{2145\,a^9}-\frac{6\,b^8}{\sqrt{ax^{\frac{1}{3}}+b}a^9}+\frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{126}-3960\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{126}b+16380\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{126}b^2-40040\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{126}b^3\,-\right.}{214}$$

input
```
integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
65536/2145*b^(15/2)/a^9 - 6*b^8/(sqrt(a*x^(1/3) + b)*a^9) + 2/2145*(429*(a
*x^(1/3) + b)^(15/2)*a^126 - 3960*(a*x^(1/3) + b)^(13/2)*a^126*b + 16380*(
a*x^(1/3) + b)^(11/2)*a^126*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^126*b^3 +
64350*(a*x^(1/3) + b)^(7/2)*a^126*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^126*
b^5 + 60060*(a*x^(1/3) + b)^(3/2)*a^126*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^
126*b^7)/a^135
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

3.196 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.196.1 Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}$$

output
```
-6*x^2/a/(b*x^(2/3)+a*x)^(1/2)-256/21*b^3*(b*x^(2/3)+a*x)^(1/2)/a^5+512/21
*b^4*(b*x^(2/3)+a*x)^(1/2)/a^6/x^(1/3)+64/7*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1
/2)/a^4-160/21*b*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+20/3*x*(b*x^(2/3)+a*x)^
(1/2)/a^2
```

3.196.2 Mathematica [A] (verified)

Time = 5.69 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{512b^5\sqrt[3]{x}+256ab^4x^{2/3}-64a^2b^3x+32a^3b^2x^{4/3}-20a^4bx^{5/3}+14a^5x^2}{21a^6\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) -
 20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*Sqrt[b*x^(2/3) + a*x])
```

3.196.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1921, 1922, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\int \frac{x}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\int \frac{x^{2/3}}{\sqrt{x^{2/3}b+ax}}dx}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\int \frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}dx}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\int \frac{1}{\sqrt{x^{2/3}b+ax}}dx}{5a}\right)}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1908}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int\frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{10\left(\frac{2x\sqrt{ax+bx^{2/3}}}{3a}-\frac{8b\left(\frac{6x^{2/3}\sqrt{ax+bx^{2/3}}}{7a}-\frac{6b\left(\frac{6\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{5a}-\frac{4b\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{a}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

input
```
Int[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

3.196. $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

output
```
(-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) + (10*((2*x*Sqrt[b*x^(2/3) + a*x])/(3*a
) - (8*b*((6*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(7*a) - (6*b*((6*x^(1/3)*Sqrt[
b*x^(2/3) + a*x])/(5*a) - (4*b*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*
x^(2/3) + a*x])/(a^2*x^(1/3))))/(5*a)))/(7*a)))/(9*a)))/a
```

3.196.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.196.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(7a^{5}x^{\frac{5}{3}}-10a^{4}b\,x^{\frac{4}{3}}+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128a\,b^{4}x^{\frac{1}{3}}+256b^{5}\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{6}}$	77
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(7a^{5}x^{\frac{5}{3}}-10a^{4}b\,x^{\frac{4}{3}}+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128a\,b^{4}x^{\frac{1}{3}}+256b^{5}\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{6}}$	77

input
```
int(x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*x*(b+a*x^(1/3))*(7*a^5*x^(5/3)-10*a^4*b*x^(4/3)+16*a^3*b^2*x-32*a^2*b
^3*x^(2/3)+128*a*b^4*x^(1/3)+256*b^5)/(b*x^(2/3)+a*x)^(3/2)/a^6
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. 2(120) = 240.

Time = 125.66 (sec) , antiderivative size = 1598, normalized size of antiderivative = 9.99

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/21*((3145728*a^3*b^13 + 2621440*a^3*b^12 - 983040*a^3*b^11 - 10192*a^12
 + 196608*(17*a^6 - 3*a^3)*b^10 + 4096*(464*a^6 + 53*a^3)*b^9 - 6144*(246*
a^6 + a^3)*b^8 + 768*(1120*a^9 - 2560*a^6 - 3*a^3)*b^7 - 256*(548*a^9 - 15
69*a^6)*b^6 - 768*(1477*a^9 + 31*a^6)*b^5 - 48*(2304*a^12 + 21176*a^9 + 33
*a^6)*b^4 - 4032*(96*a^12 - 23*a^9)*b^3 - 12*(27648*a^12 + 527*a^9)*b^2 +
3*(39296*a^12 + 51*a^9)*b)*x^2 + (3145728*b^16 + 2621440*b^15 + 196608*(17
*a^3 - 3)*b^13 - 983040*b^14 + 4096*(464*a^3 + 53)*b^12 - 10192*a^9*b^3 -
6144*(246*a^3 + 1)*b^11 + 768*(1120*a^6 - 2560*a^3 - 3)*b^10 - 256*(548*a^
6 - 1569*a^3)*b^9 - 768*(1477*a^6 + 31*a^3)*b^8 - 48*(2304*a^9 + 21176*a^6
 + 33*a^3)*b^7 - 4032*(96*a^9 - 23*a^6)*b^6 - 12*(27648*a^9 + 527*a^6)*b^5
 + 3*(39296*a^9 + 51*a^6)*b^4)*x - 2*(7*(4096*a^7*b^9 + 6144*a^7*b^8 + 768
*a^7*b^7 - 4096*a^13 - 144*a^10*b^2 + 216*a^10*b - 27*a^10 + 256*(16*a^10
- 7*a^7)*b^6 + 48*(128*a^10 - 3*a^7)*b^5 + 24*(32*a^10 + 9*a^7)*b^4 - (588
8*a^10 + 27*a^7)*b^3)*x^3 - 58*(4096*a^4*b^12 + 6144*a^4*b^11 + 768*a^4*b^
10 - 144*a^7*b^5 + 216*a^7*b^4 + 256*(16*a^7 - 7*a^4)*b^9 + 48*(128*a^7 -
3*a^4)*b^8 + 24*(32*a^7 + 9*a^4)*b^7 - (5888*a^7 + 27*a^4)*b^6 - (4096*a^1
0 + 27*a^7)*b^3)*x^2 - 128*(4096*a*b^15 + 6144*a*b^14 + 768*a*b^13 + 256*(
16*a^4 - 7*a)*b^12 - 144*a^4*b^8 + 48*(128*a^4 - 3*a)*b^11 + 216*a^4*b^7 +
 24*(32*a^4 + 9*a)*b^10 - (5888*a^4 + 27*a)*b^9 - (4096*a^7 + 27*a^4)*b^6)
*x + (1048576*b^16 + 1572864*b^15 + 65536*(16*a^3 - 7)*b^13 + 196608*b^...
```

3.196.6 Sympy [F]

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)
```

3.196.7 Maxima [F]

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{512\,b^{\frac{9}{2}}}{21\,a^6}+\frac{6\,b^5}{\sqrt{ax^{\frac{1}{3}}+b}a^6} + \frac{2\left(7\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{48}-45\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{48}b+126\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{48}b^2-210\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{48}b^3+315\sqrt{ax^{\frac{1}{3}}+\cdots}\right.}{21\,a^{54}}$$

input
```
integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3)
 + b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(
5/2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) +
b)*a^48*b^4)/a^54
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

3.197 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.197.1 Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}$$

output
```
-6*x/a/(b*x^(2/3)+a*x)^(1/2)+8*(b*x^(2/3)+a*x)^(1/2)/a^2-16*b*(b*x^(2/3)+a
*x)^(1/2)/a^3/x^(1/3)
```

3.197.2 Mathematica [A] (verified)

Time = 5.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{2\left(-8b^2\sqrt[3]{x}-4abx^{2/3}+a^2x\right)}{a^3\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[x/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(2*(-8*b^2*x^(1/3) - 4*a*b*x^(2/3) + a^2*x))/(a^3*Sqrt[b*x^(2/3) + a*x])
```

3.197.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1921, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{1}{\sqrt{x^{2/3}b+ax}}dx}{a}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1908}$$

$$\frac{4\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{2b\int \frac{1}{\sqrt[3]{x}\sqrt{x^{2/3}b+ax}}dx}{3a}\right)}{a}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}\right)}{a}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

input
```
Int[x/(b*x^(2/3) + a*x)^(3/2),x]
```

output
```
(-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (4*((2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*
Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))))/a
```

3.197.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.197.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(a^{2}x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^{2}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{3}}$	45
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(a^{2}x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^{2}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{3}}$	45

input
```
int(x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(b+a*x^(1/3))*(a^2*x^(2/3)-4*a*b*x^(1/3)-8*b^2)/(b*x^(2/3)+a*x)^(3/2)/
a^3
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(54) = 108.

Time = 123.33 (sec) , antiderivative size = 1107, normalized size of antiderivative = 16.28

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
((98304*a^3*b^10 + 81920*a^3*b^9 - 30720*a^3*b^8 + 1456*a^9 + 6144*(16*a^6
 - 3*a^3)*b^7 + 6784*(8*a^6 + a^3)*b^6 - 192*(236*a^6 + a^3)*b^5 + 24*(153
6*a^9 - 2512*a^6 - 3*a^3)*b^4 + 32*(576*a^9 + 379*a^6)*b^3 - 12*(2304*a^9
+ 61*a^6)*b^2 - 3*(10112*a^9 + 15*a^6)*b)*x^2 + (98304*b^13 + 81920*b^12 +
 6144*(16*a^3 - 3)*b^10 - 30720*b^11 + 6784*(8*a^3 + 1)*b^9 + 1456*a^6*b^3
 - 192*(236*a^3 + 1)*b^8 + 24*(1536*a^6 - 2512*a^3 - 3)*b^7 + 32*(576*a^6
+ 379*a^3)*b^6 - 12*(2304*a^6 + 61*a^3)*b^5 - 3*(10112*a^6 + 15*a^3)*b^4)*
x + 2*((4096*a^4*b^9 + 6144*a^4*b^8 + 768*a^4*b^7 - 4096*a^10 - 144*a^7*b^
2 + 216*a^7*b - 27*a^7 + 256*(16*a^7 - 7*a^4)*b^6 + 48*(128*a^7 - 3*a^4)*b
^5 + 24*(32*a^7 + 9*a^4)*b^4 - (5888*a^7 + 27*a^4)*b^3)*x^2 - 3*(4096*a^2*
b^11 + 6144*a^2*b^10 + 768*a^2*b^9 - 144*a^5*b^4 + 256*(16*a^5 - 7*a^2)*b^
8 + 216*a^5*b^3 + 48*(128*a^5 - 3*a^2)*b^7 + 24*(32*a^5 + 9*a^2)*b^6 - (58
88*a^5 + 27*a^2)*b^5 - (4096*a^8 + 27*a^5)*b^2)*x^(4/3) + 4*(4096*a*b^12 +
 6144*a*b^11 + 768*a*b^10 + 256*(16*a^4 - 7*a)*b^9 - 144*a^4*b^5 + 48*(128
*a^4 - 3*a)*b^8 + 216*a^4*b^4 + 24*(32*a^4 + 9*a)*b^7 - (5888*a^4 + 27*a)*
b^6 - (4096*a^7 + 27*a^4)*b^3)*x - (32768*b^13 + 49152*b^12 + 2048*(16*a^3
 - 7)*b^10 + 6144*b^11 + 384*(128*a^3 - 3)*b^9 - 1152*a^3*b^6 + 192*(32*a^
3 + 9)*b^8 + 1728*a^3*b^5 - 8*(5888*a^3 + 27)*b^7 - 8*(4096*a^6 + 27*a^3)*
b^4 + 5*(4096*a^3*b^10 + 6144*a^3*b^9 + 768*a^3*b^8 - 144*a^6*b^3 + 216*a^
6*b^2 + 256*(16*a^6 - 7*a^3)*b^7 + 48*(128*a^6 - 3*a^3)*b^6 + 24*(32*a^...
```

3.197.6 Sympy [F]

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(x/(a*x + b*x**(2/3))**(3/2), x)
```

3.197.7 Maxima [F]

$$\int \frac{x}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a*x + b*x^(2/3))^(3/2), x)
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x}{(bx^{2/3}+ax)^{3/2}}\,dx = \frac{16\,b^{\frac{3}{2}}}{a^3} - \frac{6\,b^2}{\sqrt{ax^{\frac{1}{3}}+b}a^3} + \frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6 - 6\,\sqrt{ax^{\frac{1}{3}}+b}a^6b\right)}{a^9}$$

input
```
integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
16*b^(3/2)/a^3 - 6*b^2/(sqrt(a*x^(1/3) + b)*a^3) + 2*((a*x^(1/3) + b)^(3/2
)*a^6 - 6*sqrt(a*x^(1/3) + b)*a^6*b)/a^9
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{x}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
int(x/(a*x + b*x^(2/3))^(3/2), x)
```

3.198 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.198.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

output
```
-6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)+6*x^(1/3)/b/(b*x
^(2/3)+a*x)^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\sqrt{bx^{2/3}+ax}}{b\left(b+a\sqrt[3]{x}\right)\sqrt[3]{x}}-\frac{6\text{arctanh}\left(\frac{\sqrt{bx^{2/3}+ax}}{\sqrt{b}\sqrt[3]{x}}\right)}{b^{3/2}}$$

input
```
Integrate[(b*x^(2/3) + a*x)^(-3/2),x]
```

output
```
(6*Sqrt[b*x^(2/3) + a*x])/(b*(b + a*x^(1/3))*x^(1/3)) - (6*ArcTanh[Sqrt[b*
x^(2/3) + a*x]/(Sqrt[b]*x^(1/3))])/b^(3/2)
```

3.198.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1912, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

input
```
Int[(b*x^(2/3) + a*x)^(-3/2),x]
```

output
```
(6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/b^(3/2)
```

3.198.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.198.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{6x\left(b+a\,x^{\frac{1}{3}}\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\sqrt{b+a\,x^{\frac{1}{3}}}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56
default	$-\frac{6x\left(b+a\,x^{\frac{1}{3}}\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\sqrt{b+a\,x^{\frac{1}{3}}}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56

input
```
int(1/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*x*(b+a*x^(1/3))*(arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*(b+a*x^(1/3))^(
1/2)-b^(3/2))/(b*x^(2/3)+a*x)^(3/2)/b^(5/2)
```

3.198.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.198.6 Sympy [F]

$$\int \frac{1}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*x**(2/3))**(-3/2), x)
```

3.198.7 Maxima [F]

$$\int \frac{1}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^(2/3))^(-3/2), x)
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{(bx^{2/3}+ax)^{3/2}}\,dx = \frac{6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{6\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)}{\sqrt{-b}b^{\frac{3}{2}}} + \frac{6}{\sqrt{ax^{\frac{1}{3}}+b}b}$$

input
```
integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sq
rt(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{b}{a\,x^{1/3}}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{3}{2};\ \frac{5}{2};\ -\frac{b}{a\,x^{1/3}}\right)}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}$$

input
```
int(1/(a*x + b*x^(2/3))^(3/2),x)
```

output
```
-(2*x*(b/(a*x^(1/3)) + 1)^(3/2)*hypergeom([3/2, 3/2], 5/2, -b/(a*x^(1/3)))
)/(a*x + b*x^(2/3))^(3/2)
```

3.199 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.199.1 Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}+\frac{105a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{9/2}}$$

output
```
105/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)+6/b/x^(2/
3)/(b*x^(2/3)+a*x)^(1/2)-7*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)+35/4*a*(b*x^(
2/3)+a*x)^(1/2)/b^3/x-105/8*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

3.199.2 Mathematica [A] (verified)

Time = 5.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{-\sqrt{b}\left(8b^3-14ab^2\sqrt[3]{x}+35a^2bx^{2/3}+105a^3x\right)+105a^3\sqrt{b+a\sqrt[3]{x}}\,x\operatorname{arctanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)}{8b^{9/2}x^{2/3}\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(-(Sqrt[b]*(8*b^3 - 14*a*b^2*x^(1/3) + 35*a^2*b*x^(2/3) + 105*a^3*x)) + 10
5*a^3*Sqrt[b + a*x^(1/3)]*x*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/(8*b^(9/
2)*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
```

3.199.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1929, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5a\left(-\frac{3a\int \frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int \frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$\Big\downarrow$ 1935

$$\frac{7\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

$\Big\downarrow$ 219

3.199. $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\frac{7\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{b}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

input
```
Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) + (7*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/
3))) - (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3)
 + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/b^(3/2)))/(4*b)))/(6*b)))/b
```

3.199.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.199.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

method	result	size
derivativedivides	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(105\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{3}x+14b^{\frac{5}{2}}a\,x^{\frac{1}{3}}-35b^{\frac{3}{2}}a^{2}x^{\frac{2}{3}}-105\sqrt{b}\,a^{3}x-8b^{\frac{7}{2}}\right)}{8\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{9}{2}}}$	88
default	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(105\sqrt{b}\,a^{3}x+35b^{\frac{3}{2}}a^{2}x^{\frac{2}{3}}-14b^{\frac{5}{2}}a\,x^{\frac{1}{3}}-105\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{3}x+8b^{\frac{7}{2}}\right)}{8\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{9}{2}}}$	88

input
```
int(1/x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(b+a*x^(1/3))*(105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*(b+a*x^(1/3))^
(1/2)*a^3*x+14*b^(5/2)*a*x^(1/3)-35*b^(3/2)*a^2*x^(2/3)-105*b^(1/2)*a^3*x-
8*b^(7/2))/(b*x^(2/3)+a*x)^(3/2)/b^(9/2)
```

3.199.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.199.6 Sympy [F]

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)
```

3.199.7 Maxima [F]

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{1}{x(bx^{2/3}+ax)^{3/2}}\,dx = -\frac{105\,a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8\sqrt{-b}b^4} - \frac{6\,a^3}{\sqrt{ax^{\frac{1}{3}}+b}b^4} - \frac{57\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^3 - 136\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3b + 87\sqrt{ax^{\frac{1}{3}}+b}a^3b^2}{8\,a^3b^4x}$$

input
```
integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sq
rt(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3
) + b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{1}{x\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(2/3))^(3/2)), x)
```

3.200 $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.200.1 Optimal result

Integrand size = 19, antiderivative size = 236

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}-\frac{3003a^4\sqrt{bx^{2/3}+ax}}{256b^6x}+\frac{9009a^5\sqrt{bx^{2/3}+ax}}{512b^7x^{2/3}}-\frac{9009a^6\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{15/2}}$$

output
```
-9009/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)+6/b/
x^(5/3)/(b*x^(2/3)+a*x)^(1/2)-13/2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)+143/2
0*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2-1287/160*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x
^(5/3)+3003/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)-3003/256*a^4*(b*x^(2
/3)+a*x)^(1/2)/b^6/x+9009/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

3.200.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6a^6\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},7,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^7\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sq
rt[b*x^(2/3) + a*x])
```

3.200.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1929, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{13\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{13\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{13\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int \frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1931}$$

3.200. $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\frac{13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{b}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1931

3.200. $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$13\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\cdots\right.$$

3.200. $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.200. $\downarrow$ 1935

3.200. $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int\frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)\right.$$

3.200. $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$\downarrow$ 219

3.200. $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(\left.11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right.\right.\right.$$

3.200. $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

input
```
Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) + (13*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x
^(7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x
^(2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3)))
- (5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*
x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b
^(3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/b
```

3.200.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.200.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{\left(b+a x^{\frac{1}{3}}\right)\left(2288 b^{\frac{9}{2}} a^{2} x^{\frac{2}{3}}-3432 b^{\frac{7}{2}} a^{3} x+6006 b^{\frac{5}{2}} a^{4} x^{\frac{4}{3}}-15015 b^{\frac{3}{2}} a^{5} x^{\frac{5}{3}}+1280 b^{\frac{13}{2}}-45045 a^{6} x^{2} \sqrt{b}+45045 \sqrt{b+a x^{\frac{1}{3}}}\ \mathrm{ar}\ldots\right.}{2560 x\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{15}{2}}}$
default	$\frac{\left(b+a x^{\frac{1}{3}}\right)\left(-2288 b^{\frac{9}{2}} a^{2} x^{\frac{2}{3}}-1280 b^{\frac{13}{2}}-45045 \sqrt{b+a x^{\frac{1}{3}}}\ \operatorname{arctanh}\left(\frac{\sqrt{b+a x^{\frac{1}{3}}}}{\sqrt{b}}\right) a^{6} x^{2}+3432 b^{\frac{7}{2}} a^{3} x-6006 b^{\frac{5}{2}} a^{4} x^{\frac{4}{3}}+15015 b\ldots\right.}{2560 x\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{15}{2}}}$

input
```
int(1/x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2560*(b+a*x^(1/3))*(2288*b^(9/2)*a^2*x^(2/3)-3432*b^(7/2)*a^3*x+6006*b^
(5/2)*a^4*x^(4/3)-15015*b^(3/2)*a^5*x^(5/3)+1280*b^(13/2)-45045*a^6*x^2*b^
(1/2)+45045*(b+a*x^(1/3))^(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^6*x
^2-1664*b^(11/2)*a*x^(1/3))/x/(b*x^(2/3)+a*x)^(3/2)/b^(15/2)
```

3.200.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.200.6 Sympy [F]

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)
```

3.200.7 Maxima [F]

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{9009\,a^6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{512\,\sqrt{-b}b^7}+\frac{6\,a^6}{\sqrt{ax^{\frac{1}{3}}+b}\,b^7}+\frac{29685\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^6-163095\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6b+364194\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b^2-416094\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^3+246}{2560\,a^6b^7x^2}$$

input
```
integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(
sqrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 1630
95*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 41
6094*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4
- 62475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}} \, dx = \int \frac{1}{x^2 \left(a\, x+b\, x^{2/3}\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(2/3))^(3/2)), x)
```

3.201 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.201.1 Optimal result

Integrand size = 19, antiderivative size = 324

$$\begin{aligned}\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} \\ &+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}} \\ &-\frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2}+\frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}}-\frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}} \\ &+\frac{230945a^7\sqrt{bx^{2/3}+ax}}{16384b^9x}-\frac{692835a^8\sqrt{bx^{2/3}+ax}}{32768b^{10}x^{2/3}}+\frac{692835a^9\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{21/2}}\end{aligned}$$

output
```
692835/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)+6
/b/x^(8/3)/(b*x^(2/3)+a*x)^(1/2)-19/3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)+3
23/48*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3-1615/224*a^2*(b*x^(2/3)+a*x)^(1/2)/b
^4/x^(8/3)+20995/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)-46189/5376*a^4
*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2+138567/14336*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/
x^(5/3)-46189/4096*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)+230945/16384*a^7*
(b*x^(2/3)+a*x)^(1/2)/b^9/x-692835/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^10/x^
(2/3)
```

3.201.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6a^9\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},10,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{10}\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(-6*a^9*x^(1/3)*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^(1/3))/b])/(b^10
*Sqrt[b*x^(2/3) + a*x])
```

3.201.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1929, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1929}$$

$$\frac{19\int \frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{19\left(-\frac{17a\int \frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{19\left(-\frac{17a\left(-\frac{15a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1931

$$\frac{19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int \frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1931

$$\frac{19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int \frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$\big\downarrow$ 1931

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\frac{19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int \frac{1}{x^2\sqrt{x^{2/3}b+ax}}\,dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{b}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

$\downarrow$ 1931

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$
\left(19\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}\,dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}\right.\right.
$$

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1931

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(\quad 17a \left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int\frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}\,dx}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b} \right. \right.$$

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1931

3.201. $\int \frac{1}{x^3 \left(b x^{2/3}+a x\right)^{3/2}} \, dx$

$$\left(\int \left(15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}\,dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+b\ldots}}{2bx^{7\ldots}}\right)}{14b}\ldots\right.\right.\right.$$

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$ ↓ 1931

$$\left(\left(\left(\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\right.}{12b}\right.\right.\right.\right.\right.$$

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1935

3.201. $\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}} \, dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}$$

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$\downarrow$ 219

3.201. $\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}} \, dx$

$$11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right.}{10b}\right.$$

3.201. $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

input
```
Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) + (19*(-1/3*Sqrt[b*x^(2/3) + a*x]/(b*x
^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((-3*Sqrt[b
*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(7
/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(2
/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) - (
5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x])
/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(3
/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b)))
/b
```

3.201.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.201.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.49

method	result
derivativedivides	$\frac{\left(b+a x^{\frac{1}{3}}\right)\left(-229376 b^{\frac{19}{2}}+272384 b^{\frac{17}{2}} a x^{\frac{1}{3}}-330752 b^{\frac{15}{2}} a^{2} x^{\frac{2}{3}}+413440 b^{\frac{13}{2}} a^{3} x-537472 b^{\frac{11}{2}} a^{4} x^{\frac{4}{3}}+739024 b^{\frac{9}{2}} a^{5} x^{\frac{5}{3}}-1108\right.}{688128 x^{2}\left(b x\right.}$
default	$-\frac{\left(b+a x^{\frac{1}{3}}\right)\left(-272384 b^{\frac{17}{2}} a x^{\frac{1}{3}}+229376 b^{\frac{19}{2}}+330752 b^{\frac{15}{2}} a^{2} x^{\frac{2}{3}}-413440 b^{\frac{13}{2}} a^{3} x+537472 b^{\frac{11}{2}} a^{4} x^{\frac{4}{3}}-739024 b^{\frac{9}{2}} a^{5} x^{\frac{5}{3}}+1\right.}{688128 x^{2}\left(\right.}$

input
```
int(1/x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/688128*(b+a*x^(1/3))*(-229376*b^(19/2)+272384*b^(17/2)*a*x^(1/3)-330752*
b^(15/2)*a^2*x^(2/3)+413440*b^(13/2)*a^3*x-537472*b^(11/2)*a^4*x^(4/3)+739
024*b^(9/2)*a^5*x^(5/3)-1108536*b^(7/2)*a^6*x^2+1939938*b^(5/2)*a^7*x^(7/3
)-4849845*b^(3/2)*a^8*x^(8/3)-14549535*a^9*x^3*b^(1/2)+14549535*arctanh((b
+a*x^(1/3))^(1/2)/b^(1/2))*(b+a*x^(1/3))^(1/2)*a^9*x^3)/x^2/(b*x^(2/3)+a*x
)^(3/2)/b^(21/2)
```

3.201.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.201.6 Sympy [F]

$$\int \frac{1}{x^3 (bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{1}{x^3 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)
```

3.201.7 Maxima [F]

$$\int \frac{1}{x^3 (bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3 (bx^{2/3}+ax)^{3/2}}\,dx = -\frac{692835\,a^9 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768\sqrt{-b}b^{10}} - \frac{6\,a^9}{\sqrt{ax^{\frac{1}{3}}+b}b^{10}} - \frac{10420767\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^9 - 88937058\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^9 b + 334408914\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^9 b^2 - 724860666\left(ax^{\frac{1}{3}}+\right.}{}$$

input
```
integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6
*a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2
)*a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^
(13/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x
^(1/3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 5037
30990*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*
b^7 + 26738145*sqrt(a*x^(1/3) + b)*a^9*b^8)/(a^9*b^10*x^3)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(2/3))^(3/2)), x)
```

3.202 $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.202.1 Optimal result

Integrand size = 19, antiderivative size = 412

$$\begin{aligned}\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx =& \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}\\ &-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3}\\ &+\frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}}-\frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}}+\frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2}\\ &-\frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}}+\frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}}-\frac{16900975a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^{12}x}\\ &+\frac{50702925a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{13}x^{2/3}}-\frac{50702925a^{12}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{27/2}}\end{aligned}$$

output
```
-50702925/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(2
7/2)+6/b/x^(11/3)/(b*x^(2/3)+a*x)^(1/2)-25/4*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(
13/3)+575/88*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^4-2415/352*a^2*(b*x^(2/3)+a*x)^
(1/2)/b^4/x^(11/3)+15295/2112*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(10/3)-26001
5/33792*a^4*(b*x^(2/3)+a*x)^(1/2)/b^6/x^3+185725/22528*a^5*(b*x^(2/3)+a*x)
^(1/2)/b^7/x^(8/3)-2414425/270336*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(7/3)+48
2885/49152*a^7*(b*x^(2/3)+a*x)^(1/2)/b^9/x^2-1448655/131072*a^8*(b*x^(2/3)
+a*x)^(1/2)/b^10/x^(5/3)+3380195/262144*a^9*(b*x^(2/3)+a*x)^(1/2)/b^11/x^(
4/3)-16900975/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^12/x+50702925/2097152*a
^11*(b*x^(2/3)+a*x)^(1/2)/b^13/x^(2/3)
```

3.202.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6a^{12}\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},13,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{13}\sqrt{bx^{2/3}+ax}}$$

input
```
Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
(6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13
*Sqrt[b*x^(2/3) + a*x])
```

3.202.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {1929, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1931, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(ax+bx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{25\int \frac{1}{x^{14/3}\sqrt{x^{2/3}b+ax}}dx}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{25\left(-\frac{23a\int \frac{1}{x^{13/3}\sqrt{x^{2/3}b+ax}}dx}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\int\frac{1}{x^4\sqrt{x^{2/3}b+ax}}dx}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\int\frac{1}{x^{11/3}\sqrt{x^{2/3}b+ax}}dx}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \; 1931$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\int\frac{1}{x^{10/3}\sqrt{x^{2/3}b+ax}}dx}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\Big\downarrow \text{1931}$$

$$\frac{25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\int \frac{1}{x^3\sqrt{x^{2/3}b+ax}}dx}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}-\frac{\sqrt{ax+bx^{2/3}}}{4bx^{13/3}}\right)}{b}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

$$\Big\downarrow \text{1931}$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(25\left(-\frac{23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\int\frac{1}{x^{8/3}\sqrt{x^{2/3}b+ax}}dx}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}\right)}{22b}-\frac{3\sqrt{ax+bx^{2/3}}}{11bx^4}\right)}{24b}\right.\right.$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1931

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\Bigg(\;\Bigg(\;23a\left(-\frac{21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\int\frac{1}{x^{7/3}\sqrt{x^{2/3}b+ax}}\,dx}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^{3}}\right)}{18b}-\frac{\sqrt{ax+bx^{2/3}}}{3bx^{10/3}}\right)}{20b}-\frac{3\sqrt{ax+b}}{10bx^{11}}\cdots\right)}{22b}\cdots$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1931

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$21a\left(-\frac{19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\int\frac{1}{x^2\sqrt{x^{2/3}b+ax}}\,dx}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^2}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\right)}{16b}-\frac{3\sqrt{ax+bx^{2/3}}}{8bx^3}\right)}{18b}-\frac{\sqrt{a\cdots}}{3\cdots}\right)}{20b}\cdots\right)$$

3.202. $\int\frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

3.202. $\downarrow$ 1931

3.202. $\int \frac{1}{x^4 \left(b x^{2/3}+a x\right)^{3/2}} \, dx$

$$\left(\left(\left(\left(19a\left(-\frac{17a\left(-\frac{15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\int\frac{1}{x^{5/3}\sqrt{x^{2/3}b+ax}}\,dx}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b}-\frac{\sqrt{ax+bx^{2/3}}}{2bx^{7/3}}\right)}{14b}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}} \cdots}{16b} \cdots}{18b} \cdots$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1931

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(\quad\left(\quad\left(\quad\left(\quad\left(\quad 17a\left(\quad 15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\int \frac{1}{x^{4/3}\sqrt{x^{2/3}b+ax}}dx}{6b} - \frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b} - \frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b} - \frac{3\sqrt{ax+bx^{2/3}}}{5bx^{2}}\right)}{12b} - \frac{\sqrt{ax+\cdots}}{2bx\cdots}}{14b}\cdots\right.\right.\right.\right.\right.\right.\right.$$

$$\cdots -\frac{\cdots}{16b}$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$\downarrow$ 1931

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(\left(\left(\left(\left(15a\left(-\frac{13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\int\frac{1}{x\sqrt{x^{2/3}b+ax}}dx}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\right)}{8b}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}\right)}{10b}-\frac{3\sqrt{\cdots}}{\cdots}}{12b}\cdots}{14b}\cdots\right.\right.\right.\right.\right.\right.$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1931

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(\left(\left(\left(\left(\left(13a\left(-\frac{11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(-\frac{a\int\frac{1}{x^{2/3}\sqrt{x^{2/3}b+ax}}\,dx}{2b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2/3}}}{bx^{4/3}}\cdots}{8b}\cdots}{10b}\cdots}{12b}\cdots$$

3.202. $\int\frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

↓ 1935

3.202. $\int \frac{1}{x^4 \left(b x^{2/3}+a x\right)^{3/2}}\, dx$

$$\frac{3a\left(\frac{3a\int \frac{1}{1-\frac{bx^{2/3}}{x^{2/3}b+ax}}\,d\frac{\sqrt[3]{x}}{\sqrt{x^{2/3}b+ax}}}{b}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}$$

$$5a\left(-\cdots\right)$$

$$7a\left(-\frac{5a(\cdots)}{6b}-\frac{\sqrt{ax+\cdots}}{bx^{4}\cdots}\right)$$

$$9a\left(-\frac{7a(\cdots)}{8b}\cdots\right)$$

$$11a\left(-\frac{9a(\cdots)}{10b}\cdots\right)$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$\downarrow$ 219

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

$$\left(\left(\left(\left(\left(\left(\left(\left(\left(11a\left(-\frac{9a\left(-\frac{7a\left(-\frac{5a\left(-\frac{3a\left(\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}\right)}{4b}-\frac{3\sqrt{ax+bx^{2/3}}}{2bx}\right)}{6b}-\frac{\sqrt{ax+bx^{2}}}{bx^{4/3}}}{8b}}{10b}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

3.202. $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

input
```
Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

output
```
6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) + (25*(-1/4*Sqrt[b*x^(2/3) + a*x]/(b*
x^(13/3)) - (23*a*((-3*Sqrt[b*x^(2/3) + a*x])/(11*b*x^4) - (21*a*((-3*Sqrt
[b*x^(2/3) + a*x])/(10*b*x^(11/3)) - (19*a*(-1/3*Sqrt[b*x^(2/3) + a*x]/(b*
x^(10/3)) - (17*a*((-3*Sqrt[b*x^(2/3) + a*x])/(8*b*x^3) - (15*a*((-3*Sqrt[
b*x^(2/3) + a*x])/(7*b*x^(8/3)) - (13*a*(-1/2*Sqrt[b*x^(2/3) + a*x]/(b*x^(
7/3)) - (11*a*((-3*Sqrt[b*x^(2/3) + a*x])/(5*b*x^2) - (9*a*((-3*Sqrt[b*x^(
2/3) + a*x])/(4*b*x^(5/3)) - (7*a*(-(Sqrt[b*x^(2/3) + a*x]/(b*x^(4/3))) -
(5*a*((-3*Sqrt[b*x^(2/3) + a*x])/(2*b*x) - (3*a*((-3*Sqrt[b*x^(2/3) + a*x]
)/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/b^(
3/2)))/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b))
)/(20*b)))/(22*b)))/(24*b)))/b
```

3.202.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.202.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.47

method	result
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(17301504 b^{\frac{25}{2}}+1673196525 \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\, a^{12} x^{4}-1673196525 a^{12} x^{4}\sqrt{b}-19660800 b^{\frac{23}{2}} a\right.}{}$
default	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(1673196525 a^{12} x^{4}\sqrt{b}-17301504 b^{\frac{25}{2}}-1673196525 \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\, a^{12} x^{4}+19660800 b^{\frac{23}{2}} a\, x\right.}{}$

input
```
int(1/x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/69206016*(b+a*x^(1/3))*(17301504*b^(25/2)+1673196525*arctanh((b+a*x^(1/
3))^(1/2)/b^(1/2))*(b+a*x^(1/3))^(1/2)*a^12*x^4-1673196525*a^12*x^4*b^(1/2
)-19660800*b^(23/2)*a*x^(1/3)+22609920*b^(21/2)*a^2*x^(2/3)-26378240*b^(19
/2)*a^3*x+31324160*b^(17/2)*a^4*x^(4/3)-38036480*b^(15/2)*a^5*x^(5/3)+4754
5600*b^(13/2)*a^6*x^2-61809280*b^(11/2)*a^7*x^(7/3)+84987760*b^(9/2)*a^8*x
^(8/3)-127481640*b^(7/2)*a^9*x^3+223092870*b^(5/2)*a^10*x^(10/3)-557732175
*b^(3/2)*a^11*x^(11/3))/x^3/(b*x^(2/3)+a*x)^(3/2)/b^(27/2)
```

3.202.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.202.6 Sympy [F]

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**4*(a*x + b*x**(2/3))**(3/2)), x)
```

3.202.7 Maxima [F]

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{50702925\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{2097152\,\sqrt{-b}b^{13}} + \frac{6\,a^{12}}{\sqrt{ax^{\frac{1}{3}}+b}b^{13}} + \frac{1257960429\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{12} - 14537792973\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}b + 76667241519\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b^2 - 2437176}{}$$

input
```
integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

output
```
50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13)
 + 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) +
 b)^(23/2)*a^12 - 14537792973*(a*x^(1/3) + b)^(21/2)*a^12*b + 76667241519*
(a*x^(1/3) + b)^(19/2)*a^12*b^2 - 243717614415*(a*x^(1/3) + b)^(17/2)*a^12
*b^3 + 519393101810*(a*x^(1/3) + b)^(15/2)*a^12*b^4 - 780150847218*(a*x^(1
/3) + b)^(13/2)*a^12*b^5 + 844265343246*(a*x^(1/3) + b)^(11/2)*a^12*b^6 -
659969685518*(a*x^(1/3) + b)^(9/2)*a^12*b^7 + 366679446705*(a*x^(1/3) + b)
^(7/2)*a^12*b^8 - 138840292305*(a*x^(1/3) + b)^(5/2)*a^12*b^9 + 3266070993
9*(a*x^(1/3) + b)^(3/2)*a^12*b^10 - 3724872723*sqrt(a*x^(1/3) + b)*a^12*b^
11)/(a^12*b^13*x^4)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(2/3))^(3/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(2/3))^(3/2)), x)
```

3.203 $\int x^2(ax^2+bx^3)\,dx$

3.203.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

output
```
1/5*a*x^5+1/6*b*x^6
```

3.203.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6
```

3.203.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a x^2+b x^3\right) \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^4 (a+b x)dx$$

$$\Big\downarrow \; 49$$

$$\int \left(a x^4+b x^5\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a x^5}{5}+\frac{b x^6}{6}$$

input
```
Int[x^2*(a*x^2 + b*x^3),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6
```

3.203.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.203.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^5(5bx+6a)}{30}$	14
default	$\frac{1}{5}x^5a+\frac{1}{6}b\,x^6$	14
norman	$\frac{1}{5}x^5a+\frac{1}{6}b\,x^6$	14
risch	$\frac{1}{5}x^5a+\frac{1}{6}b\,x^6$	14
parallelrisch	$\frac{1}{5}x^5a+\frac{1}{6}b\,x^6$	14

input
```
int(x^2*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^5*(5*b*x+6*a)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/5*a*x^5
```

3.203.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

input
```
integrate(x**2*(b*x**3+a*x**2),x)
```

output
```
a*x**5/5 + b*x**6/6
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3)\ dx = \frac{1}{6}\,bx^6 + \frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/6*b*x^6 + 1/5*a*x^5
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3)\ dx = \frac{1}{6}\,bx^6 + \frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/6*b*x^6 + 1/5*a*x^5
```

3.203.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3)\ dx = \frac{x^5\,(6\,a+5\,b\,x)}{30}$$

input
```
int(x^2*(a*x^2 + b*x^3),x)
```

output
```
(x^5*(6*a + 5*b*x))/30
```

3.204 $\int x(ax^2 + bx^3) \, dx$

3.204.1 Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x(ax^2 + bx^3) \, dx = \frac{ax^4}{4} + \frac{bx^5}{5}$$

output
```
1/4*a*x^4+1/5*b*x^5
```

3.204.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(ax^2 + bx^3) \, dx = \frac{ax^4}{4} + \frac{bx^5}{5}$$

input
```
Integrate[x*(a*x^2 + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5
```

3.204.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(ax^2+bx^3)\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^3(a+bx)dx$$

$$\Big\downarrow \; 49$$

$$\int (ax^3+bx^4)\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

input
```
Int[x*(a*x^2 + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5
```

3.204.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.204.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^4(4bx+5a)}{20}$	14
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
parallelrisch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14

input
```
int(x*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/20*x^4*(4*b*x+5*a)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3)\ dx = \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

input
```
integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

3.204.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(ax^2+bx^3)\ dx = \frac{ax^4}{4} + \frac{bx^5}{5}$$

input
```
integrate(x*(b*x**3+a*x**2),x)
```

output
```
a*x**4/4 + b*x**5/5
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x*(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

3.204.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3)\,dx=\frac{x^4\,(5\,a+4\,b\,x)}{20}$$

input
```
int(x*(a*x^2 + b*x^3),x)
```

output
```
(x^4*(5*a + 4*b*x))/20
```

3.205 $\int (ax^2 + bx^3) \, dx$

3.205.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

output
```
1/3*a*x^3+1/4*b*x^4
```

3.205.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
Integrate[a*x^2 + b*x^3,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

3.205.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

input
```
Int[a*x^2 + b*x^3,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

3.205.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.205.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^3(3bx+4a)}{12}$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
parts	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14

input
```
int(b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(3*b*x+4*a)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3)\, dx = \frac{1}{4}\, bx^4 + \frac{1}{3}\, ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.205.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int (ax^2 + bx^3)\, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
integrate(b*x**3+a*x**2,x)
```

output
```
a*x**3/3 + b*x**4/4
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3)\, dx = \frac{1}{4}\, bx^4 + \frac{1}{3}\, ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.205.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right)\,dx=\frac{x^3\,(4\,a+3\,b\,x)}{12}$$

input
```
int(a*x^2 + b*x^3,x)
```

output
```
(x^3*(4*a + 3*b*x))/12
```

3.206 $\int \frac{ax^2+bx^3}{x} \, dx$

3.206.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{ax^2 + bx^3}{x} \, dx = \frac{ax^2}{2} + \frac{bx^3}{3}$$

output
```
1/2*a*x^2+1/3*b*x^3
```

3.206.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3}{x} \, dx = \frac{ax^2}{2} + \frac{bx^3}{3}$$

input
```
Integrate[(a*x^2 + b*x^3)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3
```

3.206.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3}{x} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x(a+bx)dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(ax+bx^2\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

input
```
Int[(a*x^2 + b*x^3)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3
```

3.206.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.206.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^2(2bx+3a)}{6}$	14
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14

input
```
int((b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2*(2*b*x+3*a)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3}{x}\,dx = \frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.206.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{ax^2+bx^3}{x}\,dx = \frac{ax^2}{2}+\frac{bx^3}{3}$$

input
```
integrate((b*x**3+a*x**2)/x,x)
```

output
```
a*x**2/2 + b*x**3/3
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3}{x} \, dx = \frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

input
```
integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3}{x} \, dx = \frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

input
```
integrate((b*x^3+a*x^2)/x,x, algorithm="giac")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3}{x} \, dx = \frac{x^2 (3 a + 2 b x)}{6}$$

input
```
int((a*x^2 + b*x^3)/x,x)
```

output
```
(x^2*(3*a + 2*b*x))/6
```

3.207 $\int \frac{ax^2+bx^3}{x^2}\,dx$

3.207.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{ax^2+bx^3}{x^2}\,dx = ax+\frac{bx^2}{2}$$

output
```
a*x+1/2*b*x^2
```

3.207.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3}{x^2}\,dx = ax+\frac{bx^2}{2}$$

input
```
Integrate[(a*x^2 + b*x^3)/x^2,x]
```

output
```
a*x + (b*x^2)/2
```

3.207.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3}{x^2} \, dx$$
$$\Big\downarrow \; 9$$
$$\int (a+bx)dx$$
$$\Big\downarrow \; 17$$
$$\frac{(a+bx)^2}{2b}$$

input
```
Int[(a*x^2 + b*x^3)/x^2,x]
```

output
```
(a + b*x)^2/(2*b)
```

3.207.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.207.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x(bx+2a)}{2}$	11
default	$ax+\frac{1}{2}b\,x^2$	11
risch	$ax+\frac{1}{2}b\,x^2$	11
parallelrisch	$ax+\frac{1}{2}b\,x^2$	11
parts	$ax+\frac{1}{2}b\,x^2$	11
norman	$\frac{a\,x^2+\frac{1}{2}b\,x^3}{x}$	17

input
```
int((b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x+2*a)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3}{x^2}\,dx = \frac{1}{2}bx^2+ax$$

input
```
integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*x
```

3.207.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{ax^2+bx^3}{x^2}\,dx = ax+\frac{bx^2}{2}$$

input
```
integrate((b*x**3+a*x**2)/x**2,x)
```

output
```
a*x + b*x**2/2
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2 + bx^3}{x^2} \, dx = \frac{1}{2} bx^2 + ax$$

input
```
integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2 + bx^3}{x^2} \, dx = \frac{1}{2} bx^2 + ax$$

input
```
integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

3.207.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2 + bx^3}{x^2} \, dx = \frac{b\,x^2}{2} + a\,x$$

input
```
int((a*x^2 + b*x^3)/x^2,x)
```

output
```
a*x + (b*x^2)/2
```

3.208 $\int x^2 (ax^2 + bx^3)^2 \, dx$

3.208.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int x^2 \left(ax^2 + bx^3\right)^2 \, dx = \frac{a^2 x^7}{7} + \frac{1}{4} a b x^8 + \frac{b^2 x^9}{9}$$

output
```
1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

3.208.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2 \left(ax^2 + bx^3\right)^2 \, dx = \frac{a^2 x^7}{7} + \frac{1}{4} a b x^8 + \frac{b^2 x^9}{9}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

3.208.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3\right)^2 \, dx$$
$$\Big\downarrow \; 9$$
$$\int x^6 (a+bx)^2 dx$$
$$\Big\downarrow \; 49$$
$$\int \left(a^2x^6+2abx^7+b^2x^8\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

input
```
Int[x^2*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

3.208.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.208.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^7(28b^2x^2+63abx+36a^2)}{252}$	25
default	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
parallelrisch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25

input
```
int(x^2*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/252*x^7*(28*b^2*x^2+63*a*b*x+36*a^2)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(ax^2+bx^3)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

3.208.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(ax^2+bx^3)^2\,dx=\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{b^2x^9}{9}$$

input
```
integrate(x**2*(b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(ax^2+bx^3)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(ax^2+bx^3)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

3.208.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(ax^2+bx^3)^2\,dx=\frac{a^2\,x^7}{7}+\frac{a\,b\,x^8}{4}+\frac{b^2\,x^9}{9}$$

input
```
int(x^2*(a*x^2 + b*x^3)^2,x)
```

output
```
(a^2*x^7)/7 + (b^2*x^9)/9 + (a*b*x^8)/4
```

3.209 $\int x(ax^2+bx^3)^2\,dx$

3.209.1 Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

output
```
1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

3.209.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

input
```
Integrate[x*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

3.209.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(ax^2+bx^3)^2\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^5(a+bx)^2dx$$

$$\Big\downarrow \; 49$$

$$\int \left(a^2x^5+2abx^6+b^2x^7\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

input
```
Int[x*(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

3.209.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.209.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^6(21b^2x^2+48abx+28a^2)}{168}$	25
default	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25

input
```
int(x*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/168*x^6*(21*b^2*x^2+48*a*b*x+28*a^2)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2+bx^3)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

3.209.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(ax^2+bx^3)^2\,dx=\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{b^2x^8}{8}$$

input
```
integrate(x*(b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2+bx^3)^2\,dx = \frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2+bx^3)^2\,dx = \frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(ax^2+bx^3)^2\,dx = \frac{a^2\,x^6}{6}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^8}{8}$$

input
```
int(x*(a*x^2 + b*x^3)^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*x^8)/8 + (2*a*b*x^7)/7
```

3.210 $\int (ax^2+bx^3)^2\, dx$

3.210.1 Optimal result

Integrand size = 13, antiderivative size = 30

$$\int (ax^2+bx^3)^2\, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

output
```
1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

3.210.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (ax^2+bx^3)^2\, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

input
```
Integrate[(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

3.210.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2027, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^2 \, dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^4(a+bx)^2dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2x^4+2abx^5+b^2x^6\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

input
```
Int[(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

3.210.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.210.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^5(15b^2x^2+35abx+21a^2)}{105}$	25
default	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25
parallelrisch	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25

input
```
int((b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x^5*(15*b^2*x^2+35*a*b*x+21*a^2)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

3.210.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{b^2x^7}{7}$$

input
```
integrate((b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2 \, dx = \frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2 \, dx = \frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

3.210.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2 \, dx = \frac{a^2\,x^5}{5}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^7}{7}$$

input
```
int((a*x^2 + b*x^3)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^7)/7 + (a*b*x^6)/3
```

3.211 $\int \frac{(ax^2+bx^3)^2}{x}\,dx$

3.211.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

3.211.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

input
```
Integrate[(a*x^2 + b*x^3)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

3.211.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^2}{x} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^3(a+bx)^2dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^2x^3+2abx^4+b^2x^5\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

input
```
Int[(a*x^2 + b*x^3)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

3.211.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.211.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^4(10b^2x^2+24abx+15a^2)}{60}$	25
default	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
norman	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25

input
```
int((b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^4*(10*b^2*x^2+24*a*b*x+15*a^2)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

3.211.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

input
```
integrate((b*x**3+a*x**2)**2/x,x)
```

output
```
a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx=\frac{a^2\,x^4}{4}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^6}{6}$$

input
```
int((a*x^2 + b*x^3)^2/x,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

3.212 $\int \frac{(ax^2+bx^3)^2}{x^2}\,dx$

3.212.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

3.212.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

input
```
Integrate[(a*x^2 + b*x^3)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

3.212.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^2(a+bx)^2dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(a^2x^2+2abx^3+b^2x^4\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

input
```
Int[(a*x^2 + b*x^3)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

3.212.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.212.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$	25
default	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
norman	$\frac{\frac{1}{3}a^2x^4+\frac{1}{5}b^2x^6+\frac{1}{2}ab\,x^5}{x}$	29

input
```
int((b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^3*(6*b^2*x^2+15*a*b*x+10*a^2)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

3.212.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

input
```
integrate((b*x**3+a*x**2)**2/x**2,x)
```

output
```
a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

3.212.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2 + bx^3)^2}{x^2} \, dx = \frac{a^2 x^3}{3} + \frac{a b x^4}{2} + \frac{b^2 x^5}{5}$$

input
```
int((a*x^2 + b*x^3)^2/x^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

3.213 $\int \frac{x^6}{ax^2+bx^3} \, dx$

3.213.1 Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^6}{ax^2+bx^3} \, dx = -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

output
```
-a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

3.213.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{ax^2+bx^3} \, dx = -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Lo
g[a + b*x])/b^5
```

3.213.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{ax^2+bx^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^4}{a+bx}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^4}{b^4(a+bx)}-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

input
```
Int[x^6/(a*x^2 + b*x^3),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Lo
g[a + b*x])/b^5
```

3.213.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.213.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{1}{3}a\,b^2x^3-\frac{1}{2}a^2b\,x^2+a^3x}{b^4}+\frac{a^4\ln(bx+a)}{b^5}$	52
risch	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52
parallelrisch	$\frac{3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx}{12b^5}$	53
norman	$\frac{\frac{x^5}{4b}-\frac{a\,x^4}{3b^2}+\frac{a^2x^3}{2b^3}-\frac{a^3x^2}{b^4}}{x}+\frac{a^4\ln(bx+a)}{b^5}$	59

input
```
int(x^6/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/4*b^3*x^4+1/3*a*b^2*x^3-1/2*a^2*b*x^2+a^3*x)+a^4*ln(b*x+a)/b^5
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{ax^2+bx^3}\,dx=\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

input
```
integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*
x + a))/b^5
```

3.213.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{a^4\log(a+bx)}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

input
```
integrate(x**6/(b*x**3+a*x**2),x)
```

output
```
a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2
) + x**4/(4*b)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{a^4\log(bx+a)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

input
```
integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^
3*x)/b^4
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{a^4\log(|bx+a|)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

input
```
integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 -
12*a^3*x)/b^4
```

3.213.9 Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{x^4}{4\,b} + \frac{a^4\,\ln(a+b\,x)}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

input
```
int(x^6/(a*x^2 + b*x^3),x)
```

output
```
x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*
x^2)/(2*b^3)
```

3.214 $\int \frac{x^5}{ax^2+bx^3} dx$

3.214.1 Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{x^5}{ax^2+bx^3} dx = \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3\log(a+bx)}{b^4}$$

output
```
a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

3.214.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{ax^2+bx^3} dx = \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3\log(a+bx)}{b^4}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3),x]
```

output
```
(a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

3.214.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^3}{a+bx}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

input
```
Int[x^5/(a*x^2 + b*x^3),x]
```

output
```
(a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

3.214.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.214.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x}{b^3}-\frac{a^3\ln(bx+a)}{b^4}$	41
risch	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41
parallelrisch	$-\frac{-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx}{6b^4}$	42
norman	$\frac{\frac{a^2x^2}{b^3}+\frac{x^4}{3b}-\frac{a\,x^3}{2b^2}}{x}-\frac{a^3\ln(bx+a)}{b^4}$	48

input
```
int(x^5/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2+a^2*x)-a^3*ln(b*x+a)/b^4
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{ax^2+bx^3}\,dx=\frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

input
```
integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

3.214.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{ax^2+bx^3}\,dx = -\frac{a^3\log{(a+bx)}}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

input
```
integrate(x**5/(b*x**3+a*x**2),x)
```

output
```
-a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{ax^2+bx^3}\,dx = -\frac{a^3\log{(bx+a)}}{b^4} + \frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

input
```
integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
-a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{ax^2+bx^3}\,dx = -\frac{a^3\log{(|bx+a|)}}{b^4} + \frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

input
```
integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

3.214.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{ax^2 + bx^3}\,dx = \frac{x^3}{3\,b} - \frac{a^3\,\ln\left(a + b\,x\right)}{b^4} - \frac{a\,x^2}{2\,b^2} + \frac{a^2\,x}{b^3}$$

input
```
int(x^5/(a*x^2 + b*x^3),x)
```

output
```
x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

3.215 $\int \frac{x^4}{ax^2+bx^3} dx$

3.215.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{x^4}{ax^2+bx^3} \, dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

output
```
-a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

3.215.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{ax^2+bx^3} \, dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

3.215.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax^2+bx^3}dx$$

$$\Big\downarrow 9$$

$$\int \frac{x^2}{a+bx}dx$$

$$\Big\downarrow 49$$

$$\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

input
```
Int[x^4/(a*x^2 + b*x^3),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

3.215.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.215.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}b\,x^2+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
parallelrisch	$\frac{b^2x^2+2a^2\ln(bx+a)-2abx}{2b^3}$	30
norman	$\frac{\frac{x^3}{2b}-\frac{a\,x^2}{b^2}}{x}+\frac{a^2\ln(bx+a)}{b^3}$	37

input
```
int(x^4/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*x^2+a*x)+a^2*ln(b*x+a)/b^3
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{b^2x^2-2\,abx+2\,a^2\log(bx+a)}{2\,b^3}$$

input
```
integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

3.215.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{a^2\log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

input
```
integrate(x**4/(b*x**3+a*x**2),x)
```

output
```
a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{a^2\log(bx+a)}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{a^2\log(|bx+a|)}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.215.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{2\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

input
```
int(x^4/(a*x^2 + b*x^3),x)
```

output
```
(2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.216 $\int \frac{x^3}{ax^2+bx^3}\,dx$

3.216.1 Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

output
```
x/b-a*ln(b*x+a)/b^2
```

3.216.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

3.216.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax^2+bx^3}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x}{a+bx}dx$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

input
```
Int[x^3/(a*x^2 + b*x^3),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

3.216.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.216.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
parallelrisch	$-\frac{a\ln(bx+a)-bx}{b^2}$	19

input
```
int(x^3/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x/b-a*ln(b*x+a)/b^2
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{bx-a\log(bx+a)}{b^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
(b*x - a*log(b*x + a))/b^2
```

3.216.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{ax^2+bx^3}\,dx=-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}$$

input
```
integrate(x**3/(b*x**3+a*x**2),x)
```

output
```
-a*log(a + b*x)/b**2 + x/b
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx = \frac{x}{b} - \frac{a\log(bx+a)}{b^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
x/b - a*log(b*x + a)/b^2
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{ax^2+bx^3}\,dx = \frac{x}{b} - \frac{a\log(|bx+a|)}{b^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
x/b - a*log(abs(b*x + a))/b^2
```

3.216.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx = -\frac{a\,\ln(a+b\,x)-b\,x}{b^2}$$

input
```
int(x^3/(a*x^2 + b*x^3),x)
```

output
```
-(a*log(a + b*x) - b*x)/b^2
```

3.217 $\int \frac{x^2}{ax^2+bx^3} dx$

3.217.1 Optimal result

Integrand size = 17, antiderivative size = 10

$$\int \frac{x^2}{ax^2+bx^3} dx = \frac{\log(a+bx)}{b}$$

output
```
ln(b*x+a)/b
```

3.217.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3} dx = \frac{\log(a+bx)}{b}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3),x]
```

output
```
Log[a + b*x]/b
```

3.217.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax^2+bx^3}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{1}{a+bx}dx$$

$$\Big\downarrow \ 16$$

$$\frac{\log(a+bx)}{b}$$

input
```
Int[x^2/(a*x^2 + b*x^3),x]
```

output
```
Log[a + b*x]/b
```

3.217.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.217.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11
parallelrisch	$\frac{\ln(bx+a)}{b}$	11

input
```
int(x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)/b
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx = \frac{\log(bx+a)}{b}$$

input
```
integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
log(b*x + a)/b
```

3.217.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{ax^2+bx^3}\,dx = \frac{\log(a+bx)}{b}$$

input
```
integrate(x**2/(b*x**3+a*x**2),x)
```

output
```
log(a + b*x)/b
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2 + bx^3}\,dx = \frac{\log(bx + a)}{b}$$

input
```
integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
log(b*x + a)/b
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{ax^2 + bx^3}\,dx = \frac{\log(|bx + a|)}{b}$$

input
```
integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b
```

3.217.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2 + bx^3}\,dx = \frac{\ln(a + b\,x)}{b}$$

input
```
int(x^2/(a*x^2 + b*x^3),x)
```

output
```
log(a + b*x)/b
```

3.218 $\int \frac{x}{ax^2+bx^3}\,dx$

3.218.1 Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x}{ax^2+bx^3}\,dx=\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

output
```
ln(x)/a-ln(b*x+a)/a
```

3.218.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2+bx^3}\,dx=\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

input
```
Integrate[x/(a*x^2 + b*x^3),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

3.218.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^2+bx^3}dx$$
$$\Big\downarrow 9$$
$$\int \frac{1}{x(a+bx)}dx$$
$$\Big\downarrow 47$$
$$\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}$$
$$\Big\downarrow 14$$
$$\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}$$
$$\Big\downarrow 16$$
$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

input
```
Int[x/(a*x^2 + b*x^3),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

3.218.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

3.218.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(bx+a)}{a}$	16
default	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
risch	$\frac{\ln(-x)}{a}-\frac{\ln(bx+a)}{a}$	21

input
```
int(x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)-ln(b*x+a))/a
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{ax^2+bx^3}\,dx = -\frac{\log(bx+a)-\log(x)}{a}$$

input
```
integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
-(log(b*x + a) - log(x))/a
```

3.218.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{x}{ax^2 + bx^3} \, dx = \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

input
```
integrate(x/(b*x**3+a*x**2),x)
```

output
```
(log(x) - log(a/b + x))/a
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2 + bx^3} \, dx = -\frac{\log(bx + a)}{a} + \frac{\log(x)}{a}$$

input
```
integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
-log(b*x + a)/a + log(x)/a
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x}{ax^2 + bx^3} \, dx = -\frac{\log(|bx + a|)}{a} + \frac{\log(|x|)}{a}$$

input
```
integrate(x/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a + log(abs(x))/a
```

3.218.9 Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x}{ax^2 + bx^3}\, dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a}$$

input
```
int(x/(a*x^2 + b*x^3),x)
```

output
```
-(2*atanh((2*b*x)/a + 1))/a
```

3.219 $\int \frac{1}{ax^2+bx^3}\,dx$

3.219.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{ax^2+bx^3}\,dx = -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx)}{a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

3.219.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^3}\,dx = -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx)}{a^2}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-1),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

3.219.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{1}{x^2(a+bx)}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

input
```
Int[(a*x^2 + b*x^3)^(-1),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

3.219.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.219.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

input
```
int(1/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{ax^2+bx^3}\,dx=\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

3.219.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

input
```
integrate(1/(b*x**3+a*x**2),x)
```

output
```
-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^3}\,dx = \frac{b\log(bx+a)}{a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{ax^2+bx^3}\,dx = \frac{b\log(|bx+a|)}{a^2} - \frac{b\log(|x|)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

3.219.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{ax^2+bx^3}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2} - \frac{1}{a\,x}$$

input
```
int(1/(a*x^2 + b*x^3),x)
```

output
```
(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.220 $\int \frac{1}{x(ax^2+bx^3)}\,dx$

3.220.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{x\,(ax^2+bx^3)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

3.220.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(ax^2+bx^3)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

3.220.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(ax^2+bx^3)}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^3(a+bx)}dx$$
$$\Big\downarrow \; 54$$
$$\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

input
```
Int[1/(x*(a*x^2 + b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

3.220.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.220.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b}{x\,a^{2}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	41
norman	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	41
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln\left(-x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	43
parallelrisch	$\frac{2b^{2}\ln\left(x\right)x^{2}-2b^{2}\ln\left(bx+a\right)x^{2}+2abx-a^{2}}{2a^{3}x^{2}}$	44

input
```
int(1/x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+b/x/a^2+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(ax^{2}+bx^{3}\right)}\,dx=-\frac{2\,b^{2}x^{2}\log\left(bx+a\right)-2\,b^{2}x^{2}\log\left(x\right)-2\,abx+a^{2}}{2\,a^{3}x^{2}}$$

input
```
integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

3.220.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx=\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

input
```
integrate(1/x/(b*x**3+a*x**2),x)
```

output
```
(-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx=-\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log\left(x\right)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

input
```
integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
-b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx=-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^
3*x^2)
```

3.220.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\,(ax^2+bx^3)}\,dx = -\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

input
```
int(1/(x*(a*x^2 + b*x^3)),x)
```

output
```
- (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.221 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$

3.221.1 Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

output
```
-1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

3.221.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log
[a + b*x])/a^4
```

3.221.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3)} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^4(a+bx)}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(\frac{b^4}{a^4(a+bx)}-\frac{b^3}{a^4x}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^3}+\frac{1}{ax^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^2*(a*x^2 + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log
[a + b*x])/a^4
```

3.221.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.221.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b}{2a^{2}x^{2}}-\frac{b^{2}}{a^{3}x}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}$	53
norman	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}$	53
parallelrisch	$-\frac{6b^{3}\ln\left(x\right)x^{3}-6b^{3}\ln\left(bx+a\right)x^{3}+6a\,b^{2}x^{2}-3a^{2}bx+2a^{3}}{6a^{4}x^{3}}$	55
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(-bx-a\right)}{a^{4}}$	56

input
```
int(1/x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\,(ax^2+bx^3)}\,dx=\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log\left(x\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
 2*a^3)/(a^4*x^3)
```

3.221.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx=\frac{-2a^2+3abx-6b^2x^2}{6a^3x^3}+\frac{b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

input
```
integrate(1/x**2/(b*x**3+a*x**2),x)
```

output
```
(-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b
+ x))/a**4
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx=\frac{b^3\log(bx+a)}{a^4}-\frac{b^3\log(x)}{a^4}-\frac{6\,b^2x^2-3\,abx+2\,a^2}{6\,a^3x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/
(a^3*x^3)
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(ax^2+bx^3)}\,dx=\frac{b^3\log(|bx+a|)}{a^4}-\frac{b^3\log(|x|)}{a^4}-\frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2
*b*x + 2*a^3)/(a^4*x^3)
```

3.221.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx=\frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{a^3}{3}-\frac{a^2\,b\,x}{2}+a\,b^2\,x^2}{a^4\,x^3}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)),x)
```

output
```
(2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*
x^3)
```

3.222 $\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx$

3.222.1 Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx=\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}$$

output
```
3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

3.222.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx=\frac{9a^2bx-3ab^2x^2+b^3x^3-\frac{3a^4}{a+bx}-12a^3\log(a+bx)}{3b^5}$$

input
```
Integrate[x^8/(a*x^2 + b*x^3)^2,x]
```

output
```
(9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*
x])/(3*b^5)
```

3.222.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(ax^2+bx^3)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^4}{(a+bx)^2}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}+\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

input
```
Int[x^8/(a*x^2 + b*x^3)^2,x]
```

output
```
(3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*L
og[a + b*x])/b^5
```

3.222.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.222.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x}{b^4}-\frac{4a^3\ln(bx+a)}{b^5}-\frac{a^4}{b^5(bx+a)}$	57
risch	$\frac{3a^2x}{b^4}-\frac{a\,x^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{x^7}{3b}-\frac{2a\,x^6}{3b^2}+\frac{2a^2x^5}{b^3}-\frac{4a^4x^3}{b^5}}{x^3(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	67
parallelrisch	$-\frac{-b^4x^4+2a\,b^3x^3+12\ln(bx+a)a^3bx-6a^2b^2x^2+12a^4\ln(bx+a)+12a^4}{3b^5(bx+a)}$	71

input
```
int(x^8/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x)-4*a^3*ln(b*x+a)/b^5-a^4/b^5/(b*x+a)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,(a^3bx+a^4)\log(bx+a)}{3\,(b^6x+ab^5)}$$

input
```
integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b
*x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

3.222.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{a^4}{ab^5+b^6x} - \frac{4a^3\log{(a+bx)}}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

input
```
integrate(x**8/(b*x**3+a*x**2)**2,x)
```

output
```
-a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**
2/b**3 + x**3/(3*b**2)
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{a^4}{b^6x+ab^5} - \frac{4\,a^3\log{(bx+a)}}{b^5} + \frac{b^2x^3-3\,abx^2+9\,a^2x}{3\,b^4}$$

input
```
integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
 9*a^2*x)/b^4
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{4\,a^3\log{(|bx+a|)}}{b^5} - \frac{a^4}{(bx+a)b^5} + \frac{b^4x^3-3\,ab^3x^2+9\,a^2b^2x}{3\,b^6}$$

input
```
integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^
3*x^2 + 9*a^2*b^2*x)/b^6
```

3.222.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln\left(a+b\,x\right)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5+a\,b^4\right)}$$

input
```
int(x^8/(a*x^2 + b*x^3)^2,x)
```

output
```
x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4
/(b*(a*b^4 + b^5*x))
```

3.223 $\int \frac{x^7}{(ax^2+bx^3)^2} dx$

3.223.1 Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{x^7}{(ax^2+bx^3)^2} dx = -\frac{2ax}{b^3} + \frac{x^2}{2b^2} + \frac{a^3}{b^4(a+bx)} + \frac{3a^2\log(a+bx)}{b^4}$$

output
```
-2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

3.223.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{(ax^2+bx^3)^2} dx = \frac{-4abx + b^2x^2 + \frac{2a^3}{a+bx} + 6a^2\log(a+bx)}{2b^4}$$

input
```
Integrate[x^7/(a*x^2 + b*x^3)^2,x]
```

output
```
(-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

3.223.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3}{(a+bx)^2}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}-\frac{2a}{b^3}+\frac{x}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

input
```
Int[x^7/(a*x^2 + b*x^3)^2,x]
```

output
```
(-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^
4
```

3.223.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.223.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{3a^2\ln(bx+a)}{b^4}+\frac{a^3}{b^4(bx+a)}$	46
norman	$\frac{\frac{3a^3x^3}{b^4}+\frac{x^6}{2b}-\frac{3a\,x^5}{2b^2}}{x^3(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	56
parallelrisch	$\frac{b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)+6a^3}{2b^4(bx+a)}$	59

input
```
int(x^7/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx=\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log(bx+a)}{2\,(b^5x+ab^4)}$$

input
```
integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
 + a))/(b^5*x + a*b^4)
```

3.223.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{a^3}{ab^4+b^5x} + \frac{3a^2\log(a+bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

input
```
integrate(x**7/(b*x**3+a*x**2)**2,x)
```

output
```
a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b
**2)
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{a^3}{b^5x+ab^4} + \frac{3\,a^2\log(bx+a)}{b^4} + \frac{bx^2-4\,ax}{2\,b^3}$$

input
```
integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{3\,a^2\log(|bx+a|)}{b^4} + \frac{a^3}{(bx+a)b^4} + \frac{b^2x^2-4\,abx}{2\,b^4}$$

input
```
integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x
)/b^4
```

3.223.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx=\frac{x^2}{2\,b^2}+\frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4}+\frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

input
```
int(x^7/(a*x^2 + b*x^3)^2,x)
```

output
```
x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)
/b^3
```

3.224 $\int \frac{x^6}{(ax^2+bx^3)^2} dx$

3.224.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x^6}{(ax^2+bx^3)^2} dx = \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}$$

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

3.224.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{(ax^2+bx^3)^2} dx = \frac{bx - \frac{a^2}{a+bx} - 2a\log(a+bx)}{b^3}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3)^2,x]
```

output
```
(b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

3.224.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^2}{(a+bx)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

input
```
Int[x^6/(a*x^2 + b*x^3)^2,x]
```

output
```
x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

3.224.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.224.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
risch	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
norman	$\frac{\frac{x^5}{b}-\frac{2a^2x^3}{b^3}}{x^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	44
parallelrisch	$-\frac{2\ln(bx+a)xab-b^2x^2+2a^2\ln(bx+a)+2a^2}{b^3(bx+a)}$	49

input
```
int(x^6/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx=\frac{b^2x^2+abx-a^2-2\,(abx+a^2)\log{(bx+a)}}{b^4x+ab^3}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

3.224.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = -\frac{a^2}{ab^3+b^4x} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

input
```
integrate(x**6/(b*x**3+a*x**2)**2,x)
```

output
```
-a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = -\frac{a^2}{b^4x+ab^3} + \frac{x}{b^2} - \frac{2\,a\log(bx+a)}{b^3}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{x}{b^2} - \frac{2\,a\log(|bx+a|)}{b^3} - \frac{a^2}{(bx+a)b^3}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{x}{b^2} - \frac{a^2}{x\,b^4+a\,b^3} - \frac{2\,a\,\ln\left(a+b\,x\right)}{b^3}$$

input
```
int(x^6/(a*x^2 + b*x^3)^2,x)
```

output
```
x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

3.225 $\int \frac{x^5}{(ax^2+bx^3)^2} dx$

3.225.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x^5}{(ax^2+bx^3)^2} dx = \frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

3.225.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(ax^2+bx^3)^2} dx = \frac{\frac{a}{a+bx} + \log(a+bx)}{b^2}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3)^2,x]
```

output
```
(a/(a + b*x) + Log[a + b*x])/b^2
```

3.225.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx$$
$$\Big\downarrow 9$$
$$\int \frac{x}{(a+bx)^2}dx$$
$$\Big\downarrow 49$$
$$\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx$$
$$\Big\downarrow 2009$$
$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

input
```
Int[x^5/(a*x^2 + b*x^3)^2,x]
```

output
```
a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

3.225.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.225.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
parallelrisch	$\frac{b\ln(bx+a)x+a\ln(bx+a)+a}{b^2(bx+a)}$	31

input
```
int(x^5/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx=\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

3.225.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{a}{ab^2+b^3x}+\frac{\log(a+bx)}{b^2}$$

input
```
integrate(x**5/(b*x**3+a*x**2)**2,x)
```

output
```
a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{a}{b^3x+ab^2}+\frac{\log(bx+a)}{b^2}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{\log(|bx+a|)}{b^2}+\frac{a}{(bx+a)b^2}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{\ln(a+b\,x)}{b^2} + \frac{a}{b^2\,(a+b\,x)}$$

input
```
int(x^5/(a*x^2 + b*x^3)^2,x)
```

output
```
log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

3.226 $\int \frac{x^4}{(ax^2+bx^3)^2} dx$

3.226.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{x^4}{(ax^2+bx^3)^2} dx = -\frac{1}{b(a+bx)}$$

output
```
-1/b/(b*x+a)
```

3.226.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2} dx = -\frac{1}{b(a+bx)}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3)^2,x]
```

output
```
-(1/(b*(a + b*x)))
```

3.226.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{(a+bx)^2}dx$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{b(a+bx)}$$

input
```
Int[x^4/(a*x^2 + b*x^3)^2,x]
```

output
```
-(1/(b*(a + b*x)))
```

3.226.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.226.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13

input
```
int(x^4/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x+a)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b^2x+ab}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

3.226.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{ab+b^2x}$$

input
```
integrate(x**4/(b*x**3+a*x**2)**2,x)
```

output
```
-1/(a*b + b**2*x)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b^2x+ab}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-1/(b^2*x + a*b)
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

3.226.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b\,(a+b\,x)}$$

input
```
int(x^4/(a*x^2 + b*x^3)^2,x)
```

output
```
-1/(b*(a + b*x))
```

3.227 $\int \frac{x^3}{(ax^2+bx^3)^2}\,dx$

3.227.1 Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{a(a+bx)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx)}{a^2}$$

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

3.227.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{\frac{a}{a+bx} + \log(x) - \log(a+bx)}{a^2}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3)^2,x]
```

output
```
(a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

3.227.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^2} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x(a+bx)^2} dx$$
$$\Big\downarrow \; 54$$
$$\int \left(-\frac{b}{a^2(a+bx)} + \frac{1}{a^2x} - \frac{b}{a(a+bx)^2}\right) dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{\log(a+bx)}{a^2} + \frac{\log(x)}{a^2} + \frac{1}{a(a+bx)}$$

input
```
Int[x^3/(a*x^2 + b*x^3)^2,x]
```

output
```
1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

3.227.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.227.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}+\frac{\ln(-x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33
parallelrisch	$\frac{b\ln(x)x-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)-bx}{a^2(bx+a)}$	45

input
```
int(x^3/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = -\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

3.227.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{a^2+abx} + \frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

input
```
integrate(x**3/(b*x**3+a*x**2)**2,x)
```

output
```
1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{abx+a^2} - \frac{\log(bx+a)}{a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = -\frac{\log(|bx+a|)}{a^2} + \frac{\log(|x|)}{a^2} + \frac{1}{(bx+a)a}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{a^2+b\,x\,a} - \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

input
```
int(x^3/(a*x^2 + b*x^3)^2,x)
```

output
```
1/(a^2 + a*b*x) - (2*atanh((2*b*x)/a + 1))/a^2
```

3.228 $\int \frac{x^2}{(ax^2+bx^3)^2}\,dx$

3.228.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = -\frac{1}{a^2x} - \frac{b}{a^2(a+bx)} - \frac{2b\log(x)}{a^3} + \frac{2b\log(a+bx)}{a^3}$$

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

3.228.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = -\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3)^2,x]
```

output
```
-((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

3.228.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^2}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^2(a+bx)^2}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{2b^2}{a^3(a+bx)}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{1}{a^2x^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

input
```
Int[x^2/(a*x^2 + b*x^3)^2,x]
```

output
```
-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a
^3
```

3.228.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.228.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{a^{2}x}-\frac{b}{a^{2}(bx+a)}-\frac{2b\ln(x)}{a^{3}}+\frac{2b\ln(bx+a)}{a^{3}}$	43
risch	$\frac{-\frac{2bx}{a^{2}}-\frac{1}{a}}{x(bx+a)}-\frac{2b\ln(x)}{a^{3}}+\frac{2b\ln(-bx-a)}{a^{3}}$	49
norman	$\frac{\frac{2b^{2}x^{4}}{a^{3}}-\frac{x^{2}}{a}}{x^{3}(bx+a)}-\frac{2b\ln(x)}{a^{3}}+\frac{2b\ln(bx+a)}{a^{3}}$	53
parallelrisch	$-\frac{2b^{2}\ln(x)x^{2}-2b^{2}\ln(bx+a)x^{2}+2ab\ln(x)x-2\ln(bx+a)xab-2b^{2}x^{2}+a^{2}}{a^{3}x(bx+a)}$	70

input
```
int(x^2/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx=-\frac{2\,abx+a^2-2\,(b^2x^2+abx)\log(bx+a)+2\,(b^2x^2+abx)\log(x)}{a^3bx^2+a^4x}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*l
og(x))/(a^3*b*x^2 + a^4*x)
```

3.228.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = \frac{-a-2bx}{a^3x+a^2bx^2} + \frac{2b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

input
```
integrate(x**2/(b*x**3+a*x**2)**2,x)
```

output
```
(-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = -\frac{2\,bx+a}{a^2bx^2+a^3x} + \frac{2\,b\log(bx+a)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = \frac{2\,b\log(|bx+a|)}{a^3} - \frac{2\,b\log(|x|)}{a^3} - \frac{2\,bx+a}{(bx^2+ax)a^2}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*
x)*a^2)
```

3.228.9 Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx = \frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3} - \frac{\frac{1}{a}+\frac{2\,b\,x}{a^2}}{b\,x^2+a\,x}$$

input
```
int(x^2/(a*x^2 + b*x^3)^2,x)
```

output
```
(4*b*atanh((2*b*x)/a + 1))/a^3 - (1/a + (2*b*x)/a^2)/(a*x + b*x^2)
```

3.229 $\int \frac{x}{(ax^2+bx^3)^2} dx$

3.229.1 Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{x}{(ax^2+bx^3)^2} dx = -\frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{b^2}{a^3(a+bx)} + \frac{3b^2\log(x)}{a^4} - \frac{3b^2\log(a+bx)}{a^4}$$

output
```
-1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

3.229.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x}{(ax^2+bx^3)^2} dx = \frac{a\left(-\frac{a}{x^2} + \frac{4b}{x} + \frac{2b^2}{a+bx}\right) + 6b^2\log(x) - 6b^2\log(a+bx)}{2a^4}$$

input
```
Integrate[x/(a*x^2 + b*x^3)^2,x]
```

output
```
(a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
 b*x])/(2*a^4)
```

3.229.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^3(a+bx)^2}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{3b^3}{a^4(a+bx)}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

input
```
Int[x/(a*x^2 + b*x^3)^2,x]
```

output
```
-1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^
4 - (3*b^2*Log[a + b*x])/a^4
```

3.229.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.229.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	57
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)}+\frac{3b^2\ln(-x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	63
norman	$\frac{-\frac{3b^3x^4}{a^4}-\frac{x}{2a}+\frac{3b\,x^2}{2a^2}}{x^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	64
parallelrisch	$\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2\ln(x)x^2-6\ln(bx+a)x^2a\,b^2-6b^3x^3+3a^2bx-a^3}{2a^4x^2(bx+a)}$	87

input
```
int(x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{6\,ab^2x^2+3\,a^2bx-a^3-6\,(b^3x^3+ab^2x^2)\log(bx+a)+6\,(b^3x^3+ab^2x^2)\log(x)}{2\,(a^4bx^3+a^5x^2)}$$

input
```
integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a)
+ 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

3.229.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{-a^2+3abx+6b^2x^2}{2a^4x^2+2a^3bx^3} + \frac{3b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

input
```
integrate(x/(b*x**3+a*x**2)**2,x)
```

output
```
(-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(lo
g(x) - log(a/b + x))/a**4
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{6\,b^2x^2+3\,abx-a^2}{2\,(a^3bx^3+a^4x^2)} - \frac{3\,b^2\log(bx+a)}{a^4} + \frac{3\,b^2\log(x)}{a^4}$$

input
```
integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)
/a^4 + 3*b^2*log(x)/a^4
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = -\frac{3\,b^2\log(|bx+a|)}{a^4} + \frac{3\,b^2\log(|x|)}{a^4} + \frac{6\,ab^2x^2+3\,a^2bx-a^3}{2\,(bx+a)a^4x^2}$$

input
```
integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 +
3*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x}{(ax^2 + bx^3)^2}\,dx = \frac{\frac{3\,b^2\,x^2}{a^3} - \frac{1}{2\,a} + \frac{3\,b\,x}{2\,a^2}}{b\,x^3 + a\,x^2} - \frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^4}$$

input
```
int(x/(a*x^2 + b*x^3)^2,x)
```

output
```
((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*ata
nh((2*b*x)/a + 1))/a^4
```

3.230 $\int \frac{1}{(ax^2+bx^3)^2} dx$

3.230.1 Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{(ax^2+bx^3)^2} dx = -\frac{1}{3a^2x^3} + \frac{b}{a^3x^2} - \frac{3b^2}{a^4x} - \frac{b^3}{a^4(a+bx)} - \frac{4b^3\log(x)}{a^5} + \frac{4b^3\log(a+bx)}{a^5}$$

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

3.230.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{(ax^2+bx^3)^2} dx = -\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)} + 12b^3\log(x) - 12b^3\log(a+bx)}{3a^5}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-2),x]
```

output
```
-1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 1
2*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

3.230.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^2}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^4(a+bx)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{4b^4}{a^5(a+bx)}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{3b^2}{a^4x^2}-\frac{2b}{a^3x^3}+\frac{1}{a^2x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

input
```
Int[(a*x^2 + b*x^3)^(-2),x]
```

output
```
-1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (
4*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

3.230.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.230.4 Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	68
norman	$\frac{\frac{4b^4x^4}{a^5}-\frac{1}{3a}+\frac{2bx}{3a^2}-\frac{2b^2x^2}{a^3}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	72
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{2b^2x^2}{a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(-bx-a)}{a^5}$	75
parallelrisch	$-\frac{12\ln(x)x^4b^4-12\ln(bx+a)x^4b^4+12\ln(x)x^3a\,b^3-12\ln(bx+a)x^3a\,b^3-12b^4x^4+6a^2b^2x^2-2a^3bx+a^4}{3a^5x^3(bx+a)}$	96

input
```
int(1/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = -\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\,(b^4x^4+ab^3x^3)\log(bx+a)+12\,(b^4x^4+ab^3x^3)\log(x)}{3\,(a^5bx^4+a^6x^3)}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3
*x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3
)
```

3.230.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4} + \frac{4b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

input
```
integrate(1/(b*x**3+a*x**2)**2,x)
```

output
```
(-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*
b*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = -\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\,(a^4bx^4+a^5x^3)} + \frac{4\,b^3\log(bx+a)}{a^5} - \frac{4\,b^3\log(x)}{a^5}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) +
4*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{4\,b^3\log(|bx+a|)}{a^5} - \frac{4\,b^3\log(|x|)}{a^5} - \frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4}{3\,(bx+a)a^5x^3}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 +
6*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

input
```
int(1/(a*x^2 + b*x^3)^2,x)
```

output
```
(8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3
)/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

3.231 $\int \frac{1}{x(ax^2+bx^3)^2} \, dx$

3.231.1 Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{1}{x(ax^2+bx^3)^2} \, dx = -\frac{1}{4a^2x^4} + \frac{2b}{3a^3x^3} - \frac{3b^2}{2a^4x^2} + \frac{4b^3}{a^5x} + \frac{b^4}{a^5(a+bx)} + \frac{5b^4\log(x)}{a^6} - \frac{5b^4\log(a+bx)}{a^6}$$

output
```
-1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b
^4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

3.231.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(ax^2+bx^3)^2} \, dx = \frac{\frac{a(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4)}{x^4(a+bx)} + 60b^4\log(x) - 60b^4\log(a+bx)}{12a^6}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3)^2),x]
```

output
```
((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^
4*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

3.231.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(ax^2+bx^3)^2}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{1}{x^5(a+bx)^2}dx$$
$$\Big\downarrow \ 54$$
$$\int \left(-\frac{5b^5}{a^6(a+bx)}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{4b^3}{a^5x^2}+\frac{3b^2}{a^4x^3}-\frac{2b}{a^3x^4}+\frac{1}{a^2x^5}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

input
```
Int[1/(x*(a*x^2 + b*x^3)^2),x]
```

output
```
-1/4*1/(a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*
x) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

3.231.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.231.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{4a^{2}x^{4}}+\frac{2b}{3a^{3}x^{3}}-\frac{3b^{2}}{2a^{4}x^{2}}+\frac{4b^{3}}{a^{5}x}+\frac{b^{4}}{a^{5}(bx+a)}+\frac{5b^{4}\ln(x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	79
norman	$\frac{-\frac{5b^{5}x^{5}}{a^{6}}-\frac{1}{4a}+\frac{5bx}{12a^{2}}-\frac{5b^{2}x^{2}}{6a^{3}}+\frac{5b^{3}x^{3}}{2a^{4}}}{x^{4}(bx+a)}+\frac{5b^{4}\ln(x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	83
risch	$\frac{\frac{5b^{4}x^{4}}{a^{5}}+\frac{5b^{3}x^{3}}{2a^{4}}-\frac{5b^{2}x^{2}}{6a^{3}}+\frac{5bx}{12a^{2}}-\frac{1}{4a}}{x^{4}(bx+a)}+\frac{5b^{4}\ln(-x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	85
parallelrisch	$\frac{60\ln(x)x^{5}b^{5}-60\ln(bx+a)x^{5}b^{5}+60\ln(x)x^{4}a\,b^{4}-60\ln(bx+a)x^{4}a\,b^{4}-60b^{5}x^{5}+30a^{2}b^{3}x^{3}-10a^{3}b^{2}x^{2}+5a^{4}bx-3a^{5}}{12a^{6}x^{4}(bx+a)}$	109

input
```
int(1/x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b
^4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\,(ax^2+bx^3)^2}\,dx$$
$$=\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5-60\,(b^5x^5+ab^4x^4)\log(bx+a)+60\,(b^5x^5+ab^4x^4)\log(}{12\,(a^6bx^5+a^7x^4)}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
 60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/
(a^6*b*x^5 + a^7*x^4)
```

3.231.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = \frac{-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4}{12a^6x^4+12a^5bx^5} + \frac{5b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

input
```
integrate(1/x/(b*x**3+a*x**2)**2,x)
```

output
```
(-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)
/(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = \frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\left(a^5bx^5+a^6x^4\right)} - \frac{5\,b^4\log\left(bx+a\right)}{a^6} + \frac{5\,b^4\log\left(x\right)}{a^6}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5
*b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = -\frac{5\,b^4\log\left(|bx+a|\right)}{a^6} + \frac{5\,b^4\log\left(|x|\right)}{a^6} + \frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5}{12\left(bx+a\right)a^6x^4}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4
+ 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx=\frac{\frac{5\,b^3\,x^3}{2\,a^4}-\frac{5\,b^2\,x^2}{6\,a^3}-\frac{1}{4\,a}+\frac{5\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{12\,a^2}}{b\,x^5+a\,x^4}-\frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

input
```
int(1/(x*(a*x^2 + b*x^3)^2),x)
```

output
```
((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (
5*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

3.232 $\int x^2\sqrt{ax^2+bx^3}\,dx$

3.232.1 Optimal result

Integrand size = 19, antiderivative size = 105

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3}+\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2}-\frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}$$

output
```
2/9*(b*x^3+a*x^2)^(3/2)/b-32/315*a^3*(b*x^3+a*x^2)^(3/2)/b^4/x^3+16/105*a^
2*(b*x^3+a*x^2)^(3/2)/b^3/x^2-4/21*a*(b*x^3+a*x^2)^(3/2)/b^2/x
```

3.232.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2(x^2(a+bx))^{3/2}\,(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3)}{315b^4x^3}$$

input
```
Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3
))/(315*b^4*x^3)
```

3.232.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\int x\sqrt{bx^3+ax^2}dx}{3b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\int \sqrt{bx^3+ax^2}dx}{7b}\right)}{3b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}\right)}{3b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}\right)}{7b}\right)}{3b}$$

input
```
Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (2*a*((2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
- (4*a*((-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/
2))/(5*b*x^2)))/(7*b)))/(3*b)
```

3.232.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.232.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}\left(15b^2x^2-12abx+8a^2\right)}{105b^3}$	32
gosper	$-\frac{2(bx+a)\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
default	$-\frac{2(bx+a)\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
risch	$-\frac{2\sqrt{x^2(bx+a)}\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)}{315x\,b^4}$	61
trager	$-\frac{2\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	63

input
```
int(x^2*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x^3 + a*x^2)/(b^4*x)
```

3.232.6 Sympy [F]

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \int x^2\sqrt{x^2\,(a+bx)}\,dx$$

input
```
integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x + a)/b^4
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{32\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^4} + \frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a\mathrm{sgn}(x)}{b^3} + \frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3\right.}{b^3}\right.}{315\,b}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b^3 + (35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*
x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^3)/b
```

3.232.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

input
```
int(x^2*(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^
2 + 8*a^3*b*x))/(315*b^4*x)
```

3.233 $\int x\sqrt{ax^2+bx^3}\,dx$

3.233.1 Optimal result

Integrand size = 17, antiderivative size = 80

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3}-\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}$$

output
```
16/105*a^2*(b*x^3+a*x^2)^(3/2)/b^3/x^3-8/35*a*(b*x^3+a*x^2)^(3/2)/b^2/x^2+
2/7*(b*x^3+a*x^2)^(3/2)/b/x
```

3.233.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2(x^2(a+bx))^{3/2}\,(8a^2-12abx+15b^2x^2)}{105b^3x^3}$$

input
```
Integrate[x*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

3.233.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\int \sqrt{bx^3+ax^2}dx}{7b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}\right)}{7b}$$

input
```
Int[x*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(7*b*x) - (4*a*((-4*a*(a*x^2 + b*x^3)^(3/2))/(15
*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*x^2)))/(7*b)
```

3.233.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.233.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^{2}}$	21
gosper	$\frac{2(bx+a)(15b^{2}x^{2}-12abx+8a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{3}x}$	46
default	$\frac{2(bx+a)(15b^{2}x^{2}-12abx+8a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{3}x}$	46
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3})}{105x\,b^{3}}$	50
trager	$\frac{2(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3})\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{3}x}$	52

input
```
int(x*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

input
```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(
b^3*x)
```

3.233.6 Sympy [F]

$$\int x\sqrt{ax^2+bx^3}\,dx=\int x\sqrt{x^2\,(a+bx)}\,dx$$

input
```
integrate(x*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(a + b*x)), x)
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

input
```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int x\sqrt{ax^2+bx^3}\,dx = -\frac{16\,a^{\frac{7}{2}}\text{sgn}(x)}{105\,b^3} + \frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a\text{sgn}(x)}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)\text{sgn}(x)}{b^2}\right)}{105\,b}$$

input
```
integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x)/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*
x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b^2
)/b
```

3.233.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3\right)}{105\,b^3\,x}$$

input
```
int(x*(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x))/(
105*b^3*x)
```

3.234 $\int \sqrt{ax^2+bx^3}\,dx$

3.234.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \sqrt{ax^2+bx^3}\,dx=-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}+\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}$$

output
```
-4/15*a*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*(b*x^3+a*x^2)^(3/2)/b/x^2
```

3.234.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{x^2(a+bx)}(-2a^2+abx+3b^2x^2)}{15b^2x}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2*x)
```

3.234.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{2a\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b
*x^2)
```

3.234.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.234.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.25

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
gosper	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
default	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,(-3b^{2}x^{2}-abx+2a^{2})}{15x\,b^{2}}$	39
trager	$-\frac{2(-3b^{2}x^{2}-abx+2a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	41

input
```
int((b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

3.234.6 Sympy [F]

$$\int \sqrt{ax^2+bx^3}\,dx = \int \sqrt{ax^2+bx^3}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(a*x**2 + b*x**3), x)
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \sqrt{ax^2 + bx^3}\, dx = \frac{2\,(3\,b^2x^2 + abx - 2\,a^2)\sqrt{bx + a}}{15\,b^2}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \sqrt{ax^2 + bx^3}\, dx = \frac{4\,a^{\frac{5}{2}}\operatorname{sgn}(x)}{15\,b^2} + \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a\operatorname{sgn}(x)}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)\operatorname{sgn}(x)}{b}\right)}{15\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*
sgn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*sgn(x)/b)/b
```

3.234.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2 + bx^3}\, dx = \frac{2\,\sqrt{b\,x^3 + a\,x^2}\,\left(-2\,a^2 + a\,b\,x + 3\,b^2\,x^2\right)}{15\,b^2\,x}$$

input
```
int((a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

3.235 $\int \frac{\sqrt{ax^2+bx^3}}{x} dx$

3.235.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{ax^2+bx^3}}{x} dx = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

output
```
2/3*(b*x^3+a*x^2)^(3/2)/b/x^3
```

3.235.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ax^2+bx^3}}{x} dx = \frac{2(x^2(a+bx))^{3/2}}{3bx^3}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
```

output
```
(2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

3.235.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x,x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

3.235.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.235.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{x^2(bx+a)}\,(bx+a)}{3xb}$	25
gosper	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
default	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
trager	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
pseudoelliptic	$2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28

input
```
int((b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(x^2*(b*x+a))^(1/2)/x*(b*x+a)/b
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2\sqrt{bx^3+ax^2}(bx+a)}{3\,bx}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

3.235.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x, x)
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx$$
$$= -\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b} + \frac{2\left(3\,\sqrt{bx+a}a\mathrm{sgn}(x) + \left((bx+a)^{\frac{3}{2}} - 3\,\sqrt{bx+a}a\right)\mathrm{sgn}(x)\right)}{3\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-2/3*a^(3/2)*sgn(x)/b + 2/3*(3*sqrt(b*x + a)*a*sgn(x) + ((b*x + a)^(3/2) -
 3*sqrt(b*x + a)*a)*sgn(x))/b
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}}{x}\,dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x, x)
```

3.236 $\int \frac{\sqrt{ax^2+bx^3}}{x^2} dx$

3.236.1 Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2} dx = \frac{2\sqrt{ax^2+bx^3}}{x} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

output
```
-2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2)+2*(b*x^3+a*x^2)^(1/2)/x
```

3.236.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2} dx = \frac{2x\left(a+bx-\sqrt{a}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

output
```
(2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqr
t[x^2*(a + b*x)]
```

3.236.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$$

$$\Big\downarrow \text{1927}$$

$$a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

output
```
(2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^3]]
```

3.236.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.236.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) bx+\sqrt{bx+a}\,\sqrt{a}}{x\sqrt{a}}$	36
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\sqrt{bx+a}\right)}{x\sqrt{bx+a}}$	52

input
```
int((b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x+(b*x+a)^(1/2)*a^(1/2))/x/a^(1/2)
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \left[\frac{\sqrt{a}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x}, \frac{2\left(\sqrt{-a}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*s
qrt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(
a*x)) + sqrt(b*x^3 + a*x^2))/x]
```

3.236.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}}{x^2}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

3.236.7 Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^2}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^2, x)
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\sqrt{bx+a}\,\mathrm{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}(x)}{\sqrt{-a}}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x
) - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\sqrt{b\,x^3+a\,x^2}}{x}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{\frac{1}{x}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{b\,x^3+a\,x^2}\,{\left(\frac{1}{x}\right)}^{3/2}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^2,x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2))/x + (a^(1/2)*asin((a^(1/2)*(1/x)^(1/2)*1i)/b^(1/
2))*(a*x^2 + b*x^3)^(1/2)*(1/x)^(3/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

3.237 $\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx$

3.237.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

output
```
-b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)-(b*x^3+a*x^2)^(1/2)/x^2
```

3.237.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=-\frac{\sqrt{a+bx}\left(\sqrt{a}\sqrt{a+bx}+bx\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

output
```
-((Sqrt[a + b*x]*(Sqrt[a]*Sqrt[a + b*x] + b*x*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]]))/(Sqrt[a]*Sqrt[x^2*(a + b*x)]))
```

3.237.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}dx$$

$$\Big\downarrow \ 1926$$

$$\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

$$\Big\downarrow \ 1914$$

$$-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

output
```
-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/
Sqrt[a]
```

3.237.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.237.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{2} x^{2}-\left(2 a^{\frac{3}{2}}+\sqrt{a}\, b x \right) \sqrt{bx+a}}{4 a^{\frac{3}{2}} x^{2}}$	50
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b x +\sqrt{bx+a}\,\sqrt{a}\right)}{x^{2} \sqrt{bx+a}\,\sqrt{a}}$	56
risch	$-\frac{\sqrt{x^{2} \left(bx+a \right)}}{x^{2}}-\frac{b \,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(bx+a \right)}}{\sqrt{a}\, x \sqrt{bx+a}}$	57

input
```
int((b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-(2*a^(3/2)+a^(1/2)*b*x)*(b*x+a
)^(1/2))/a^(3/2)/x^2
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,ax^2}, \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}a}{ax^2}\right]$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^
2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 +
 a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

3.237.6 Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\, dx = \int \frac{\sqrt{x^2 (a + bx)}}{x^3}\, dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

3.237.7 Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\, dx = \int \frac{\sqrt{bx^3 + ax^2}}{x^3}\, dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^3}\, dx = \frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} - \frac{\sqrt{bx+a}\, b \operatorname{sgn}(x)}{x}}{b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
(b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - sqrt(b*x + a)*b*sgn(
x)/x)/b
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^3}\,dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^3,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x^3, x)
```

3.238 $\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx$

3.238.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx = -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}$$

output
```
1/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-1/2*(b*x^3+a*x^2)^(
1/2)/x^3-1/4*b*(b*x^3+a*x^2)^(1/2)/a/x^2
```

3.238.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(2a+bx) + b^2x^2 \text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(2*a + b*x)) + b^2*x^2*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(3/2)*x^3*Sqrt[a + b*x])
```

3.238.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/4
```

3.238.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.238.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{3} x^{3}-\left(\sqrt{a}\, b^{2} x^{2}-\frac{2 a^{\frac{3}{2}} b x}{3}-\frac{8 a^{\frac{5}{2}}}{3}\right) \sqrt{bx+a}}{8 a^{\frac{5}{2}} x^{3}}$	61
risch	$-\frac{(bx+2a)\sqrt{x^{2}(bx+a)}}{4x^{3}a}+\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{x^{2}(bx+a)}}{4a^{\frac{3}{2}} x\sqrt{bx+a}}$	69
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left((bx+a)^{\frac{3}{2}} a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{2} x^{2}+\sqrt{bx+a}\, a^{\frac{5}{2}}\right)}{4x^{3}\sqrt{bx+a}\, a^{\frac{5}{2}}}$	73

input
```
int((b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^(5/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^3*x^3-(a^(1/2)*b^2*x^2-2/3*
a^(3/2)*b*x-8/3*a^(5/2))*(b*x+a)^(1/2))/x^3
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \left[\frac{\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(abx+2\,a^2)}{8\,a^2x^3}, -\frac{\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}(abx+2\,a^2)}{4\,a^2x^3}\right]$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/
x^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^
2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(a*
b*x + 2*a^2))/(a^2*x^3)]
```

3.238.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x***4,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

3.238.7 Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^4}\,dx = \int \frac{\sqrt{bx^3 + ax^2}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^4}\,dx = -\frac{\frac{b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a} + \frac{(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)+\sqrt{bx+a}ab^3\mathrm{sgn}(x)}{ab^2x^2}}{4\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + ((b*x + a)^
(3/2)*b^3*sgn(x) + sqrt(b*x + a)*a*b^3*sgn(x))/(a*b^2*x^2))/b
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3}}{x^4}\,dx = \int \frac{\sqrt{b\,x^3 + a\,x^2}}{x^4}\,dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x^4, x)
```

3.239 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$

3.239.1 Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=-\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}$$

output
```
-1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/3*(b*x^3+a*x^2)^
(1/2)/x^4-1/12*b*(b*x^3+a*x^2)^(1/2)/a/x^3+1/8*b^2*(b*x^3+a*x^2)^(1/2)/a^2
/x^2
```

3.239.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(8a^2+2abx-3b^2x^2)+3b^3x^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{5/2}x^4\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

output
```
-1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2
*x^2) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2)*x^4*Sqrt[a + b
*x])
```

3.239.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

input
```
Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b
*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^3]])/a^(3/2)))/(4*a)))/6
```

3.239.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.239.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{5\left(-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4+\sqrt{bx+a}\left(\sqrt{a}\,b^3x^3-\frac{2a^{\frac{3}{2}}b^2x^2}{3}+\frac{8a^{\frac{5}{2}}bx}{15}+\frac{16a^{\frac{7}{2}}}{5}\right)\right)}{64a^{\frac{7}{2}}x^4}$	72
risch	$-\frac{\left(-3b^2x^2+2abx+8a^2\right)\sqrt{x^2(bx+a)}}{24x^4a^2}-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{5}{2}}x\sqrt{bx+a}}$	81
default	$\frac{\sqrt{b\,x^3+a\,x^2}\left(3(bx+a)^{\frac{5}{2}}a^{\frac{5}{2}}-8(bx+a)^{\frac{3}{2}}a^{\frac{7}{2}}-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^3x^3-3\sqrt{bx+a}\,a^{\frac{9}{2}}\right)}{24x^4\sqrt{bx+a}\,a^{\frac{9}{2}}}$	89

input
```
int((b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-5/64*(-arctanh((b*x+a)^(1/2)/a^(1/2))*b^4*x^4+(b*x+a)^(1/2)*(a^(1/2)*b^3*
x^3-2/3*a^(3/2)*b^2*x^2+8/15*a^(5/2)*b*x+16/5*a^(7/2)))/a^(7/2)/x^4
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$$
$$=\left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^3x^4},\ \frac{3\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+}}{}\right.}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^
4), 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) +
(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

3.239.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \int \frac{\sqrt{x^2(a+bx)}}{x^5}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

3.239.7 Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{5}{2}}b^4\mathrm{sgn}(x)-8\,(bx+a)^{\frac{3}{2}}ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}a^2b^4\mathrm{sgn}(x)}{a^2b^3x^3}}{24\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
 + a)^(5/2)*b^4*sgn(x) - 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*
a^2*b^4*sgn(x))/(a^2*b^3*x^3))/b
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^5}\,dx$$

input
```
int((a*x^2 + b*x^3)^(1/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3)^(1/2)/x^5, x)
```

3.240 $\int x^2(ax^2+bx^3)^{3/2}\,dx$

3.240.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^2(ax^2+bx^3)^{3/2}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{15b} - \frac{512a^5(ax^2+bx^3)^{5/2}}{45045b^6x^5} + \frac{256a^4(ax^2+bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3(ax^2+bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^2} - \frac{4a(ax^2+bx^3)^{5/2}}{39b^2x}$$

output
```
2/15*(b*x^3+a*x^2)^(5/2)/b-512/45045*a^5*(b*x^3+a*x^2)^(5/2)/b^6/x^5+256/9
009*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^4-64/1287*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^
3+32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^2-4/39*a*(b*x^3+a*x^2)^(5/2)/b^2/x
```

3.240.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int x^2(ax^2+bx^3)^{3/2}\,dx = \frac{2x(a+bx)^3(-256a^5+640a^4bx-1120a^3b^2x^2+1680a^2b^3x^3-2310ab^4x^4+3003b^5x^5)}{45045b^6\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3
*x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

3.240.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2 \left(ax^2+bx^3\right)^{5/2}}{15 b}-\frac{2 a \int x \left(bx^3+ax^2\right)^{3/2}dx}{3 b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2 \left(ax^2+bx^3\right)^{5/2}}{15 b}-\frac{2 a \left(\frac{2 \left(ax^2+bx^3\right)^{5/2}}{13 b x}-\frac{8 a \int \left(bx^3+ax^2\right)^{3/2}dx}{13 b}\right)}{3 b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2 \left(ax^2+bx^3\right)^{5/2}}{15 b}-\frac{2 a \left(\frac{2 \left(ax^2+bx^3\right)^{5/2}}{13 b x}-\frac{8 a \left(\frac{2 \left(ax^2+bx^3\right)^{5/2}}{11 b x^2}-\frac{6 a \int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11 b}\right)}{13 b}\right)}{3 b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2 \left(ax^2+bx^3\right)^{5/2}}{15 b}-\frac{2 a \left(\frac{2 \left(ax^2+bx^3\right)^{5/2}}{13 b x}-\frac{8 a \left(\frac{2 \left(ax^2+bx^3\right)^{5/2}}{11 b x^2}-\frac{6 a \left(\frac{2 \left(ax^2+bx^3\right)^{5/2}}{9 b x^3}-\frac{4 a \int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9 b}\right)}{11 b}\right)}{13 b}\right)}{3 b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{15b} - \frac{2a\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx} - \frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2} - \frac{6a\left(\frac{2(ax^2+bx^3)^{5/2}}{9bx^3} - \frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4} - \frac{2a\int\frac{(bx^3+ax^2)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2(ax^2+bx^3)^{5/2}}{15b} - \frac{2a\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx} - \frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2} - \frac{6a\left(\frac{2(ax^2+bx^3)^{5/2}}{9bx^3} - \frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4} - \frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}$$

3.240. $\int x^2(ax^2+bx^3)^{3/2}\, dx$

input
```
Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (2*a*((2*(a*x^2 + b*x^3)^(5/2))/(13*b*x
) - (8*a*((2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^
(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a
*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)))/(13*b)))/(3*b)
```

3.240.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.240.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}\left(35b^{2}x^{2}-20abx+8a^{2}\right)}{315b^{3}}$	32
gosper	$-\frac{2(bx+a)\left(-3003b^{5}x^{5}+2310a\,b^{4}x^{4}-1680a^{2}b^{3}x^{3}+1120a^{3}b^{2}x^{2}-640a^{4}bx+256a^{5}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{45045b^{6}x^{3}}$	79
default	$-\frac{2(bx+a)\left(-3003b^{5}x^{5}+2310a\,b^{4}x^{4}-1680a^{2}b^{3}x^{3}+1120a^{3}b^{2}x^{2}-640a^{4}bx+256a^{5}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{45045b^{6}x^{3}}$	79
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-3003b^{7}x^{7}-3696a\,b^{6}x^{6}-63a^{2}b^{5}x^{5}+70a^{3}b^{4}x^{4}-80a^{4}b^{3}x^{3}+96a^{5}b^{2}x^{2}-128a^{6}bx+256a^{7}\right)}{45045x\,b^{6}}$	94
trager	$-\frac{2\left(-3003b^{7}x^{7}-3696a\,b^{6}x^{6}-63a^{2}b^{5}x^{5}+70a^{3}b^{4}x^{4}-80a^{4}b^{3}x^{3}+96a^{5}b^{2}x^{2}-128a^{6}bx+256a^{7}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{45045b^{6}x}$	96

input
```
int(x^2*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\,a\right.}{45045\,b^6x}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x
^2)/(b^6*x)
```

3.240.6 Sympy [F]

$$\int x^2 \left(ax^2+bx^3\right)^{3/2} dx = \int x^2 \left(x^2(a+bx)\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int x^2 \left(ax^2 +bx^3\right)^{3/2} dx = \frac{2\left(3003\, b^7x^7+3696\, ab^6x^6+63\, a^2b^5x^5-70\, a^3b^4x^4+80\, a^4b^3x^3-96\, a^5b^2x^2+128\, a^6bx-256\, a^7\right)\sqrt{bx+a}}{45045\, b^6}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^
6
```

3.240.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(137) = 274.

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.75

$$\int x^2 \left(ax^2+bx^3\right)^{3/2} dx = \frac{512\, a^{\frac{15}{2}} \operatorname{sgn}(x)}{45045\, b^6} + \frac{2\left(\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}a^5\right)a^2\operatorname{sgn}(x)}{b^5}+\frac{30\left(231\,(bx+a)^{\frac{13}{2}}-163\cdots\right.}{}\right.}{}$$

input
```
integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(65*(63*(b*x + a)^(11/2) - 385*(b*
x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155
*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^5 + 30*(231*(b*
x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*
(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5
+ 3003*sqrt(b*x + a)*a^6)*a*sgn(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b
*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3
+ 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^
(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^5)/b
```

3.240.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx = -\frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2\,(256\,a^5-640\,a^4\,b\,x+1120\,a^3\,b^2\,x^2-1680\,a^2\,b^3\,x^3+2310\,a\,b^4\,x^4-3003\,b^5\,x^5)}{45045\,b^6\,x}$$

input
```
int(x^2*(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(256*a^5 - 3003*b^5*x^5 + 2310*a*b^4
*x^4 + 1120*a^3*b^2*x^2 - 1680*a^2*b^3*x^3 - 640*a^4*b*x))/(45045*b^6*x)
```

3.241 $\int x(ax^2 + bx^3)^{3/2}\, dx$

3.241.1 Optimal result

Integrand size = 17, antiderivative size = 136

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx = \frac{256a^4(ax^2+bx^3)^{5/2}}{15015b^5x^5} - \frac{128a^3(ax^2+bx^3)^{5/2}}{3003b^4x^4} + \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2+bx^3)^{5/2}}{13bx}$$

output
```
256/15015*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^5-128/3003*a^3*(b*x^3+a*x^2)^(5/2)
/b^4/x^4+32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^3-16/143*a*(b*x^3+a*x^2)^(5/
2)/b^2/x^2+2/13*(b*x^3+a*x^2)^(5/2)/b/x
```

3.241.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx = \frac{2x(a+bx)^3\left(128a^4-320a^3bx+560a^2b^2x^2-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3
+ 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

3.241.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^{3/2}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\int \left(bx^3+ax^2\right)^{3/2}dx}{13b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}\right)}{13b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{13bx}-\frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}$$

input
```
Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(13*b*x) - (8*a*((2*(a*x^2 + b*x^3)^(5/2))/(11*b
*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b
*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/
(11*b)))/(13*b)
```

3.241.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.241. $\int x(ax^2+bx^3)^{3/2}\,dx$

3.241.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^{2}}$	21
gosper	$\frac{2(bx+a)\left(1155b^{4}x^{4}-840a\,b^{3}x^{3}+560a^{2}b^{2}x^{2}-320a^{3}bx+128a^{4}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{15015x^{3}b^{5}}$	68
default	$\frac{2(bx+a)\left(1155b^{4}x^{4}-840a\,b^{3}x^{3}+560a^{2}b^{2}x^{2}-320a^{3}bx+128a^{4}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{15015x^{3}b^{5}}$	68
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(1155b^{6}x^{6}+1470a\,x^{5}b^{5}+35a^{2}x^{4}b^{4}-40a^{3}x^{3}b^{3}+48a^{4}x^{2}b^{2}-64a^{5}xb+128a^{6}\right)}{15015x\,b^{5}}$	83
trager	$\frac{2\left(1155b^{6}x^{6}+1470a\,x^{5}b^{5}+35a^{2}x^{4}b^{4}-40a^{3}x^{3}b^{3}+48a^{4}x^{2}b^{2}-64a^{5}xb+128a^{6}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15015b^{5}x}$	85

input
```
int(x*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

$$\int x\left(ax^{2}+bx^{3}\right)^{3/2}\,dx=\frac{2\left(1155\,b^{6}x^{6}+1470\,ab^{5}x^{5}+35\,a^{2}b^{4}x^{4}-40\,a^{3}b^{3}x^{3}+48\,a^{4}b^{2}x^{2}-64\,a^{5}bx+128\,a^{6}\right)\sqrt{bx^{3}+ax^{2}}}{15015\,b^{5}x}$$

input
```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

3.241.6 Sympy [F]

$$\int x(ax^2+bx^3)^{3/2}\,dx=\int x\left(x^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x*(x**2*(a + b*x))**(3/2), x)
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int x(ax^2+bx^3)^{3/2}\,dx=\frac{2\,(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6)\sqrt{bx+a}}{15015\,b^5}$$

input
```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

3.241.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(116) = 232.

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.81

$$\int x(ax^2+bx^3)^{3/2}\,dx=-\frac{256\,a^{\frac{13}{2}}\mathrm{sgn}(x)}{15015\,b^5}+\frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)a^2\mathrm{sgn}(x)}{b^4}+\frac{130\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx\right.}{}\right.}{}$$

input
```
integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b
*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
sqrt(b*x + a)*a^4)*a^2*sgn(x)/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x +
a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*
x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*sgn(x)/b^4 + 15*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
 a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003
*sqrt(b*x + a)*a^6)*sgn(x)/b^4)/b
```

3.241.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x(ax^2+bx^3)^{3/2}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2\,(128\,a^4-320\,a^3\,b\,x+560\,a^2\,b^2\,x^2-840\,a\,b^3\,x^3+1155\,b^4\,x^4)}{15015\,b^5\,x}$$

input
```
int(x*(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(128*a^4 + 1155*b^4*x^4 - 840*a*b^3*x
^3 + 560*a^2*b^2*x^2 - 320*a^3*b*x))/(15015*b^5*x)
```

3.242 $\int (ax^2+bx^3)^{3/2} \, dx$

3.242.1 Optimal result

Integrand size = 15, antiderivative size = 108

$$\int (ax^2+bx^3)^{3/2} \, dx = -\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2+bx^3)^{5/2}}{11bx^2}$$

output
```
-32/1155*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^5+16/231*a^2*(b*x^3+a*x^2)^(5/2)/b^
3/x^4-4/33*a*(b*x^3+a*x^2)^(5/2)/b^2/x^3+2/11*(b*x^3+a*x^2)^(5/2)/b/x^2
```

3.242.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int (ax^2+bx^3)^{3/2} \, dx = \frac{2x(a+bx)^3 (-16a^3+40a^2bx-70ab^2x^2+105b^3x^3)}{1155b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(115
5*b^4*Sqrt[x^2*(a + b*x)])
```

3.242.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*
b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x
^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)
```

3.242.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.242.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
gosper	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
default	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)}{1155x\,b^{4}}$	72
trager	$-\frac{2\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{1155b^{4}x}$	74

input
```
int((b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)^(5/2)/b
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int (ax^2 + bx^3)^{3/2} \, dx = \frac{2 (105 \, b^5 x^5 + 140 \, a b^4 x^4 + 5 \, a^2 b^3 x^3 - 6 \, a^3 b^2 x^2 + 8 \, a^4 b x - 16 \, a^5) \sqrt{b x^3 + a x^2}}{1155 \, b^4 x}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

3.242.6 Sympy [F]

$$\int (ax^2 + bx^3)^{3/2} \, dx = \int (ax^2 + bx^3)^{\frac{3}{2}} \, dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(3/2), x)
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int (ax^2 + bx^3)^{3/2} \, dx = \frac{2 (105 \, b^5 x^5 + 140 \, a b^4 x^4 + 5 \, a^2 b^3 x^3 - 6 \, a^3 b^2 x^2 + 8 \, a^4 b x - 16 \, a^5) \sqrt{b x + a}}{1155 \, b^4}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

3.242.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(92) = 184.

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.94

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{32\,a^{\frac{11}{2}}\,\mathrm{sgn}(x)}{1155\,b^4}+\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^2\mathrm{sgn}(x)}{b^3}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3\right.}{b^3}\right.}{3465}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 +
 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
- 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b
*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(
x)/b^3)/b
```

3.242.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2\,(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3)}{1155\,b^4\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
 - 40*a^2*b*x))/(1155*b^4*x)
```

3.243 $\int \frac{(ax^2+bx^3)^{3/2}}{x} dx$

3.243.1 Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{(ax^2+bx^3)^{3/2}}{x} dx = \frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5} - \frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

output
```
16/315*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^5-8/63*a*(b*x^3+a*x^2)^(5/2)/b^2/x^4+
2/9*(b*x^3+a*x^2)^(5/2)/b/x^3
```

3.243.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x} dx = \frac{2x(a+bx)^3(8a^2-20abx+35b^2x^2)}{315b^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
```

output
```
(2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b
*x)])
```

3.243.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\int \frac{(bx^3+ax^2)^{3/2}}{x^2}dx}{9b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{2a\int \frac{(bx^3+ax^2)^{3/2}}{x^3}dx}{7b}\right)}{9b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}\right)}{9b}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(
35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)
```

3.243.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.243.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 1.82 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
pseudoelliptic	$-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(bx+4a)}{3}$	35
gosper	$\frac{2(bx+a)(35b^2x^2-20abx+8a^2)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{315b^3x^3}$	46
default	$\frac{2(bx+a)(35b^2x^2-20abx+8a^2)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{315b^3x^3}$	46
risch	$\frac{2\sqrt{x^2(bx+a)}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315x\,b^3}$	61
trager	$\frac{2(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)\sqrt{b\,x^3+a\,x^2}}{315b^3x}$	63

input
```
int((b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/3*(b*x+a)^(1/2)*(b*x+4*a)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx^3+ax^2}}{315\,b^3x}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x^3 + a*x^2)/(b^3*x)
```

3.243.6 Sympy [F]

$$\int \frac{(ax^2 + bx^3)^{3/2}}{x}\, dx = \int \frac{(x^2(a + bx))^{\frac{3}{2}}}{x}\, dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x, x)
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{(ax^2 + bx^3)^{3/2}}{x}\, dx = \frac{2\,(35\,b^4x^4 + 50\,ab^3x^3 + 3\,a^2b^2x^2 - 4\,a^3bx + 8\,a^4)\sqrt{bx + a}}{315\,b^3}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x + a)/b^3
```

3.243.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \frac{(ax^2 + bx^3)^{3/2}}{x}\, dx = -\frac{16\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^3} + \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)a^2\mathrm{sgn}(x)}{b^2} + \frac{18\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)a\mathrm{sgn}(x)}{b^2} + \frac{\left(35\,(bx\cdots\right.}{}\right.}{315\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
-16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(
3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*sgn(x)/b^2 + 18*(5*(b*x + a)^(7/2) - 21
*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(
x)/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)
*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^2)/b
```

3.243.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2\,(8\,a^2-20\,a\,b\,x+35\,b^2\,x^2)}{315\,b^3\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x,x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*b*x))/(315
*b^3*x)
```

3.244 $\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx$

3.244.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = -\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5} + \frac{2(ax^2+bx^3)^{5/2}}{7bx^4}$$

output
```
-4/35*a*(b*x^3+a*x^2)^(5/2)/b^2/x^5+2/7*(b*x^3+a*x^2)^(5/2)/b/x^4
```

3.244.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2(x^2(a+bx))^{5/2}(-2a+5bx)}{35b^2x^5}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

output
```
(2*(x^2*(a + b*x))^(5/2)*(-2*a + 5*b*x))/(35*b^2*x^5)
```

3.244.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

output
```
(-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b
*x^4)
```

3.244.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.244.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
default	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)}{35x\,b^{2}}$	50
trager	$-\frac{2\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35b^{2}x}$	52
pseudoelliptic	$\frac{2bx\sqrt{bx+a}\,\sqrt{a}-3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx-\sqrt{bx+a}\,a^{\frac{3}{2}}}{x\sqrt{a}}$	52

input
```
int((b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*
x)
```

3.244.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}}{35\,b^2}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(44) = 88.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.62

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{4\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^2} + \frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^2\mathrm{sgn}(x)}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a\mathrm{sgn}(x)}{b}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a\cdots\right.}{b}\right.}{105\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*
a^2*sgn(x)/b + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*a*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b)/b
```

3.244.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = -\frac{2\,(2\,a-5\,b\,x)\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2}{35\,b^2\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^2,x)
```

output
```
-(2*(2*a - 5*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(35*b^2*x)
```

3.245 $\int \frac{(ax^2+bx^3)^{3/2}}{x^3} \, dx$

3.245.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3} \, dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

output
```
2/5*(b*x^3+a*x^2)^(5/2)/b/x^5
```

3.245.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3} \, dx = \frac{2(x^2(a+bx))^{5/2}}{5bx^5}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

output
```
(2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

3.245.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \; 1920$$

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

3.245.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.245.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{2(bx+a)(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}{5b\,x^{3}}$	27
default	$\frac{2(bx+a)(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}{5b\,x^{3}}$	27
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,(b^{2}x^{2}+2abx+a^{2})}{5xb}$	36
trager	$\frac{2(b^{2}x^{2}+2abx+a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{5bx}$	38
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-2\sqrt{bx+a}\,a^{\frac{3}{2}}-5bx\sqrt{bx+a}\,\sqrt{a}}{4x^{2}\sqrt{a}}$	56

input
```
int((b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)/b*(b*x^3+a*x^2)^(3/2)/x^3
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx^3+ax^2}}{5\,bx}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

3.245.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b}+\frac{2\left(15\sqrt{bx+a}a^2\mathrm{sgn}(x)+10\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a\mathrm{sgn}(x)+\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx}\right.\right.}{15\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-2/5*a^(5/2)*sgn(x)/b + 2/15*(15*sqrt(b*x + a)*a^2*sgn(x) + 10*((b*x + a)^
(3/2) - 3*sqrt(b*x + a)*a)*a*sgn(x) + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*sgn(x))/b
```

3.245.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx=\frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^2}{5\,b\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^3,x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(5*b*x)
```

3.246 $\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx$

3.246.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx = \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3} - 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

output
```
2/3*(b*x^3+a*x^2)^(3/2)/x^3-2*a^(3/2)*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2
))+2*a*(b*x^3+a*x^2)^(1/2)/x
```

3.246.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4} dx = \frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}(4a+bx) - 3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

output
```
(2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

3.246.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{ 1927}$$

$$a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 1927}$$

$$a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 1914}$$

$$a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 219}$$

$$a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

output
```
(2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) + a*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[
a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])
```

3.246.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.246.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{3} x^{3}-\sqrt{bx+a}\,\left(\sqrt{a}\, b^{2} x^{2}+\frac{14 a^{\frac{3}{2}} b x}{3}+\frac{8 a^{\frac{5}{2}}}{3}\right)}{8 a^{\frac{3}{2}} x^{3}}$	61
default	$-\frac{2 \left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(3 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\left(bx+a\right)^{\frac{3}{2}}-3 \sqrt{bx+a}\, a\right)}{3 x^{3} \left(bx+a\right)^{\frac{3}{2}}}$	63

input
```
int((b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(3/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^3*x^3-(b*x+a)^(1/2)*(a^(1/2)
*b^2*x^2+14/3*a^(3/2)*b*x+8/3*a^(5/2)))/x^3
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \left[\frac{3\,a^{\frac{3}{2}}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(bx+4\,a)}{3\,x}, \frac{2\left(3\,\sqrt{-a}ax\arctan\left(\right.\right.}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
 + 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b
*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x]
```

3.246.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

3.246.7 Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + \frac{2}{3}\,(bx+a)^{\frac{3}{2}}\mathrm{sgn}(x) + 2\,\sqrt{bx+a}a\mathrm{sgn}(x) - \frac{2\left(3\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + 4\,\sqrt{-a}a^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)
*sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) +
 4*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^4, x)
```

3.247 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx$

3.247.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx = \frac{3b\sqrt{ax^2+bx^3}}{x} - \frac{(ax^2+bx^3)^{3/2}}{x^4} - 3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

output
```
-(b*x^3+a*x^2)^(3/2)/x^4-3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2
)+3*b*(b*x^3+a*x^2)^(1/2)/x
```

3.247.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx = -\frac{\sqrt{a+bx}\left((a-2bx)\sqrt{a+bx}+3\sqrt{a}bx\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

output
```
-((Sqrt[a + b*x]*((a - 2*b*x)*Sqrt[a + b*x] + 3*Sqrt[a]*b*x*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]))/Sqrt[x^2*(a + b*x)])
```

3.247.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1926, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

$$\Big\downarrow \text{1927}$$

$$\frac{3}{2}b\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

output
```
-((a*x^2 + b*x^3)^(3/2)/x^4) + (3*b*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]))/2
```

3.247.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.247.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{a\sqrt{x^2(bx+a)}}{x^2}+\frac{b\left(4\sqrt{bx+a}-6\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{x^2(bx+a)}}{2x\sqrt{bx+a}}$	70
default	$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-2bx\sqrt{bx+a}\,\sqrt{a}+3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{x^4(bx+a)^{\frac{3}{2}}\sqrt{a}}$	72
pseudoelliptic	$\frac{-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4}{64}+\frac{3\sqrt{bx+a}\left(\sqrt{a}\,b^3x^3-\frac{2a^{\frac{3}{2}}b^2x^2}{3}-8a^{\frac{5}{2}}bx-\frac{16a^{\frac{7}{2}}}{3}\right)}{64}}{a^{\frac{5}{2}}x^4}$	72

input
```
int((b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-a/x^2*(x^2*(b*x+a))^(1/2)+1/2*b*(4*(b*x+a)^(1/2)-6*a^(1/2)*arctanh((b*x+a
)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \left[\frac{3\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{2\,x^2}, \frac{3\sqrt{-a}bx^2\arctan\left(\right.}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/
x^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sq
rt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2]
```

3.247.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

3.247.7 Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\text{sgn}(x)}{\sqrt{-a}} + 2\,\sqrt{bx+a}b^2\text{sgn}(x) - \frac{\sqrt{bx+a}ab\text{sgn}(x)}{x}}{b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
(3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*
b^2*sgn(x) - sqrt(b*x + a)*a*b*sgn(x)/x)/b
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^5}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^5, x)
```

3.248 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx$

3.248.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx = -\frac{3b\sqrt{ax^2+bx^3}}{4x^2} - \frac{(ax^2+bx^3)^{3/2}}{2x^5} - \frac{3b^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}$$

output
```
-1/2*(b*x^3+a*x^2)^(3/2)/x^5-3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)/a^(1/2)-3/4*b*(b*x^3+a*x^2)^(1/2)/x^2
```

3.248.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(2a+5bx)+3b^2x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}x^3\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

output
```
-1/4*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(2*a + 5*b*x) + 3*b^2*x^2
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^3*Sqrt[a + b*x])
```

3.248.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1926, 1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6} \, dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{4}b\int \frac{\sqrt{bx^3+ax^2}}{x^3}dx-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3}{4}b\left(-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

output
```
-1/2*(a*x^2 + b*x^3)^(3/2)/x^5 + (3*b*(-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]))/4
```

3.248.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

3.248.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{(5bx+2a)\sqrt{x^2(bx+a)}}{4x^3}-\frac{3b^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{4\sqrt{a}\,x\sqrt{bx+a}}$	67
default	$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2+5\sqrt{a}\,(bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{4x^5(bx+a)^{\frac{3}{2}}\sqrt{a}}$	74
pseudoelliptic	$-\frac{-\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^5b^5}{128}+\sqrt{bx+a}\left(\frac{15\sqrt{a}\,b^4x^4}{128}-\frac{5a^{\frac{3}{2}}b^3x^3}{64}+\frac{a^{\frac{5}{2}}b^2x^2}{16}+\frac{11a^{\frac{7}{2}}bx}{8}+a^{\frac{9}{2}}\right)}{5a^{\frac{7}{2}}x^5}$	82

input
```
int((b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(5*b*x+2*a)/x^3*(x^2*(b*x+a))^(1/2)-3/4*b^2/a^(1/2)*arctanh((b*x+a)^(
1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \left[\frac{3\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(5\,abx+2\,a^2)}{8\,ax^3}, \frac{3\,\sqrt{-a}b^2x^3\,\mathrm{arct}}{}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*
(5*a*b*x + 2*a^2))/(a*x^3)]
```

3.248.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

3.248.7 Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} - \frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)-3\,\sqrt{bx+a}ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(
3/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^6}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^6,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^6, x)
```

3.249 $\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx$

3.249.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2} - \frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}$$

output
```
-1/3*(b*x^3+a*x^2)^(3/2)/x^6+1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)/a^(3/2)-1/4*b*(b*x^3+a*x^2)^(1/2)/x^3-1/8*b^2*(b*x^3+a*x^2)^(1/2)/a/x^2
```

3.249.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(8a^2+14abx+3b^2x^2)+3b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{3/2}x^4\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^
2)) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(24*a^(3/2)*x^4*Sqrt[a +
b*x])
```

3.249.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1926, 1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^4}dx-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1926}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1931}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

output
```
-1/3*(a*x^2 + b*x^3)^(3/2)/x^6 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(
Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)))/4))/2
```

3.249.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.249.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\left(3 b^{2} x^{2}+14 a b x +8 a^{2}\right) \sqrt{x^{2} \left(b x +a \right)}}{24 x^{4} a}+\frac{b^{3} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{8 a^{\frac{3}{2}} x \sqrt{b x +a}}$	81
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(3 a^{\frac{3}{2}} \left(b x +a \right)^{\frac{5}{2}}-3 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a \,b^{3} x^{3}+8 a^{\frac{5}{2}} \left(b x +a \right)^{\frac{3}{2}}-3 \sqrt{b x +a}\, a^{\frac{7}{2}}\right)}{24 x^{6} \left(b x +a \right)^{\frac{3}{2}} a^{\frac{5}{2}}}$	87
pseudoelliptic	$-\frac{13 \left(\frac{105 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{6} x^{6}}{1664}+\sqrt{b x +a}\, \left(-\frac{105 \sqrt{a}\, b^{5} x^{5}}{1664}+\frac{35 a^{\frac{3}{2}} b^{4} x^{4}}{832}-\frac{7 a^{\frac{5}{2}} b^{3} x^{3}}{208}+\frac{3 b^{2} x^{2} a^{\frac{7}{2}}}{104}+a^{\frac{9}{2}} b x +\frac{10 a^{\frac{11}{2}}}{13}\right)\right)}{60 a^{\frac{9}{2}} x^{6}}$	94

input
```
int((b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(3*b^2*x^2+14*a*b*x+8*a^2)/x^4/a*(x^2*(b*x+a))^(1/2)+1/8*b^3/a^(3/2)
*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.61

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^2x^4},\right.$$
$$\left.-\frac{3\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{24\,a^2x^4}\right]$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x
^4), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))
+ (3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4)]
```

3.249.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

3.249.7 Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = -\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a} + \frac{3\,(bx+a)^{\frac{5}{2}}b^4\mathrm{sgn}(x)+8\,(bx+a)^{\frac{3}{2}}ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}a^2b^4\mathrm{sgn}(x)}{ab^3x^3}}{24\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x
+ a)^(5/2)*b^4*sgn(x) + 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a
^2*b^4*sgn(x))/(a*b^3*x^3))/b
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^7}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^7,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^7, x)
```

3.250 $\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx$

3.250.1 Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}$$

output
```
-1/4*(b*x^3+a*x^2)^(3/2)/x^7-3/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2
))/a^(5/2)-1/8*b*(b*x^3+a*x^2)^(1/2)/x^4-1/32*b^2*(b*x^3+a*x^2)^(1/2)/a/x^
3+3/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

3.250.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(16a^3+24a^2bx+2ab^2x^2-3b^3x^3)+3b^4x^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{64a^{5/2}x^5\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

output
```
-1/64*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(16*a^3 + 24*a^2*b*x + 2
*a*b^2*x^2 - 3*b^3*x^3) + 3*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5
/2)*x^5*Sqrt[a + b*x])
```

3.250.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1926, 1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{8}b\int \frac{\sqrt{bx^3+ax^2}}{x^5}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

$$\Big\downarrow \; 219$$

$$\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

output
```
-1/4*(a*x^2 + b*x^3)^(3/2)/x^7 + (3*b*(-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(
-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) +
(b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/6))/8
```

3.250.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.250.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

method	result	si
risch	$-\frac{\left(-3b^{3}x^{3}+2a\,b^{2}x^{2}+24a^{2}bx+16a^{3}\right)\sqrt{x^{2}\left(bx+a\right)}}{64x^{5}a^{2}}-\frac{3b^{4}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^{2}\left(bx+a\right)}}{64a^{\frac{5}{2}}x\sqrt{bx+a}}$	9
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(3\left(bx+a\right)^{\frac{7}{2}}a^{\frac{5}{2}}-11\left(bx+a\right)^{\frac{5}{2}}a^{\frac{7}{2}}-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}x^{4}b^{4}-11\left(bx+a\right)^{\frac{3}{2}}a^{\frac{9}{2}}+3\sqrt{bx+a}\,a^{\frac{11}{2}}\right)}{64x^{7}\left(bx+a\right)^{\frac{3}{2}}a^{\frac{9}{2}}}$	1
pseudoelliptic	$-\frac{5\left(-\frac{63\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^{7}b^{7}}{1280}+\sqrt{bx+a}\left(\frac{63\sqrt{a}\,b^{6}x^{6}}{1280}-\frac{21a^{\frac{3}{2}}b^{5}x^{5}}{640}+\frac{21a^{\frac{5}{2}}b^{4}x^{4}}{800}-\frac{9a^{\frac{7}{2}}b^{3}x^{3}}{400}+\frac{a^{\frac{9}{2}}b^{2}x^{2}}{50}+a^{\frac{11}{2}}bx+\frac{4a^{\frac{13}{2}}}{5}\right)\right)}{28a^{\frac{11}{2}}x^{7}}$	1

input
```
int((b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/64*(-3*b^3*x^3+2*a*b^2*x^2+24*a^2*b*x+16*a^3)/x^5/a^2*(x^2*(b*x+a))^(1/
2)-3/64*b^4/a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(
b*x+a)^(1/2)
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.44

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \left[\frac{3\sqrt{a}b^4x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16\,a^4)\sqrt{bx}}{128\,a^3x^5}\right.$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^
3 + a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)
*sqrt(-a)/(a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqr
t(b*x^3 + a*x^2))/(a^3*x^5)]
```

3.250.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

3.250.7 Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \frac{\frac{3\,b^5 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}a^3b^5\mathrm{s}}{a^2b^4x^4}}{64\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
 + a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3
/2)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^8}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^8,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^8, x)
```

3.251 $\int \frac{(ax^2+bx^3)^{3/2}}{x^9} dx$

3.251.1 Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{3b^5\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}$$

output
```
-1/5*(b*x^3+a*x^2)^(3/2)/x^8+3/128*b^5*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/
2))/a^(7/2)-3/40*b*(b*x^3+a*x^2)^(1/2)/x^5-1/80*b^2*(b*x^3+a*x^2)^(1/2)/a/
x^4+1/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^3-3/128*b^4*(b*x^3+a*x^2)^(1/2)/a^3
/x^2
```

3.251.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.70

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(128a^4+176a^3bx+8a^2b^2x^2-10ab^3x^3+15b^4x^4)+15b^5\right.}{640a^{7/2}x^6\sqrt{a+bx}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(128*a^4 + 176*a^3*b*x + 8*a
^2*b^2*x^2 - 10*a*b^3*x^3 + 15*b^4*x^4)) + 15*b^5*x^5*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]]))/(640*a^(7/2)*x^6*Sqrt[a + b*x])
```

3.251.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1926, 1926, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{10}b\int \frac{\sqrt{bx^3+ax^2}}{x^6}dx-\frac{(ax^2+bx^3)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1926}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{(ax^2+bx^3)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{(ax^2+bx^3)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{(ax^2+bx^3)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$\Big\downarrow$ 1914

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$\Big\downarrow$ 219

$$\frac{3}{10}b\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3}}{4x^5}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

3.251. $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx$

output
```
-1/5*(a*x^2 + b*x^3)^(3/2)/x^8 + (3*b*(-1/4*Sqrt[a*x^2 + b*x^3]/x^5 + (b*(
-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3)
- (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2
 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/8))/10
```

3.251.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.251. $\int \frac{(ax^2+bx^3)^{3/2}}{x^9} dx$

3.251.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{\left(15 b^{4} x^{4}-10 a \,b^{3} x^{3}+8 a^{2} b^{2} x^{2}+176 a^{3} b x +128 a^{4}\right) \sqrt{x^{2} \left(b x +a \right)}}{640 x^{6} a^{3}}+\frac{3 b^{5} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{128 a^{\frac{7}{2}} x \sqrt{b x +a}}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(15 (b x +a)^{\frac{9}{2}} a^{\frac{7}{2}}-70 (b x +a)^{\frac{7}{2}} a^{\frac{9}{2}}+128 (b x +a)^{\frac{5}{2}} a^{\frac{11}{2}}-15 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{3} x^{5} b^{5}+70 (b x +a)^{\frac{3}{2}} a^{\frac{13}{2}}-15 \sqrt{b x}\right.}{640 x^{8} (b x +a)^{\frac{3}{2}} a^{\frac{13}{2}}}$
pseudoelliptic	$-\frac{\frac{693 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{8} x^{8}}{256}+\sqrt{b x +a}\, \left(-\frac{693 \sqrt{a}\, b^{7} x^{7}}{256}+\frac{231 a^{\frac{3}{2}} b^{6} x^{6}}{128}-\frac{231 a^{\frac{5}{2}} b^{5} x^{5}}{160}+\frac{99 a^{\frac{7}{2}} b^{4} x^{4}}{80}-\frac{11 a^{\frac{9}{2}} b^{3} x^{3}}{10}+a^{\frac{11}{2}} b^{2} x^{2}+68 a^{\frac{13}{2}} b x\right.}{448 a^{\frac{13}{2}} x^{8}}$

input
```
int((b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(15*b^4*x^4-10*a*b^3*x^3+8*a^2*b^2*x^2+176*a^3*b*x+128*a^4)/x^6/a^3
*(x^2*(b*x+a))^(1/2)+3/128*b^5/a^(7/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/x/(b
*x+a)^(1/2)*(x^2*(b*x+a))^(1/2)
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.33

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \left[\frac{15\,\sqrt{a}b^5x^6\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\,(15\,ab^4x^4 - 10\,a^2b^3x^3 + 8\,a^3b^2x^2 + 176\,a^4}{1280\,a^4x^6}\right.$$

$$\left. -\frac{15\,\sqrt{-a}b^5x^6\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + (15\,ab^4x^4 - 10\,a^2b^3x^3 + 8\,a^3b^2x^2 + 176\,a^4bx + 128\,a^5)\sqrt{bx^3+ax^2}}{640\,a^4x^6}\right]$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqr
t(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*
x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*a
rctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3
 + 8*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]
```

3.251.6 Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2)/x**9,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)/x**9, x)
```

3.251.7 Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx =$$
$$-\frac{\frac{15\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^3} + \frac{15\,(bx+a)^{\frac{9}{2}}b^6\mathrm{sgn}(x) - 70\,(bx+a)^{\frac{7}{2}}ab^6\mathrm{sgn}(x) + 128\,(bx+a)^{\frac{5}{2}}a^2b^6\mathrm{sgn}(x) + 70\,(bx+a)^{\frac{3}{2}}a^3b^6\mathrm{sgn}(x) - 15\,\sqrt{bx+a}a^4b^6\mathrm{s}}{a^3b^5x^5}}{640\,b}$$

input
```
integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
-1/640*(15*b^6*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*
(b*x + a)^(9/2)*b^6*sgn(x) - 70*(b*x + a)^(7/2)*a*b^6*sgn(x) + 128*(b*x +
a)^(5/2)*a^2*b^6*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b^6*sgn(x) - 15*sqrt(b*x
+ a)*a^4*b^6*sgn(x))/(a^3*b^5*x^5))/b
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^9}\,dx$$

input
```
int((a*x^2 + b*x^3)^(3/2)/x^9,x)
```

output
```
int((a*x^2 + b*x^3)^(3/2)/x^9, x)
```

3.252 $\int \frac{x^4}{\sqrt{ax^2+bx^3}} dx$

3.252.1 Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}} dx = \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

output
```
16/35*a^2*(b*x^3+a*x^2)^(1/2)/b^3-32/35*a^3*(b*x^3+a*x^2)^(1/2)/b^4/x-12/3
5*a*x*(b*x^3+a*x^2)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x^2)^(1/2)/b
```

3.252.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}} dx = \frac{2\sqrt{x^2(a+bx)}(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4x}$$

input
```
Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(3
5*b^4*x)
```

3.252.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{7b}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{7b}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{7b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{7b}$$

input
```
Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a*x^2 + b*x^3])/(7*b) - (6*a*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
- (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x
)))/(5*b)))/(7*b)
```

3.252.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.252.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

method	result	size
trager	$-\frac{2\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35b^{4}x}$	52
risch	$-\frac{2x\left(bx+a\right)\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35\sqrt{x^{2}\left(bx+a\right)}\,b^{4}}$	53
pseudoelliptic	$\frac{2\sqrt{bx+a}\,\left(35b^{4}x^{4}-40a\,b^{3}x^{3}+48a^{2}b^{2}x^{2}-64a^{3}bx+128a^{4}\right)}{315b^{5}}$	54
gosper	$-\frac{2\left(bx+a\right)\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)x}{35b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	55
default	$-\frac{2\left(bx+a\right)\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)x}{35b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	55

input
```
int(x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4/x*(b*x^3+a*x^2)^(1/2)
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3)\sqrt{bx^3+ax^2}}{35\,b^4x}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b
^4*x)
```

3.252.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^4}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)}{35\,\sqrt{bx+a}b^4}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*
x + a)*b^4)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{32\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^4} + \frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)}{35\,b^4\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/35*a^(7/2)*sgn(x)/b^4 + 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
+ 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/(b^4*sgn(x))
```

3.252.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3)}{35\,b^4\,x}$$

input
```
int(x^4/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2 - 8*a^2*b*x))/
(35*b^4*x)
```

3.253 $\int \frac{x^3}{\sqrt{ax^2+bx^3}} dx$

3.253.1 Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}} dx = -\frac{8a\sqrt{ax^2+bx^3}}{15b^2} + \frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x} + \frac{2x\sqrt{ax^2+bx^3}}{5b}$$

output
```
-8/15*a*(b*x^3+a*x^2)^(1/2)/b^2+16/15*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x+2/5*x*
(b*x^3+a*x^2)^(1/2)/b
```

3.253.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}} dx = \frac{2\sqrt{x^2(a+bx)}(8a^2-4abx+3b^2x^2)}{15b^3x}$$

input
```
Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

3.253.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1922$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}$$

$$\Big\downarrow \ 1922$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}$$

input
```
Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4
*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*b)
```

3.253.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.253.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
trager	$\frac{2\left(3b^{2}x^{2}-4abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{3}x}$	41
risch	$\frac{2x(bx+a)\left(3b^{2}x^{2}-4abx+8a^{2}\right)}{15\sqrt{x^{2}(bx+a)}\,b^{3}}$	42
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35b^{4}}$	43
gosper	$\frac{2(bx+a)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44
default	$\frac{2(bx+a)\left(3b^{2}x^{2}-4abx+8a^{2}\right)x}{15b^{3}\sqrt{b\,x^{3}+a\,x^{2}}}$	44

input
```
int(x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3/x*(b*x^3+a*x^2)^(1/2)
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

3.253.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^3x^3 - ab^2x^2 + 4\,a^2bx + 8\,a^3)}{15\,\sqrt{bx+a}b^3}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = -\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^3} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-16/15*a^(5/2)*sgn(x)/b^3 + 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a
 + 15*sqrt(b*x + a)*a^2)/(b^3*sgn(x))
```

3.253.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(8\,a^2-4\,a\,b\,x+3\,b^2\,x^2)}{15\,b^3\,x}$$

input
```
int(x^3/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*b^3*x)
```

3.254 $\int \frac{x^2}{\sqrt{ax^2+bx^3}} dx$

3.254.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

output
```
2/3*(b*x^3+a*x^2)^(1/2)/b-4/3*a*(b*x^3+a*x^2)^(1/2)/b^2/x
```

3.254.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2(-2a+bx)\sqrt{x^2(a+bx)}}{3b^2x}$$

input
```
Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)
```

3.254.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1922$$

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

input
```
Int[x^2/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

3.254.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.254.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
trager	$-\frac{2(-bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{3b^{2}x}$	30
risch	$-\frac{2x(bx+a)(-bx+2a)}{3\sqrt{x^{2}(bx+a)}\,b^{2}}$	31
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3b^{2}x^{2}-4abx+8a^{2})}{15b^{3}}$	32
gosper	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
default	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33

input
```
int(x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x+2*a)/b^2/x*(b*x^3+a*x^2)^(1/2)
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx^3+ax^2}(bx-2\,a)}{3\,b^2x}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

3.254.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\,(b^2x^2-abx-2\,a^2)}{3\,\sqrt{bx+a}b^2}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^2}+\frac{2\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)}{3\,b^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/3*a^(3/2)*sgn(x)/b^2 + 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/(b^2*sg
n(x))
```

3.254.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=-\frac{\left(\frac{4\,a}{3\,b^2}-\frac{2\,x}{3\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x}$$

input
```
int(x^2/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
-(((4*a)/(3*b^2) - (2*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

3.255 $\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$

3.255.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{bx}$$

output
```
2*(b*x^3+a*x^2)^(1/2)/b/x
```

3.255.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}}{bx}$$

input
```
Integrate[x/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)])/(b*x)
```

3.255.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

input
```
Int[x/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

3.255.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.255.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
trager	$\frac{2\sqrt{b\,x^3+a\,x^2}}{bx}$	22
risch	$\frac{2x(bx+a)}{\sqrt{x^2(bx+a)}\,b}$	23
gosper	$\frac{2x(bx+a)}{b\sqrt{b\,x^3+a\,x^2}}$	25
default	$\frac{2x(bx+a)}{b\sqrt{b\,x^3+a\,x^2}}$	25

input
```
int(x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\, dx = \frac{2\sqrt{bx^3 + ax^2}}{bx}$$

input
```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^3 + a*x^2)/(b*x)
```

3.255.6 Sympy [F]

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\, dx = \int \frac{x}{\sqrt{x^2 (a + bx)}}\, dx$$

input
```
integrate(x/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(a + b*x)), x)
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\, dx = \frac{2\sqrt{bx + a}}{b}$$

input
```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)/b
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\,dx = -\frac{2\sqrt{a}\mathrm{sgn}(x)}{b} + \frac{2\sqrt{bx+a}}{b\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*sgn(x)/b + 2*sqrt(b*x + a)/(b*sgn(x))
```

3.255.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{ax^2 + bx^3}}\,dx = \frac{2\,|x|\,\sqrt{a+b\,x}}{b\,x}$$

input
```
int(x/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*abs(x)*(a + b*x)^(1/2))/(b*x)
```

3.256 $\int \frac{1}{\sqrt{ax^2+bx^3}} dx$

3.256.1 Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
 b*x)])
```

3.256.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \ 1914$$
$$-2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}$$
$$\Big\downarrow \ 219$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

3.256.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

3.256.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13
default	$-\frac{2x\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{a}}$	39

input
```
int(1/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)^(1/2)/b
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{\sqrt{a}}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)}{a}\right]$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(
-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))/a]
```

3.256.6 Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**3), x)
```

3.256.7 Maxima [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3 + ax^2}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^3 + a*x^2), x)
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(
-a))/(sqrt(-a)*sgn(x))
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3 + a\,x^2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(1/(a*x^2 + b*x^3)^(1/2), x)
```

3.257 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

3.257.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=-\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

output
```
b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-(b*x^3+a*x^2)^(1/2)/a/x^2
```

3.257.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}(a+bx)+bx\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)) + b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/
(a^(3/2)*Sqrt[x^2*(a + b*x)])
```

3.257.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

$$\Big\downarrow \text{1914}$$

$$\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

input
```
Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3
]])/a^(3/2)
```

3.257.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.257.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.33

method	result	size
pseudoelliptic	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{\sqrt{bx+a}\,\left(\sqrt{bx+a}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{5}{2}}}$	55
risch	$-\frac{bx+a}{a\sqrt{x^{2}(bx+a)}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{a^{\frac{3}{2}}\,\sqrt{x^{2}(bx+a)}}$	59

input
```
int(1/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.35

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,a^2x^2},\right.$$
$$\left.-\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}a}{a^2x^2}\right]$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^
2) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^
3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

3.257.6 Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\sqrt{x^2\left(a+bx\right)}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

3.257.7 Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\,x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = -\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))
/(b*sgn(x))
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

3.258 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

3.258.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx=-\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}$$

output
```
-3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/2*(b*x^3+a*x^2)^
(1/2)/a/x^3+3/4*b*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

3.258.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{a}(-2a^2+abx+3b^2x^2)-3b^2x^2\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(Sqrt[a]*(-2*a^2 + a*b*x + 3*b^2*x^2) - 3*b^2*x^2*Sqrt[a + b*x]*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a + b*x)])
```

3.258.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

input
```
Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) +
(b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)
```

3.258.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.258.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-\sqrt{bx+a}\,\sqrt{a}}{a^{\frac{3}{2}}x}$	36
risch	$-\frac{(bx+a)(-3bx+2a)}{4a^{2}x\sqrt{x^{2}(bx+a)}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{4a^{\frac{5}{2}}\sqrt{x^{2}(bx+a)}}$	73
default	$-\frac{\sqrt{bx+a}\left(-3a^{\frac{3}{2}}bx\sqrt{bx+a}+3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{2}x^{2}+2\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{4x\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{7}{2}}}$	77

input
```
int(1/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^(3/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-(b*x+a)^(1/2)*a^(1/2))/x
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \left[\frac{3\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(3\,abx-2\,a^2)}{8\,a^3x^3}, \frac{3\,\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\cdots}{4\,a^3x^3}\right.$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-
a)*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2
)*(3*a*b*x - 2*a^2))/(a^3*x^3)]
```

3.258.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

3.258.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3
/2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/(b*sgn(x))
```

3.258.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{\frac{a}{b\,x}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a}{b\,x}\right)}{5\,x\,\sqrt{b\,x^3+a\,x^2}}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
-(2*(a/(b*x) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -a/(b*x)))/(5*x*(a*x^2
+ b*x^3)^(1/2))
```

3.259 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$

3.259.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}$$

output
```
5/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)-1/3*(b*x^3+a*x^2)^(
1/2)/a/x^4+5/12*b*(b*x^3+a*x^2)^(1/2)/a^2/x^3-5/8*b^2*(b*x^3+a*x^2)^(1/2)/
a^3/x^2
```

3.259.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \frac{-\sqrt{a}(8a^3-2a^2bx+5ab^2x^2+15b^3x^3)+15b^3x^3\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(8*a^3 - 2*a^2*b*x + 5*a*b^2*x^2 + 15*b^3*x^3)) + 15*b^3*x^3*Sq
rt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a +
b*x)])
```

3.259.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{219}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

input
```
Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3)
- (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2
 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)
```

3.259.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.259.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{-3 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{2} x^{2}-2 \sqrt{bx+a}\, a^{\frac{3}{2}}+3 b x \sqrt{bx+a}\, \sqrt{a}}{4 x^{2} a^{\frac{5}{2}}}$	56
risch	$-\frac{(bx+a)\left(15 b^{2} x^{2}-10 a b x+8 a^{2}\right)}{24 a^{3} x^{2} \sqrt{x^{2}(bx+a)}}+\frac{5 b^{3} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{bx+a}\, x}{8 a^{\frac{7}{2}} \sqrt{x^{2}(bx+a)}}$	84
default	$-\frac{\sqrt{bx+a}\left(15 a^{\frac{3}{2}} b^{2} x^{2} \sqrt{bx+a}-15 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{3} x^{3}-10 a^{\frac{5}{2}} b x \sqrt{bx+a}+8 \sqrt{bx+a}\, a^{\frac{7}{2}}\right)}{24 x^{2} \sqrt{b\, x^{3}+a\, x^{2}}\, a^{\frac{9}{2}}}$	95

input
```
int(1/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-3*arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-2*(b*x+a)^(1/2)*a^(3/2)+3*b
*x*(b*x+a)^(1/2)*a^(1/2))/x^2/a^(5/2)
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \left[\frac{15\,\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^4x^4},\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{24\,a^4x^4}\right]$$

input
```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4
*x^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x
)) + (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

3.259.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

3.259.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{15\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+33\,\sqrt{bx+a}a^2b^4}{a^3b^3x^3}}{24\,b\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a
)^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^
3*x^3))/(b*sgn(x))
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

3.260 $\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

3.260.1 Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x^4}{b\sqrt{ax^2+bx^3}} - \frac{16a\sqrt{ax^2+bx^3}}{5b^3} + \frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2}$$

output
```
-2*x^4/b/(b*x^3+a*x^2)^(1/2)-16/5*a*(b*x^3+a*x^2)^(1/2)/b^3+32/5*a^2*(b*x^
3+a*x^2)^(1/2)/b^4/x+12/5*x*(b*x^3+a*x^2)^(1/2)/b^2
```

3.260.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*
x)])
```

3.260.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{6\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) + (6*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (
4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))
/(5*b)))/b
```

3.260.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.260.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(bx+a)\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)x^{3}}{5b^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	56
default	$\frac{2(bx+a)\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)x^{3}}{5b^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	56
trager	$\frac{2\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{5(bx+a)b^{4}x}$	58
risch	$\frac{2\left(b^{2}x^{2}-3abx+11a^{2}\right)(bx+a)x}{5b^{4}\sqrt{x^{2}(bx+a)}}+\frac{2a^{3}x}{b^{4}\sqrt{x^{2}(bx+a)}}$	62
pseudoelliptic	$\frac{\frac{2}{11}b^{6}x^{6}-\frac{8}{33}a\,x^{5}b^{5}+\frac{80}{231}a^{2}x^{4}b^{4}-\frac{128}{231}a^{3}x^{3}b^{3}+\frac{256}{231}a^{4}x^{2}b^{2}-\frac{1024}{231}a^{5}xb-\frac{2048}{231}a^{6}}{b^{7}\sqrt{bx+a}}$	76

input
```
int(x^6/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(
3/2)
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)\sqrt{bx^3+ax^2}}{5\,(b^5x^2+ab^4x)}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*
x^2 + a*b^4*x)
```

3.260.6 Sympy [F]

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)}{5\,\sqrt{bx+a}b^4}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{32\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b^4} + \frac{2\,a^3}{\sqrt{bx+a}b^4\mathrm{sgn}\,(x)} + \frac{2\left((bx+a)^{\frac{5}{2}}b^{16} - 5\,(bx+a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-32/5*a^(5/2)*sgn(x)/b^4 + 2*a^3/(sqrt(b*x + a)*b^4*sgn(x)) + 2/5*((b*x +
a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/(b^2
0*sgn(x))
```

3.260.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(16\,a^3+8\,a^2\,b\,x-2\,a\,b^2\,x^2+b^3\,x^3)}{5\,b^4\,x\,\left(a+b\,x\right)}$$

input
```
int(x^6/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 + b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x))/(5*
b^4*x*(a + b*x))
```

3.261 $\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx$

3.261.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{16a\sqrt{ax^2+bx^3}}{3b^3x}$$

output
```
-2*x^3/b/(b*x^3+a*x^2)^(1/2)+8/3*(b*x^3+a*x^2)^(1/2)/b^2-16/3*a*(b*x^3+a*x
^2)^(1/2)/b^3/x
```

3.261.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

3.261.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}} dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{b} - \frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{b} - \frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{b} - \frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (4*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*
a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/b
```

3.261.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.261.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
default	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
trager	$-\frac{2\left(-b^{2}x^{2}+4abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3(bx+a)b^{3}x}$	48
risch	$-\frac{2(-bx+5a)(bx+a)x}{3b^{3}\sqrt{x^{2}(bx+a)}}-\frac{2a^{2}x}{b^{3}\sqrt{x^{2}(bx+a)}}$	52
pseudoelliptic	$\frac{\frac{2}{9}b^{5}x^{5}-\frac{20}{63}a\,b^{4}x^{4}+\frac{32}{63}a^{2}b^{3}x^{3}-\frac{64}{63}a^{3}b^{2}x^{2}+\frac{256}{63}a^{4}bx+\frac{512}{63}a^{5}}{b^{6}\sqrt{bx+a}}$	65

input
```
int(x^5/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)\sqrt{bx^3+ax^2}}{3\,(b^4x^2+ab^3x)}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

3.261.6 Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)}{3\,\sqrt{bx+a}b^3}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{16\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^3} - \frac{2\,a^2}{\sqrt{bx+a}b^3\mathrm{sgn}\,(x)} + \frac{2\left(\left(bx+a\right)^{\frac{3}{2}}b^6 - 6\,\sqrt{bx+a}ab^6\right)}{3\,b^9\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
16/3*a^(3/2)*sgn(x)/b^3 - 2*a^2/(sqrt(b*x + a)*b^3*sgn(x)) + 2/3*((b*x + a
)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b^6)/(b^9*sgn(x))
```

3.261.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^2+4\,a\,b\,x-b^2\,x^2\right)}{3\,b^3\,x\,\left(a+b\,x\right)}$$

input
```
int(x^5/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*x*(a + b*x))
```

3.262 $\int \frac{x^4}{(ax^2+bx^3)^{3/2}} dx$

3.262.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}} dx = -\frac{2x^2}{b\sqrt{ax^2+bx^3}} + \frac{4\sqrt{ax^2+bx^3}}{b^2x}$$

output
```
-2*x^2/b/(b*x^3+a*x^2)^(1/2)+4*(b*x^3+a*x^2)^(1/2)/b^2/x
```

3.262.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}} dx = \frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

3.262.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{2\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

3.262.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

3.262.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2(bx+a)(bx+2a)x^{3}}{b^{2}(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	34
default	$\frac{2(bx+a)(bx+2a)x^{3}}{b^{2}(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	34
trager	$\frac{2(bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{(bx+a)b^{2}x}$	36
risch	$\frac{2(bx+a)x}{b^{2}\sqrt{x^{2}(bx+a)}}+\frac{2ax}{b^{2}\sqrt{x^{2}(bx+a)}}$	42
pseudoelliptic	$\frac{\frac{2}{7}b^{4}x^{4}-\frac{16}{35}a\,b^{3}x^{3}+\frac{32}{35}a^{2}b^{2}x^{2}-\frac{128}{35}a^{3}bx-\frac{256}{35}a^{4}}{b^{5}\sqrt{bx+a}}$	54

input
```
int(x^4/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}(bx+2\,a)}{b^3x^2+ab^2x}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

3.262.6 Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^4}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(bx+2\,a)}{\sqrt{bx+a}b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{bx+a}}{b\mathrm{sgn}(x)}+\frac{a}{\sqrt{bx+a}b\mathrm{sgn}(x)}\right)}{b} - \frac{4\,\sqrt{a}\mathrm{sgn}(x)}{b^2}$$

input
```
integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*(sqrt(b*x + a)/(b*sgn(x)) + a/(sqrt(b*x + a)*b*sgn(x)))/b - 4*sqrt(a)*sg
n(x)/b^2
```

3.262.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(2\,a+b\,x)\,\sqrt{b\,x^3+a\,x^2}}{b^2\,x\,(a+b\,x)}$$

input
```
int(x^4/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(2*a + b*x)*(a*x^2 + b*x^3)^(1/2))/(b^2*x*(a + b*x))
```

3.263 $\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx$

3.263.1 Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x}{b\sqrt{ax^2+bx^3}}$$

output
```
-2*x/b/(b*x^3+a*x^2)^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x}{b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

3.263.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1920}$$

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

input
```
Int[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

3.263.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.263.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
default	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
trager	$-\frac{2\sqrt{b\,x^3+a\,x^2}}{(bx+a)bx}$	29
pseudoelliptic	$\frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{b^4\sqrt{bx+a}}$	42

input
```
int(x^3/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x+a)/b*x^3/(b*x^3+a*x^2)^(3/2)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

3.263.6 Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+ab}}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(b*x + a)*b)
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\mathrm{sgn}(x)}{\sqrt{a}b} - \frac{2}{\sqrt{bx+a}b\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*sgn(x)/(sqrt(a)*b) - 2/(sqrt(b*x + a)*b*sgn(x))
```

3.263.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

input
```
int(x^3/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2))/(b*x*(a + b*x))
```

3.264 $\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx$

3.264.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x}{a\sqrt{ax^2+bx^3}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

output
```
-2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x/a/(b*x^3+a*x^2)^(1/2
)
```

3.264.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x\left(\sqrt{a}-\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(Sqrt[a] - Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(3/2)*Sq
rt[x^2*(a + b*x)])
```

3.264.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1929, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2x}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

input
```
Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

3.264.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

3.264.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$\frac{\frac{2}{3}b^2x^2-\frac{8}{3}abx-\frac{16}{3}a^2}{b^3\sqrt{bx+a}}$	31
default	$-\frac{2x^3(bx+a)\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\sqrt{bx+a}-a^{\frac{3}{2}}\right)}{(b\,x^3+a\,x^2)^{\frac{3}{2}}a^{\frac{5}{2}}}$	54

input
```
int(x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b^2*x^2-4*a*b*x-8*a^2)/b^3/(b*x+a)^(1/2)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.00

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{(bx^2+ax)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}a}{a^2bx^2+a^3x}, \frac{2\left((bx^2+ax)\sqrt{-a}\operatorname{arct}\right.}{}\right.$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)
/(a^2*b*x^2 + a^3*x)]
```

3.264.6 Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

3.264.7 Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^{\frac{3}{2}}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\,(x)} + \frac{2}{\sqrt{bx+a}a\mathrm{sgn}\,(x)}$$

input
```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(sqrt(a)*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a))*sgn(x)/(sqrt(-a)*a^(3/2))
 + 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) + 2/(sqrt(b*x + a)
*a*sgn(x))
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^2/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^3)^(3/2), x)
```

3.265 $\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx$

3.265.1 Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2}{a\sqrt{ax^2+bx^3}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}$$

output
```
3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)+2/a/(b*x^3+a*x^2)^(1/2)
-3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

3.265.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx=\frac{-\sqrt{a}(a+3bx)+3bx\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-(Sqrt[a]*(a + 3*b*x)) + 3*b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]])/(a^(5/2)*Sqrt[x^2*(a + b*x)])
```

3.265.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1929, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{3\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{3\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{3\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

input
```
Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
2/(a*Sqrt[a*x^2 + b*x^3]) + (3*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/a
```

3.265.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.265.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

method	result	size
pseudoelliptic	$\frac{2bx+4a}{b^{2}\sqrt{bx+a}}$	20
default	$\frac{x^{2}\left(bx+a\right)\left(3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-3\sqrt{a}\,bx-a^{\frac{3}{2}}\right)}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{5}{2}}}$	62
risch	$-\frac{bx+a}{a^{2}\sqrt{x^{2}\left(bx+a\right)}}-\frac{b\left(\frac{4}{\sqrt{bx+a}}-\frac{6\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{2a^{2}\sqrt{x^{2}\left(bx+a\right)}}$	75

input
```
int(x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b*x+4*a)/b^2/(b*x+a)^(1/2)
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.52

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{3\,(b^2x^3+abx^2)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(3\,abx+a^2)}{2\,(a^3bx^3+a^4x^2)},\right.$$
$$\left.-\frac{3\,(b^2x^3+abx^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}(3\,abx+a^2)}{a^3bx^3+a^4x^2}\right]$$

input
```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*
x^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a
^4*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(
-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2)]
```

3.265.6 Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x/(x**2*(a + b*x))**(3/2), x)
```

3.265.7 Maxima [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\,(x)} - \frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) - (3*(b*x + a)*b
 - 2*a*b)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2*sgn(x))
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(x/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int(x/(a*x^2 + b*x^3)^(3/2), x)
```

3.266 $\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

3.266.1 Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{15b^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}$$

output
```
-15/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)+2/a/x/(b*x^3+a*x^
2)^(1/2)-5/2*(b*x^3+a*x^2)^(1/2)/a^2/x^3+15/4*b*(b*x^3+a*x^2)^(1/2)/a^3/x^
2
```

3.266.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{a}(-2a^2+5abx+15b^2x^2)-15b^2x^2\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

output
```
(Sqrt[a]*(-2*a^2 + 5*a*b*x + 15*b^2*x^2) - 15*b^2*x^2*Sqrt[a + b*x]*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2)*x*Sqrt[x^2*(a + b*x)])
```

3.266.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-3/2),x]
```

output
```
2/(a*x*Sqrt[a*x^2 + b*x^3]) + (5*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*
(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/a^(3/2)))/(4*a)))/a
```

3.266.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.266.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$-\frac{2}{b\sqrt{bx+a}}$	13
default	$-\frac{x(bx+a)\left(15\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-5a^{\frac{3}{2}}bx-15\sqrt{a}\,b^{2}x^{2}+2a^{\frac{5}{2}}\right)}{4\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{7}{2}}}$	76
risch	$-\frac{(bx+a)(-7bx+2a)}{4a^{3}x\sqrt{x^{2}(bx+a)}}+\frac{b^{2}\left(\frac{16}{\sqrt{bx+a}}-\frac{30\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{8a^{3}\sqrt{x^{2}(bx+a)}}$	88

input
```
int(1/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/(b*x+a)^(1/2)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.99

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{15\left(b^3x^4+ab^2x^3\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{bx^3}}{8\left(a^4bx^4+a^5x^3\right)}\right.$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
 a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 +
a*x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3
)*sqrt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

3.266.6 Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-3/2), x)
```

3.266.7 Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-3/2), x)
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3\mathrm{sgn}\left(x\right)} + \frac{2\,b^2}{\sqrt{bx+a}a^3\mathrm{sgn}\left(x\right)} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2 - 9\,\sqrt{bx+a}ab^2}{4\,a^3b^2x^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) + 2*b^2/(sqr
t(b*x + a)*a^3*sgn(x)) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^
2)/(a^3*b^2*x^2*sgn(x))
```

3.266.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.38

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

input
```
int(1/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2
+ b*x^3)^(3/2))
```

3.267 $\int \frac{1}{x(ax^2+bx^3)^{3/2}}\,dx$

3.267.1 Optimal result

Integrand size = 19, antiderivative size = 138

$$\int \frac{1}{x\,(ax^2+bx^3)^{3/2}}\,dx = \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} - \frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} + \frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}$$

output
```
35/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(9/2)+2/a/x^2/(b*x^3+a*x
^2)^(1/2)-7/3*(b*x^3+a*x^2)^(1/2)/a^2/x^4+35/12*b*(b*x^3+a*x^2)^(1/2)/a^3/
x^3-35/8*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^2
```

3.267.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\,(ax^2+bx^3)^{3/2}}\,dx = \frac{-\sqrt{a}(8a^3-14a^2bx+35ab^2x^2+105b^3x^3)+105b^3x^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{9/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(-(Sqrt[a]*(8*a^3 - 14*a^2*b*x + 35*a*b^2*x^2 + 105*b^3*x^3)) + 105*b^3*x^
3*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(9/2)*x^2*Sqrt[x^2*(
a + b*x)])
```

3.267.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1929, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{7\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

$\downarrow$ 219

$$\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
2/(a*x^2*Sqrt[a*x^2 + b*x^3]) + (7*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*
b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2))
 + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/
a
```

3.267.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.267.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

method	result	size
pseudoelliptic	$-\frac{2 \,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a \sqrt{bx+a}}$	31
default	$-\frac{(bx+a)\left(-105 \,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\, b^{3} x^{3}-14 a^{\frac{5}{2}} bx+35 a^{\frac{3}{2}} b^{2} x^{2}+105 \sqrt{a}\, b^{3} x^{3}+8 a^{\frac{7}{2}}\right)}{24\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} a^{\frac{9}{2}}}$	86
risch	$-\frac{(bx+a)\left(57 b^{2} x^{2}-22 a b x+8 a^{2}\right)}{24 a^{4} x^{2} \sqrt{x^{2}(bx+a)}}-\frac{b^{3}\left(\frac{32}{\sqrt{bx+a}}-\frac{70 \,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\, x}{16 a^{4} \sqrt{x^{2}(bx+a)}}$	99

input
```
int(1/x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/a/(b*x+a)^(1/2)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.75

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \left[\frac{105\,(b^4x^5+ab^3x^4)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(105\,ab^3x^3+35\,a^2b^2x^2-14\right.$$

$$\left.-\frac{105\,(b^4x^5+ab^3x^4)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4)\sqrt{bx^3+ax^2}}{24\,(a^5bx^5+a^6x^4)}\right]$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x +
 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 +
a*b^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (105*a*b^
3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x
^5 + a^6*x^4)]
```

3.267.6 Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

3.267.7 Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{35\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^4\mathrm{sgn}\left(x\right)}-\frac{2\,b^3}{\sqrt{bx+a}a^4\mathrm{sgn}\left(x\right)}-\frac{57\,(bx+a)^{\frac{5}{2}}b^3-136\,(bx+a)^{\frac{3}{2}}ab^3+87\,\sqrt{bx+a}a^2b^3}{24\,a^4b^3x^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sgn(x)) - 2*b^3/(sq
rt(b*x + a)*a^4*sgn(x)) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/
2)*a*b^3 + 87*sqrt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3*sgn(x))
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

3.268 $\int \frac{1}{x^2(ax^2+bx^3)^{3/2}}\,dx$

3.268.1 Optimal result

Integrand size = 19, antiderivative size = 166

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{315b^4\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}$$

output
```
-315/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(11/2)+2/a/x^3/(b*x^3
+a*x^2)^(1/2)-9/4*(b*x^3+a*x^2)^(1/2)/a^2/x^5+21/8*b*(b*x^3+a*x^2)^(1/2)/a
^3/x^4-105/32*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^3+315/64*b^3*(b*x^3+a*x^2)^(1/
2)/a^5/x^2
```

3.268.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{a}(-16a^4+24a^3bx-42a^2b^2x^2+105ab^3x^3+315b^4x^4)-315b^4x^4\sqrt{a+bx}\,\text{arctan}}{64a^{11/2}x^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(Sqrt[a]*(-16*a^4 + 24*a^3*b*x - 42*a^2*b^2*x^2 + 105*a*b^3*x^3 + 315*b^4*
x^4) - 315*b^4*x^4*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(11
/2)*x^3*Sqrt[x^2*(a + b*x)])
```

3.268.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1929, 1931, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax^2+bx^3\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1929}$$

$$\frac{9 \int \frac{1}{x^4 \sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^3 \sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9 \left(-\frac{7 b \int \frac{1}{x^3 \sqrt{bx^3+ax^2}}dx}{8 a}-\frac{\sqrt{ax^2+bx^3}}{4 a x^5}\right)}{a}+\frac{2}{ax^3 \sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9 \left(-\frac{7 b \left(-\frac{5 b \int \frac{1}{x^2 \sqrt{bx^3+ax^2}}dx}{6 a}-\frac{\sqrt{ax^2+bx^3}}{3 a x^4}\right)}{8 a}-\frac{\sqrt{ax^2+bx^3}}{4 a x^5}\right)}{a}+\frac{2}{ax^3 \sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{9 \left(-\frac{7 b \left(-\frac{5 b \left(-\frac{3 b \int \frac{1}{x \sqrt{bx^3+ax^2}}dx}{4 a}-\frac{\sqrt{ax^2+bx^3}}{2 a x^3}\right)}{6 a}-\frac{\sqrt{ax^2+bx^3}}{3 a x^4}\right)}{8 a}-\frac{\sqrt{ax^2+bx^3}}{4 a x^5}\right)}{a}+\frac{2}{ax^3 \sqrt{ax^2+bx^3}}$$

$$\big\downarrow \text{1931}$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\big\downarrow \text{1914}$$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

$$\big\downarrow \text{219}$$

3.268. $\int \frac{1}{x^2(ax^2+bx^3)^{3/2}}\,dx$

$$\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

input
```
Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
2/(a*x^3*Sqrt[a*x^2 + b*x^3]) + (9*(-1/4*Sqrt[a*x^2 + b*x^3]/(a*x^5) - (7*
b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^
3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/a
```

3.268.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.268.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

method	result	size
pseudoelliptic	$\frac{3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-3\sqrt{a}\,bx-a^{\frac{3}{2}}}{x\,a^{\frac{5}{2}}\sqrt{bx+a}}$	51
default	$-\frac{(bx+a)\left(315\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4-24a^{\frac{7}{2}}bx+42a^{\frac{5}{2}}b^2x^2-105a^{\frac{3}{2}}b^3x^3-315b^4x^4\sqrt{a}+16a^{\frac{9}{2}}\right)}{64x\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{11}{2}}}$	100
risch	$-\frac{(bx+a)\left(-187b^3x^3+82a\,b^2x^2-40a^2bx+16a^3\right)}{64a^5x^3\sqrt{x^2(bx+a)}}+\frac{b^4\left(\frac{256}{\sqrt{bx+a}}-\frac{630\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{128a^5\sqrt{x^2(bx+a)}}$	110

input
```
int(1/x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(3*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-3*a^(1/2)*b*x-a^(3/2))
/x/a^(5/2)/(b*x+a)^(1/2)
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/2}} \, dx = \left[\frac{315 (b^5x^6 + ab^4x^5)\sqrt{a} \log \left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2} \right) + 2\,(315\,ab^4x^4 + 105\,a^2b^3x^3 - }{128\,(a^6bx^6 + a^7x^5)} \right.$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2
*x^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/
64*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(a*x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x -
 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

3.268.6 Sympy [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (x^2 (a + bx))^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

3.268.7 Maxima [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \frac{315\,b^4 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}a^5 \mathrm{sgn}\,(x)} + \frac{2\,b^4}{\sqrt{bx+a}a^5 \mathrm{sgn}\,(x)} + \frac{187\,(bx+a)^{\frac{7}{2}}b^4 - 643\,(bx+a)^{\frac{5}{2}}ab^4 + 765\,(bx+a)^{\frac{3}{2}}a^2b^4 - 325\,\sqrt{bx+a}a^3b^4}{64\,a^5b^4x^4 \mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
315/64*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*sgn(x)) + 2*b^4/(s
qrt(b*x + a)*a^5*sgn(x)) + 1/64*(187*(b*x + a)^(7/2)*b^4 - 643*(b*x + a)^(
5/2)*a*b^4 + 765*(b*x + a)^(3/2)*a^2*b^4 - 325*sqrt(b*x + a)*a^3*b^4)/(a^5
*b^4*x^4*sgn(x))
```

3.268.9 Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = -\frac{2\left(\frac{a}{b\,x}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{2};\ \frac{13}{2};\ -\frac{a}{b\,x}\right)}{11\,x\,(b\,x^3+a\,x^2)^{3/2}}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
-(2*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 11/2], 13/2, -a/(b*x)))/(11*x*(a*x
^2 + b*x^3)^(3/2))
```

3.269 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

3.269.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}$$

output
```
-5/8*a^3*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)+1/3*x^(3/2)*
(b*x^3+a*x^2)^(1/2)/b+5/8*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x^(1/2)-5/12*a*x^(1/
2)*(b*x^3+a*x^2)^(1/2)/b^2
```

3.269.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b}x^{3/2}(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{7/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[b]*x^(3/2)*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 30*a^3*x
*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(24*b
^(7/2)*Sqrt[x^2*(a + b*x)])
```

3.269.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1930, 1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\int \frac{x^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\int \frac{x^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a\left(\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}$$

input
```
Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*((Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) - (3*a*(Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2
))/Sqrt[a*x^2 + b*x^3]])/b^(3/2)))/(4*b)))/(6*b)
```

3.269.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.269.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\left(8 b^{2} x^{2}-10 a b x+15 a^{2}\right) x^{\frac{3}{2}} \left(b x+a\right)}{24 b^{3} \sqrt{x^{2} \left(b x+a\right)}}-\frac{5 a^{3} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right) \sqrt{x}\, \sqrt{x \left(b x+a\right)}}{16 b^{\frac{7}{2}} \sqrt{x^{2} \left(b x+a\right)}}$	100
default	$\frac{\sqrt{x}\, \left(16 b^{\frac{9}{2}} x^{4}-4 b^{\frac{7}{2}} a\,x^{3}+10 b^{\frac{5}{2}} a^{2} x^{2}+30 a^{3} b^{\frac{3}{2}} x -15 \sqrt{x \left(b x+a\right)}\, \ln \left(\frac{2 \sqrt{b\,x^{2}+a x}\, \sqrt{b}+2 b x+a}{2 \sqrt{b}}\right) a^{3} b\right)}{48 \sqrt{b\,x^{3}+a\,x^{2}}\, b^{\frac{9}{2}}}$	103

input
```
int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^2*x^2-10*a*b*x+15*a^2)*x^(3/2)*(b*x+a)/b^3/(x^2*(b*x+a))^(1/2)-5
/16*a^3/b^(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1
/2)*x^(1/2)*(x*(b*x+a))^(1/2)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.44

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{15\,a^3\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx^3+ax^2}\sqrt{x}}{48\,b^4x}\right.$$

input
```
integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)
*sqrt(x))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*s
qrt(x))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-
b)/(b*x^(3/2))) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*
sqrt(x))/(b^4*x)]
```

3.269.6 Sympy [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)
```

3.269.7 Maxima [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{5\,a^3\log\left(|a|\right)\operatorname{sgn}(x)}{16\,b^{\frac{7}{2}}} + \frac{\sqrt{bx+a}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}+\frac{15\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}}{24\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 1/24*(sqrt(b*x + a)*(2*x*(4*x/b - 5
*a/b^2) + 15*a^2/b^3)*sqrt(x) + 15*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
 + a)))/b^(7/2))/sgn(x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{7/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(7/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(7/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.270 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$

3.270.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx=-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

output
```
3/4*a^2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)-3/4*a*(b*x^3+
a*x^2)^(1/2)/b^2/x^(1/2)+1/2*x^(1/2)*(b*x^3+a*x^2)^(1/2)/b
```

3.270.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{b}x^{3/2}(-3a^2-abx+2b^2x^2)+6a^2x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*x*Sqrt[a + b*x]*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2)*Sqrt[x^2*(a
+ b*x)])
```

3.270.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\int \frac{x^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b}\right)}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{3a\left(\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}\right)}{4b}$$

input
```
Int[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*(Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]
) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/b^(3/2)))/(4*b)
```

3.270.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.270.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{(-2bx+3a)x^{\frac{3}{2}}(bx+a)}{4b^2\sqrt{x^2(bx+a)}}+\frac{3a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x^2(bx+a)}}$	89
default	$\frac{\sqrt{x}\left(4b^{\frac{7}{2}}x^3-2b^{\frac{5}{2}}a\,x^2-6a^2b^{\frac{3}{2}}x+3\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b\right)}{8\sqrt{b\,x^3+a\,x^2}\,b^{\frac{7}{2}}}$	92

input
```
int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*x+3*a)*x^(3/2)*(b*x+a)/b^2/(x^2*(b*x+a))^(1/2)+3/8*a^2/b^(5/2)*
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1/2)*x^(1/2)*(x*(
b*x+a))^(1/2)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{3\,a^2\sqrt{b}x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}(2\,b^2x-3\,ab)\sqrt{x}}{8\,b^3x}, -\frac{3\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)-\sqrt{bx^3+ax^2}(2\,b^2x-3\,ab)\sqrt{x}}{4\,b^3x}\right]$$

input
```
integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*s
qrt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x), -1/
4*(3*a^2*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) - sqr
t(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x)]
```

3.270.6 Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)
```

3.270.7 Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{3\,a^2\log\left(|a|\right)\operatorname{sgn}(x)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx+a}\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}}{4\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
3/8*a^2*log(abs(a))*sgn(x)/b^(5/2) + 1/4*(sqrt(b*x + a)*sqrt(x)*(2*x/b - 3
*a/b^2) - 3*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))/sgn(x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{5/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(5/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(5/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.271 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

3.271.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

output
```
-a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)+(b*x^3+a*x^2)^(1/2
)/b/x^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b}x^{3/2}(a+bx) + 2ax\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[b]*x^(3/2)*(a + b*x) + 2*a*x*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])
/(Sqrt[a] - Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

3.271.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1930$$

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b}$$

$$\Big\downarrow \ 1935$$

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

input
```
Int[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2
+ b*x^3]])/b^(3/2)
```

3.271.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.271.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{x^{\frac{3}{2}} (bx+a)}{b\sqrt{x^2 (bx+a)}} - \frac{a \ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x^2 (bx+a)}}$	78
default	$\frac{\sqrt{x}\left(2b^{\frac{5}{2}}x^2+2b^{\frac{3}{2}}ax-a\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^3+a\,x^2}\,b^{\frac{5}{2}}}$	79

input
```
int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*x^(3/2)*(b*x+a)/(x^2*(b*x+a))^(1/2)-1/2*a/b^(3/2)*ln((1/2*a+b*x)/b^(1/
2)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.18

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}b\sqrt{x}}{2\,b^2x}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right.}{b^2x}\right.$$

input
```
integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(
x))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sq
rt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) + sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(
b^2*x)]
```

3.271.6 Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

3.271.7 Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{2\,b^{\frac{3}{2}}} + \frac{\frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{\sqrt{bx+a}\sqrt{x}}{b}}{\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*log(abs(a))*sgn(x)/b^(3/2) + (a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
 + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x)/b)/sgn(x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{x^{3/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(3/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(3/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.272 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$

3.272.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2x\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*x*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt
[x^2*(a + b*x)])
```

3.272.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1935}$$

$$2\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

input
```
Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]
```

3.272.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.272.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.

Time = 1.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^{2}+a x}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{b}}$	58

input
```
int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2
)*b^(1/2)+2*b*x+a)/b^(1/2))/b^(1/2)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b}\right]$$

input
```
integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -
2*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2)))/b]
```

3.272.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)
```

3.272.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\log(|a|)\,\mathrm{sgn}(x)}{\sqrt{b}} - \frac{2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(a))*sgn(x)/sqrt(b) - 2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/
(sqrt(b)*sgn(x))
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^(1/2)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(x^(1/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.273 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$

3.273.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

output
```
-2*(b*x^3+a*x^2)^(1/2)/a/x^(3/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))
```

3.273.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1920}$$

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

3.273.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.273.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{2\sqrt{x}\,(bx+a)}{\sqrt{x^2(bx+a)}\,a}$	25
gosper	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27
default	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27

input
```
int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(x^2*(b*x+a))^(1/2)*x^(1/2)/a*(b*x+a)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{bx^3+ax^2}}{ax^{\frac{3}{2}}}$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^3 + a*x^2)/(a*x^(3/2))
```

3.273.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{x^2(a+bx)}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)
```

3.273.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \frac{4\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)\operatorname{sgn}(x)}$$

input
```
integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*sgn(x))
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.274 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$

3.274.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}$$

output
```
-2/3*(b*x^3+a*x^2)^(1/2)/a/x^(5/2)+4/3*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(3/2)
```

3.274.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))
```

3.274.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1922$$

$$-\frac{2b\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

$$\Big\downarrow \ 1920$$

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

input
```
Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*
x^(3/2))
```

3.274.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.274.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

method	result	size
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^{2}(bx+a)}\,\sqrt{x}\,a^{2}}$	31
gosper	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
default	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33

input
```
int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x^2*(b*x+a))^(1/2)/x^(1/2)*(b*x+a)*(-2*b*x+a)/a^2
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{3\,a^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))
```

3.274.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)
```

3.274.7 Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{2\,(bx+a)b^3}{a^2}-\frac{3\,b^3}{a}\right)\sqrt{bx+a}b}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}|b|\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*(2*(b*x + a)*b^3/a^2 - 3*b^3/a)*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(
3/2)*abs(b)*sgn(x))
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.275 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$

3.275.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}$$

output
```
-2/5*(b*x^3+a*x^2)^(1/2)/a/x^(7/2)+8/15*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(5/2)-
16/15*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(3/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{x^2(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^{7/2}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))
```

3.275.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\int \frac{1}{x^{3/2}\sqrt{bx^3+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) - (4*b*((-2*Sqrt[a*x^2 + b*x^3])/(3
*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^(3/2))))/(5*a)
```

3.275.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.275.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15\sqrt{x^{2}(bx+a)}\, x^{\frac{3}{2}} a^{3}}$	44
gosper	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{\frac{3}{2}} a^{3}\sqrt{b\, x^{3}+a\, x^{2}}}$	46
default	$-\frac{2(bx+a)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{\frac{3}{2}} a^{3}\sqrt{b\, x^{3}+a\, x^{2}}}$	46

input
```
int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/(x^2*(b*x+a))^(1/2)/x^(3/2)*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx^3+ax^2}}{15\,a^3x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))
```

3.275.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2(a+bx)}}\,dx$$

input
```
integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)
```

3.275.7 Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5\operatorname{sgn}(x)}$$

input
```
integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^2 + a^2)*b^(5/2)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5*
sgn(x))
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.276 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$

3.276.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} + \frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}$$

output
```
-2/7*(b*x^3+a*x^2)^(1/2)/a/x^(9/2)+12/35*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(7/2)
-16/35*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(5/2)+32/35*b^3*(b*x^3+a*x^2)^(1/2)/a
^4/x^(3/2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^{9/2}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(3
5*a^4*x^(9/2))
```

3.276.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\int \frac{1}{x^{5/2}\sqrt{bx^3+ax^2}}dx}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\int \frac{1}{x^{3/2}\sqrt{bx^3+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{x}\sqrt{bx^3+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}\right)}{5a}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) - (6*b*((-2*Sqrt[a*x^2 + b*x^3])/(5
*a*x^(7/2)) - (4*b*((-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x
^2 + b*x^3])/(3*a^2*x^(3/2))))/(5*a)))/(7*a)
```

3.276.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.276.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35\sqrt{x^{2}(bx+a)}\,x^{\frac{5}{2}}\,a^{4}}$	55
gosper	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{\frac{5}{2}}\,a^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	57
default	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{\frac{5}{2}}\,a^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	57

input
```
int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/(x^2*(b*x+a))^(1/2)/x^(5/2)*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b
*x+5*a^3)/a^4
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx^3+ax^2}}{35\,a^4x^{\frac{9}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a
^4*x^(9/2))
```

3.276.6 Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)
```

3.276.7 Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}\,x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4+7\,a^2\left(\sqrt{b}\sqrt{x}-\sqrt{bx+}\right.\right.}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sq
rt(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/
(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7*sgn(x))
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.277 $\int x^{1-3n}(ax^2+bx^3)^n\,dx$

3.277.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx = \frac{x^{2-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n \text{Hypergeometric2F1}\left(2-n,-n,3-n,-\frac{bx}{a}\right)}{2-n}$$

output
```
x^(2-3*n)*(b*x^3+a*x^2)^n*hypergeom([-n, 2-n],[3-n],-b*x/a)/(2-n)/((1+b*x/
a)^n)
```

3.277.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx = \frac{x^{2-3n}(x^2(a+bx))^n\left(1+\frac{bx}{a}\right)^{-n}\text{Hypergeometric2F1}\left(2-n,-n,3-n,-\frac{bx}{a}\right)}{2-n}$$

input
```
Integrate[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
(x^(2 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)
/a)])/((2 - n)*(1 + (b*x)/a)^n)
```

3.277.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\int x^{1-n}(a+bx)^n dx$$

$$\Big\downarrow \text{76}$$

$$x^{-2n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\int x^{1-n}\left(\frac{bx}{a}+1\right)^n dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{2-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n \text{Hypergeometric2F1}\left(2-n,-n,3-n,-\frac{bx}{a}\right)}{2-n}$$

input
```
Int[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
(x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)
/a)])/((2 - n)*(1 + (b*x)/a)^n)
```

3.277.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.277.4 Maple [F]

$$\int x^{1-3n} \left(b\,x^3+a\,x^2\right)^n dx$$

input
```
int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

output
```
int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

3.277.5 Fricas [F]

$$\int x^{1-3n} \left(ax^2+bx^3\right)^n \, dx = \int \left(bx^3+ax^2\right)^n x^{-3\,n+1} \, dx$$

input
```
integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

3.277.6 Sympy [F]

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{1-3n}\left(x^2(a+bx)\right)^n\,dx$$

input
```
integrate(x**(1-3*n)*(b*x**3+a*x**2)**n,x)
```

output
```
Integral(x**(1 - 3*n)*(x**2*(a + b*x))**n, x)
```

3.277.7 Maxima [F]

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n+1}\,dx$$

input
```
integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

3.277.8 Giac [F]

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n+1}\,dx$$

input
```
integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{1-3\,n}\,\left(b\,x^3+a\,x^2\right)^n\,dx$$

input
```
int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x)
```

output
```
int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n, x)
```

3.278 $\int x^{-3n}(ax^2+bx^3)^n\,dx$

3.278.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\frac{x^{-1-3n}(ax^2+bx^3)^{1+n}\,\text{Hypergeometric2F1}\left(1,2,2-n,-\frac{bx}{a}\right)}{a(1-n)}$$

output
```
x^(-1-3*n)*(b*x^3+a*x^2)^(1+n)*hypergeom([1, 2],[2-n],-b*x/a)/a/(1-n)
```

3.278.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\frac{x^{1-3n}(x^2(a+bx))^n\left(1+\frac{bx}{a}\right)^{-n}\text{Hypergeometric2F1}\left(1-n,-n,2-n,-\frac{bx}{a}\right)}{1-n}$$

input
```
Integrate[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

output
```
(x^(1 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)
/a)])/((1 - n)*(1 + (b*x)/a)^n)
```

3.278.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\int x^{-n}(a+bx)^n dx$$

$$\Big\downarrow \text{ 76}$$

$$x^{-2n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\int x^{-n}\left(\frac{bx}{a}+1\right)^n dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{x^{1-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n \text{Hypergeometric2F1}\left(1-n,-n,2-n,-\frac{bx}{a}\right)}{1-n}$$

input
```
Int[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

output
```
(x^(1 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)
/a)])/((1 - n)*(1 + (b*x)/a)^n)
```

3.278.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.278.4 Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{n} x^{-3n}dx$$

input
```
int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

output
```
int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

3.278.5 Fricas [F]

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\int \frac{\left(bx^3+ax^2\right)^n}{x^{3\,n}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^n/x^(3*n), x)
```

3.278.6 Sympy [F]

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{-3n}\left(x^2(a+bx)\right)^n\,dx$$

input
```
integrate((b*x**3+a*x**2)**n/(x**(3*n)),x)
```

output
```
Integral((x**2*(a + b*x))**n/x**(3*n), x)
```

3.278.7 Maxima [F]

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\int\frac{\left(bx^3+ax^2\right)^n}{x^{3\,n}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

3.278.8 Giac [F]

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\int\frac{\left(bx^3+ax^2\right)^n}{x^{3\,n}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^n}{x^{3\,n}}\,dx$$

input
```
int((a*x^2 + b*x^3)^n/x^(3*n),x)
```

output
```
int((a*x^2 + b*x^3)^n/x^(3*n), x)
```

3.279 $\int x^{-1-3n}(ax^2+bx^3)^n \, dx$

3.279.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int x^{-1-3n}(ax^2+bx^3)^n \, dx = -\frac{x^{-3n}\left(1+\frac{bx}{a}\right)^{-n}(ax^2+bx^3)^n \text{Hypergeometric2F1}\left(-n,-n,1-n,-\frac{bx}{a}\right)}{n}$$

output
```
-(b*x^3+a*x^2)^n*hypergeom([-n, -n],[1-n],-b*x/a)/n/(x^(3*n))/((1+b*x/a)^n
)
```

3.279.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int x^{-1-3n}(ax^2+bx^3)^n \, dx = -\frac{x^{-3n}(x^2(a+bx))^n\left(1+\frac{bx}{a}\right)^{-n} \text{Hypergeometric2F1}\left(-n,-n,1-n,-\frac{bx}{a}\right)}{n}$$

input
```
Integrate[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-(((x^2*(a + b*x))^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3
*n)*(1 + (b*x)/a)^n))
```

3.279.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}\left(ax^2+bx^3\right)^n dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\int x^{-n-1}(a+bx)^n dx$$

$$\Big\downarrow \text{76}$$

$$x^{-2n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\int x^{-n-1}\left(\frac{bx}{a}+1\right)^n dx$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n \text{Hypergeometric2F1}\left(-n,-n,1-n,-\frac{bx}{a}\right)}{n}$$

input
```
Int[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3
*n)*(1 + (b*x)/a)^n))
```

3.279.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.279.4 Maple [F]

$$\int x^{-1-3n}\left(b\,x^{3}+a\,x^{2}\right)^{n}dx$$

input
```
int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
```

output
```
int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
```

3.279.5 Fricas [F]

$$\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-1}\,dx$$

input
```
integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

3.279.6 Sympy [F]

$$\int x^{-1-3n} \left(ax^2+bx^3\right)^n \, dx = \int x^{-3n-1} \left(x^2(a+bx)\right)^n \, dx$$

input
```
integrate(x**(-1-3*n)*(b*x**3+a*x**2)**n,x)
```

output
```
Integral(x**(-3*n - 1)*(x**2*(a + b*x))**n, x)
```

3.279.7 Maxima [F]

$$\int x^{-1-3n} \left(ax^2+bx^3\right)^n \, dx = \int \left(bx^3+ax^2\right)^n x^{-3\,n-1} \, dx$$

input
```
integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

3.279.8 Giac [F]

$$\int x^{-1-3n} \left(ax^2+bx^3\right)^n \, dx = \int \left(bx^3+ax^2\right)^n x^{-3\,n-1} \, dx$$

input
```
integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^n}{x^{3\,n+1}}\,dx$$

input
```
int((a*x^2 + b*x^3)^n/x^(3*n + 1),x)
```

output
```
int((a*x^2 + b*x^3)^n/x^(3*n + 1), x)
```

3.280 $\int x^{-2-3n}(ax^2+bx^3)^n\,dx$

3.280.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a(1+n)}$$

output
```
-(b*x^3+a*x^2)^(1+n)/a/(1+n)/(x^(3+3*n))
```

3.280.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{x^{-3(1+n)}\left(x^2(a+bx)\right)^{1+n}}{a(1+n)}$$

input
```
Integrate[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-((x^2*(a + b*x))^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

3.280.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-2}\left(ax^2+bx^3\right)^n\,dx$$

$$\Big\downarrow \ 1920$$

$$-\frac{x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

input
```
Int[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

3.280.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.280.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
gosper	$-\frac{x^{-1-3n}(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{n}}{a(1+n)}$	36
parallelrisch	$-\frac{x^{2}x^{-2-3n}\left(x^{2}(bx+a)\right)^{n}b+x\,x^{-2-3n}\left(x^{2}(bx+a)\right)^{n}a}{a(1+n)}$	56

input
```
int(x^(-2-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
```

output
```
-x^(-1-3*n)/a/(1+n)*(b*x+a)*(b*x^3+a*x^2)^n
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int x^{-2-3n} (ax^2 + bx^3)^n \, dx = -\frac{(bx^2 + ax)(bx^3 + ax^2)^n x^{-3\,n-2}}{an + a}$$

input
```
integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

output
```
-(b*x^2 + a*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 2)/(a*n + a)
```

3.280.6 Sympy [F]

$$\int x^{-2-3n} (ax^2 + bx^3)^n \, dx = \int x^{-3n-2} (x^2(a + bx))^n \, dx$$

input
```
integrate(x**(-2-3*n)*(b*x**3+a*x**2)**n,x)
```

output
```
Integral(x**(-3*n - 2)*(x**2*(a + b*x))**n, x)
```

3.280.7 Maxima [F]

$$\int x^{-2-3n} (ax^2 + bx^3)^n \, dx = \int (bx^3 + ax^2)^n x^{-3\,n-2} \, dx$$

input
```
integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

3.280.8 Giac [F]

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-2}\,dx$$

input
```
integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x}{x^{3\,n+2}\,\left(n+1\right)}+\frac{b\,x^2}{a\,x^{3\,n+2}\,\left(n+1\right)}\right)$$

input
```
int((a*x^2 + b*x^3)^n/x^(3*n + 2),x)
```

output
```
-(a*x^2 + b*x^3)^n*(x/(x^(3*n + 2)*(n + 1)) + (b*x^2)/(a*x^(3*n + 2)*(n +
1)))
```

3.281 $\int x^{-3-3n}(ax^2+bx^3)^n \, dx$

3.281.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n \, dx=-\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^2(1+n)(2+n)}$$

output
```
-x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)/a/(2+n)+b*(b*x^3+a*x^2)^(1+n)/a^2/(1+n)/(2
+n)/(x^(3+3*n))
```

3.281.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n \, dx=-\frac{x^{-4-3n}(a+an-bx)\left(x^2(a+bx)\right)^{1+n}}{a^2(1+n)(2+n)}$$

input
```
Integrate[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-((x^(-4 - 3*n)*(a + a*n - b*x)*(x^2*(a + b*x))^(1 + n))/(a^2*(1 + n)*(2 +
 n)))
```

3.281.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-3}\left(ax^2+bx^3\right)^n\,dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{b\int x^{-3n-2}\left(bx^3+ax^2\right)^n\,dx}{a(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

$$\Big\downarrow \text{1920}$$

$$\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

input
```
Int[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-((x^(-4 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)
^(1 + n))/(a^2*(1 + n)*(2 + n)*x^(3*(1 + n)))
```

3.281.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.281.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{x^{-2-3n}(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{n}(an-bx+a)}{a^{2}(1+n)(2+n)}$	50

input
```
int(x^(-3-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
```

output
```
-x^(-2-3*n)/a^2/(1+n)/(2+n)*(b*x+a)*(b*x^3+a*x^2)^n*(a*n-b*x+a)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{(abnx^2-b^2x^3+(a^2n+a^2)x)(bx^3+ax^2)^nx^{-3\,n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

input
```
integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

output
```
-(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 3)/(a
^2*n^2 + 3*a^2*n + 2*a^2)
```

3.281.6 Sympy [F]

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{-3n-3}\left(x^2(a+bx)\right)^n\,dx$$

input
```
integrate(x**(-3-3*n)*(b*x**3+a*x**2)**n,x)
```

output
```
Integral(x**(-3*n - 3)*(x**2*(a + b*x))**n, x)
```

3.281.7 Maxima [F]

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-3}\,dx$$

input
```
integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

3.281.8 Giac [F]

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-3}\,dx$$

input
```
integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x\,(n+1)}{x^{3\,n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}\right)$$

input
```
int((a*x^2 + b*x^3)^n/x^(3*n + 3),x)
```

output
```
-(a*x^2 + b*x^3)^n*((x*(n + 1))/(x^(3*n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/
(a^2*x^(3*n + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(3*n + 3)*(3*n + n^2 +
2)))
```

3.282 $\int x^{-4-3n}(ax^2+bx^3)^n\,dx$

3.282.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int x^{-4-3n}(ax^2+bx^3)^n\,dx=-\frac{x^{-5-3n}(ax^2+bx^3)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}(ax^2+bx^3)^{1+n}}{a^2(2+n)(3+n)}-\frac{2b^2x^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a^3(1+n)(2+n)(3+n)}$$

output
```
-x^(-5-3*n)*(b*x^3+a*x^2)^(1+n)/a/(3+n)+2*b*x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)
/a^2/(2+n)/(3+n)-2*b^2*(b*x^3+a*x^2)^(1+n)/a^3/(2+n)/(n^2+4*n+3)/(x^(3+3*n
))
```

3.282.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int x^{-4-3n}(ax^2+bx^3)^n\,dx=-\frac{x^{-3(1+n)}(a+bx)\left(x^2(a+bx)\right)^n\left(a^2(2+3n+n^2)-2ab(1+n)x+2b^2x^2\right)}{a^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-(((a + b*x)*(x^2*(a + b*x))^n*(a^2*(2 + 3*n + n^2) - 2*a*b*(1 + n)*x + 2*
b^2*x^2))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n))))
```

3.282.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-4} \left(ax^2+bx^3\right)^n \, dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{2b \int x^{-3(n+1)} \left(bx^3+ax^2\right)^n dx}{a(n+3)}-\frac{x^{-3n-5} \left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{2b \left(-\frac{b \int x^{-3n-2} \left(bx^3+ax^2\right)^n dx}{a(n+2)}-\frac{x^{-3n-4} \left(ax^2+bx^3\right)^{n+1}}{a(n+2)}\right)}{a(n+3)}-\frac{x^{-3n-5} \left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{2b \left(\frac{b x^{-3(n+1)} \left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4} \left(ax^2+bx^3\right)^{n+1}}{a(n+2)}\right)}{a(n+3)}-\frac{x^{-3n-5} \left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

input
```
Int[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

output
```
-((x^(-5 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(3 + n))) - (2*b*(-((x^(-4 - 3
*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)^(1 + n))/(a
^2*(1 + n)*(2 + n)*x^(3*(1 + n)))))/(a*(3 + n))
```

3.282.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.282.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{x^{-3-3n}(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{n}\left(a^{2}n^{2}-2abnx+2b^{2}x^{2}+3a^{2}n-2abx+2a^{2}\right)}{a^{3}(1+n)(2+n)(3+n)}$	84

input
```
int(x^(-4-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
```

output
```
-x^(-3-3*n)/a^3/(1+n)/(2+n)/(3+n)*(b*x+a)*(b*x^3+a*x^2)^n*(a^2*n^2-2*a*b*n
*x+2*b^2*x^2+3*a^2*n-2*a*b*x+2*a^2)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx = \frac{(2\,ab^2nx^3-2\,b^3x^4-(a^2bn^2+a^2bn)x^2-(a^3n^2+3\,a^3n+2\,a^3)x)(bx^3+ax^2)^nx^{-3\,n-4}}{a^3n^3+6\,a^3n^2+11\,a^3n+6\,a^3}$$

input
```
integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

output
```
(2*a*b^2*n*x^3 - 2*b^3*x^4 - (a^2*b*n^2 + a^2*b*n)*x^2 - (a^3*n^2 + 3*a^3*
n + 2*a^3)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 4)/(a^3*n^3 + 6*a^3*n^2 + 11*a^3
*n + 6*a^3)
```

3.282.6 Sympy [F]

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{-3n-4}\left(x^2(a+bx)\right)^n\,dx$$

input
```
integrate(x**(-4-3*n)*(b*x**3+a*x**2)**n,x)
```

output
```
Integral(x**(-3*n - 4)*(x**2*(a + b*x))**n, x)
```

3.282.7 Maxima [F]

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3n-4}\,dx$$

input
```
integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

3.282.8 Giac [F]

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3n-4}\,dx$$

input
```
integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

$$\int x^{-4-3n}(ax^2+bx^3)^n\,dx = -(b\,x^3+a\,x^2)^n\left(\frac{x\,(n^2+3\,n+2)}{x^{3\,n+4}\,(n^3+6\,n^2+11\,n+6)}+\frac{2\,b^3\,x^4}{a^3\,x^{3\,n+4}\,(n^3+6\,n^2+11\,n+6)}-\frac{2\,b^2\,n\,x^3}{a^2\,x^{3\,n+4}\,(n^3+6\,n^2+11\,n+6)}+\frac{b\,n\,x^2\,(n+1)}{a\,x^{3\,n+4}\,(n^3+6\,n^2+11\,n+6)}\right)$$

input
```
int((a*x^2 + b*x^3)^n/x^(3*n + 4),x)
```

output
```
-(a*x^2 + b*x^3)^n*((x*(3*n + n^2 + 2))/(x^(3*n + 4)*(11*n + 6*n^2 + n^3 +
 6)) + (2*b^3*x^4)/(a^3*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*n*x
^3)/(a^2*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (b*n*x^2*(n + 1))/(a*x^(3
*n + 4)*(11*n + 6*n^2 + n^3 + 6)))
```

3.283 $\int \frac{x^{11}}{(ax^2+bx^5)^3} dx$

3.283.1 Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{x^{11}}{(ax^2+bx^5)^3} dx = \frac{x^6}{6a(a+bx^3)^2}$$

output
```
1/6*x^6/a/(b*x^3+a)^2
```

3.283.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^{11}}{(ax^2+bx^5)^3} dx = -\frac{a+2bx^3}{6b^2(a+bx^3)^2}$$

input
```
Integrate[x^11/(a*x^2 + b*x^5)^3,x]
```

output
```
-1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

3.283.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^5}{(a+bx^3)^3}dx$$

$$\Big\downarrow \; 796$$

$$\frac{x^6}{6a\,(a+bx^3)^2}$$

input
```
Int[x^11/(a*x^2 + b*x^5)^3,x]
```

output
```
x^6/(6*a*(a + b*x^3)^2)
```

3.283.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

3.283.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2b\,x^{3}+a}{6(b\,x^{3}+a)^{2}b^{2}}$	23
parallelrisch	$\frac{-2b\,x^{3}-a}{6b^{2}(b\,x^{3}+a)^{2}}$	25
risch	$\frac{-\frac{x^{3}}{3b}-\frac{a}{6b^{2}}}{(b\,x^{3}+a)^{2}}$	26
default	$\frac{a}{6b^{2}(b\,x^{3}+a)^{2}}-\frac{1}{3b^{2}(b\,x^{3}+a)}$	31
norman	$\frac{-\frac{x^{8}}{3b}-\frac{a\,x^{5}}{6b^{2}}}{x^{5}(b\,x^{3}+a)^{2}}$	32

input
```
int(x^11/(b*x^5+a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*b*x^3+a)/(b*x^3+a)^2/b^2
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

3.283.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = \frac{-a-2bx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

input
```
integrate(x**11/(b*x**5+a*x**2)**3,x)
```

output
```
(-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

3.283.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="maxima")
```

output
```
-1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="giac")
```

output
```
-1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx=-\frac{\frac{a}{6\,b^2}+\frac{x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int(x^11/(a*x^2 + b*x^5)^3,x)
```

output
```
-(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.284 $\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx$

3.284.1 Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

output
```
16/45*a^2*(b*x^5+a*x^2)^(1/2)/b^3/x-8/45*a*x^2*(b*x^5+a*x^2)^(1/2)/b^2+2/1
5*x^5*(b*x^5+a*x^2)^(1/2)/b
```

3.284.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{x^2\left(a+bx^3\right)}(8a^2-4abx^3+3b^2x^6)}{45b^3x}$$

input
```
Integrate[x^9/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

3.284.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{4a\int \frac{x^6}{\sqrt{bx^5+ax^2}}dx}{5b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{4a\left(\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{2a\int \frac{x^3}{\sqrt{bx^5+ax^2}}dx}{3b}\right)}{5b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{4a\left(\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}\right)}{5b}$$

input
```
Int[x^9/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b) - (4*a*((-4*a*Sqrt[a*x^2 + b*x^5])/(9*b
^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)))/(5*b)
```

3.284.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.284.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

method	result	size
trager	$\frac{2\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)\sqrt{b\,x^{5}+a\,x^{2}}}{45b^{3}x}$	43
gosper	$\frac{2\left(b\,x^{3}+a\right)\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)x}{45b^{3}\sqrt{b\,x^{5}+a\,x^{2}}}$	48
default	$\frac{2\left(b\,x^{3}+a\right)\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)x}{45b^{3}\sqrt{b\,x^{5}+a\,x^{2}}}$	48
risch	$\frac{2x\left(b\,x^{3}+a\right)\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)}{45\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,b^{3}}$	48

input
```
int(x^9/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(3*b^2*x^6-4*a*b*x^3+8*a^2)/b^3/x*(b*x^5+a*x^2)^(1/2)
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\,(3\,b^2x^6-4\,abx^3+8\,a^2)\sqrt{bx^5+ax^2}}{45\,b^3x}$$

input
```
integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)
```

3.284.6 Sympy [F]

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^9}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**9/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**9/sqrt(x**2*(a + b*x**3)), x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\left(3\,b^3x^9-ab^2x^6+4\,a^2bx^3+8\,a^3\right)}{45\,\sqrt{bx^3+a}b^3}$$

input
```
integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=-\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{45\,b^3}+\frac{2\,\sqrt{bx^3+a}a^2}{3\,b^3\mathrm{sgn}\left(x\right)}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}-10\left(bx^3+a\right)^{\frac{3}{2}}a\right)}{45\,b^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-16/45*a^(5/2)*sgn(x)/b^3 + 2/3*sqrt(b*x^3 + a)*a^2/(b^3*sgn(x)) + 2/45*(3
*(b*x^3 + a)^(5/2) - 10*(b*x^3 + a)^(3/2)*a)/(b^3*sgn(x))
```

3.284.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{b\,x^5+a\,x^2}\,\left(8\,a^2-4\,a\,b\,x^3+3\,b^2\,x^6\right)}{45\,b^3\,x}$$

input
```
int(x^9/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^5)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3*x)
```

3.285 $\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$

3.285.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = -\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}+\frac{2x^2\sqrt{ax^2+bx^5}}{9b}$$

output
```
-4/9*a*(b*x^5+a*x^2)^(1/2)/b^2/x+2/9*x^2*(b*x^5+a*x^2)^(1/2)/b
```

3.285.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{2(-2a+bx^3)\sqrt{x^2(a+bx^3)}}{9b^2x}$$

input
```
Integrate[x^6/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)
```

3.285.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$$
$$\Big\downarrow \text{1922}$$
$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{2a\int \frac{x^3}{\sqrt{bx^5+ax^2}}dx}{3b}$$
$$\Big\downarrow \text{1920}$$
$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

input
```
Int[x^6/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

3.285.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.285.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

method	result	size
trager	$-\frac{2\left(-b\,x^{3}+2a\right)\sqrt{b\,x^{5}+a\,x^{2}}}{9b^{2}x}$	32
gosper	$-\frac{2\left(b\,x^{3}+a\right)\left(-b\,x^{3}+2a\right)x}{9b^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
default	$-\frac{2\left(b\,x^{3}+a\right)\left(-b\,x^{3}+2a\right)x}{9b^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
risch	$-\frac{2x\left(b\,x^{3}+a\right)\left(-b\,x^{3}+2a\right)}{9\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,b^{2}}$	37

input
```
int(x^6/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-b*x^3+2*a)/b^2/x*(b*x^5+a*x^2)^(1/2)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{bx^5+ax^2}(bx^3-2\,a)}{9\,b^2x}$$

input
```
integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)
```

3.285.6 Sympy [F]

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^6}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**6/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**6/sqrt(x**2*(a + b*x**3)), x)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(b^2x^6-abx^3-2\,a^2\right)}{9\,\sqrt{bx^3+a}b^2}$$

input
```
integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b^2} + \frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b^2\mathrm{sgn}\left(x\right)} - \frac{2\,\sqrt{bx^3+a}a}{3\,b^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/9*a^(3/2)*sgn(x)/b^2 + 2/9*(b*x^3 + a)^(3/2)/(b^2*sgn(x)) - 2/3*sqrt(b*x
^3 + a)*a/(b^2*sgn(x))
```

3.285.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b\,x^5+a\,x^2}\left(\frac{4\,a}{9\,b^2}-\frac{2\,x^3}{9\,b}\right)}{x}$$

input
```
int(x^6/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
-((a*x^2 + b*x^5)^(1/2)*((4*a)/(9*b^2) - (2*x^3)/(9*b)))/x
```

3.286 $\int \frac{x^3}{\sqrt{ax^2+bx^5}} dx$

3.286.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}} dx = \frac{2\sqrt{ax^2+bx^5}}{3bx}$$

output
```
2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

3.286.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}} dx = \frac{2\sqrt{x^2(a+bx^3)}}{3bx}$$

input
```
Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)
```

3.286.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx$$
$$\Big\downarrow \text{1920}$$
$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

input
```
Int[x^3/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

3.286.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.286.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
trager	$\frac{2\sqrt{b\,x^{5}+a\,x^{2}}}{3bx}$	22
gosper	$\frac{2x\left(b\,x^{3}+a\right)}{3b\sqrt{b\,x^{5}+a\,x^{2}}}$	27
default	$\frac{2x\left(b\,x^{3}+a\right)}{3b\sqrt{b\,x^{5}+a\,x^{2}}}$	27
risch	$\frac{2x\left(b\,x^{3}+a\right)}{3\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,b}$	27

input
```
int(x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{bx^5+ax^2}}{3\,bx}$$

input
```
integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^5 + a*x^2)/(b*x)
```

3.286.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**3/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(a + b*x**3)), x)
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{bx^3+a}}{3\,b}$$

input
```
integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^3 + a)/b
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\sqrt{a}\mathrm{sgn}(x)}{3\,b} + \frac{2\,\sqrt{bx^3+a}}{3\,b\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(a)*sgn(x)/b + 2/3*sqrt(b*x^3 + a)/(b*sgn(x))
```

3.286.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{b\,x^5+a\,x^2}}{3\,b\,x}$$

input
```
int(x^3/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^5)^(1/2))/(3*b*x)
```

3.287 $\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$

3.287.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

output
```
-2/3*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(1/2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx=-\frac{2x\sqrt{a+bx^3}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^
2*(a + b*x^3)])
```

3.287.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1914}$$

$$-\frac{2}{3}\int \frac{1}{1-\frac{ax^2}{bx^5+ax^2}}d\frac{x}{\sqrt{bx^5+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

3.287.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

3.287.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{2x\sqrt{b\,x^{3}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{a}}$	43

input
```
int(1/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)
)/a^(1/2)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\log\left(\frac{bx^4+2\,ax-2\,\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right)}{3\,\sqrt{a}}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{ax}\right)}{3\,a}\right]$$

input
```
integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3
*sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x))/a]
```

3.287.6 Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$$

input
```
integrate(1/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**5), x)
```

3.287.7 Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^5 + a*x^2), x)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx=-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\,\mathrm{sgn}(x)}{3\,\sqrt{-a}}+\frac{2\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)
/sqrt(-a))/(sqrt(-a)*sgn(x))
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(1/(a*x^2 + b*x^5)^(1/2), x)
```

3.288 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$

3.288.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx=-\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}$$

output
```
1/3*b*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(3/2)-1/3*(b*x^5+a*x^2)^(1/
2)/a/x^4
```

3.288.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx=\frac{-\sqrt{a}(a+bx^3)+bx^3\sqrt{a+bx^3}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}x^2\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-(Sqrt[a]*(a + b*x^3)) + b*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(3*a^(3/2)*x^2*Sqrt[x^2*(a + b*x^3)])
```

3.288.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{b\int \frac{1}{\sqrt{bx^5+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{b\int \frac{1}{1-\frac{ax^2}{bx^5+ax^2}}d\frac{x}{\sqrt{bx^5+ax^2}}}{3a}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

input
```
Int[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^5]/(a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^5]])/(3*a^(3/2))
```

3.288.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

3.288.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{b\,x^{3}+a}\,\left(b \,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right) a\,x^{3}-\sqrt{b\,x^{3}+a}\,a^{\frac{3}{2}}\right)}{3 x^{2} \sqrt{b\,x^{5}+a\,x^{2}}\,a^{\frac{5}{2}}}$	66
risch	$-\frac{b\,x^{3}+a}{3 a\,x^{2} \sqrt{x^{2} \left(b\,x^{3}+a \right)}}+\frac{b \,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right) \sqrt{b\,x^{3}+a}\,x}{3 a^{\frac{3}{2}} \sqrt{x^{2} \left(b\,x^{3}+a \right)}}$	73

input
```
int(1/x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/x^2*(b*x^3+a)^(1/2)*(b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a*x^3-(b*x^3+a
)^(1/2)*a^(3/2))/(b*x^5+a*x^2)^(1/2)/a^(5/2)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\sqrt{a}bx^4\log\left(\frac{bx^4+2\,ax+2\,\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right) - 2\,\sqrt{bx^5+ax^2}a}{6\,a^2x^4},\right.$$
$$\left. -\frac{\sqrt{-a}bx^4\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{ax}\right) + \sqrt{bx^5+ax^2}a}{3\,a^2x^4}\right]$$

input
```
integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^
4) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(
b*x^5 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]
```

3.288.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^3\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)
```

3.288.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = -\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx^3+a}b}{ax^3}}{3\,b\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^3 + a)*
b/(a*x^3))/(b*sgn(x))
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^3*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^3*(a*x^2 + b*x^5)^(1/2)), x)
```

3.289 $\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$

3.289.1 Optimal result

Integrand size = 19, antiderivative size = 238

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
2/5*(b*x^5+a*x^2)^(1/2)/b-4/15*a*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.289.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x^2\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^4/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
 -((b*x^3)/a)]))/(5*b*Sqrt[x^2*(a + b*x^3)])
```

3.289.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1930, 1938, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{2a\int \frac{x}{\sqrt{bx^5+ax^2}}dx}{5b}$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{2ax\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}dx}{5b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[x^4/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a*x^2 + b*x^5])
```

3.289.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.289.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

method	result
default	$\frac{2x\left(ia\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}F\left(\frac{\sqrt{3}\cdots}{\cdots}\right.\right.}{15\sqrt{b\,x^{5}+a\,x^{2}}\,b^{2}}$
	$4ia\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{\cdots}}{\cdots}\right.}{\left(-a\,b^{2}\cdots\right.}}$
risch	$\frac{2x^{2}\left(b\,x^{3}+a\right)}{5b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{\cdots}{15b^{2}\sqrt{x^{2}\left(b\,x^{3}\cdots\right.}}$

input
```
int(x^4/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*x*(I*a*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b
^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/
2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+3*b^2*x^4+3*a*b*x)/(b*x^5+a*x^
2)^(1/2)/b^2
```

3.289.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(2\,a\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\sqrt{bx^5+ax^2}b\right)}{5\,b^2}$$

input
```
integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(2*a*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*x^5 + a*x^2)*
b)/b^2
```

3.289.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{x^2(a+bx^3)}}\,dx$$

input
```
integrate(x**4/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**2*(a + b*x**3)), x)
```

3.289.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

3.289.8 Giac [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^4/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^4/(a*x^2 + b*x^5)^(1/2), x)
```

3.290 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$

3.290.1 Optimal result

Integrand size = 17, antiderivative size = 212

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
2/3*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(1/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.290.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/S
qrt[x^2*(a + b*x^3)]
```

3.290.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1938, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[x/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

3.290.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.290.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.09

method	result
default	$-\frac{i x \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{-\frac{i \left(i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}-2 b x-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right) \sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{-\frac{2 \left(-b x+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}} \left(i \sqrt{3}-3\right)}}\, \sqrt{-\frac{i \left(i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}+2 b x+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right) \sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, F\left(\frac{\sqrt{3}\,\sqrt{2}\cdots}{\cdots}\right.}{3 \sqrt{b\,x^{5}+a\,x^{2}}\, b}$

input
```
int(x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/(b*x^5+a*x^2)^(1/2)*x*3^(1/2)/b*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-
a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^
(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/
(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

3.290.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.07

$$\int \frac{x}{\sqrt{ax^2 + bx^5}}\,dx = \frac{2\,\text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right)}{\sqrt{b}}$$

input
```
integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4*a/b, x)/sqrt(b)
```

3.290.6 Sympy [F]

$$\int \frac{x}{\sqrt{ax^2 + bx^5}}\,dx = \int \frac{x}{\sqrt{x^2\,(a + bx^3)}}\,dx$$

input
```
integrate(x/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(a + b*x**3)), x)
```

3.290.7 Maxima [F]

$$\int \frac{x}{\sqrt{ax^2 + bx^5}} \, dx = \int \frac{x}{\sqrt{bx^5 + ax^2}} \, dx$$

input
```
integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^5 + a*x^2), x)
```

3.290.8 Giac [F]

$$\int \frac{x}{\sqrt{ax^2 + bx^5}} \, dx = \int \frac{x}{\sqrt{bx^5 + ax^2}} \, dx$$

input
```
integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*x^5 + a*x^2), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^2 + bx^5}} \, dx = \int \frac{x}{\sqrt{b\,x^5 + a\,x^2}} \, dx$$

input
```
int(x/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x/(a*x^2 + b*x^5)^(1/2), x)
```

3.291 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$

3.291.1 Optimal result

Integrand size = 19, antiderivative size = 243

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-1/2*(b*x^5+a*x^2)^(1/2)/a/x^3-1/6*b^(2/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/(b*x^5+a*x^2)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.291.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])
/(x*Sqrt[x^2*(a + b*x^3)])
```

3.291.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1931, 1938, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1931}$$

$$-\frac{b\int \frac{x}{\sqrt{bx^5+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

$$\Big\downarrow \text{1938}$$

$$-\frac{bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}dx}{4a\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

$$\Big\downarrow \text{759}$$

$$-\frac{\sqrt{2+\sqrt{3}}b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

input
```
Int[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^5]/(a*x^3) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*x*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a*x^2 + b*x^5])
```

3.291.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.291.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02

method	result
default	$\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{-\cdots}}{\cdots}\right.}{12x\sqrt{b\,x^{5}+a\,x^{2}}\,a}$
	$i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+i\cdots\right.}{\left(-a\cdots\right.}}$
risch	$-\frac{b\,x^{3}+a}{2ax\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{\cdots}{6a\sqrt{x^{2}\left(b\cdots\right.}}$

input
```
int(1/x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/x*(I*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2
)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)
,2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2-6*b*x^3-6*a)/(b*x^5+a*x^2)^(
1/2)/a
```

3.291.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b}x^3\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\sqrt{bx^5+ax^2}}{2\,ax^3}$$

input
```
integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) + sqrt(b*x^5 + a*x^2))
/(a*x^3)
```

3.291.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**2/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x**3))), x)
```

3.291.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

3.291.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

3.291.9 Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{\frac{a}{b\,x^3}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^3}\right)}{7\,x\,\sqrt{b\,x^5+a\,x^2}}$$

input
```
int(1/(x^2*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
-(2*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 7/6], 13/6, -a/(b*x^3)))/(7*x*(a
*x^2 + b*x^5)^(1/2))
```

3.292 $\int \frac{x^5}{\sqrt{ax^2+bx^5}} dx$

3.292.1 Optimal result

Integrand size = 19, antiderivative size = 514

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}} dx = -\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{2x\sqrt{ax^2+bx^5}}{7b}$$
$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$-\frac{8\sqrt{2}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-8/7*a*x*(b*x^3+a)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(
1/2)+2/7*x*(b*x^5+a*x^2)^(1/2)/b-8/21*a^(4/3)*x*(a^(1/3)+b^(1/3)*x)*Ellipt
icF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/7*3^(1/4)*a^(4/
3)*x*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
b^(5/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

3.292.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{7b\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^5/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
 -((b*x^3)/a)]))/(7*b*Sqrt[x^2*(a + b*x^3)])
```

3.292.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1930, 1938, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4a\int \frac{x^2}{\sqrt{bx^5+ax^2}}dx}{7b}$$

$\Big\downarrow$ 1938

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4ax\sqrt{a+bx^3}\int \frac{x}{\sqrt{bx^3+a}}dx}{7b\sqrt{ax^2+bx^5}}$$

$\Big\downarrow$ 832

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4ax\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b\sqrt{ax^2+bx^5}}$$

$\Big\downarrow$ 759

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4ax\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\cdots}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\cdots}\right.}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right.}{7b\sqrt{ax^2+bx^5}}$$

$\Big\downarrow$ 2416

$$\frac{2x\sqrt{ax^2+bx^5}}{7b}-\frac{4ax\sqrt{a+bx^3}\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{\cdots}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\cdots\right.}{7b\sqrt{ax^2+bx^5}}$$

input

```
Int[x^5/Sqrt[a*x^2 + b*x^5],x]
```

3.292. $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$

output
```
(2*x*Sqrt[a*x^2 + b*x^5])/(7*b) - (4*a*x*Sqrt[a + b*x^3]*(((2*Sqrt[a + b*x
^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b*Sqrt[a*x^2 + b*x^
5])
```

3.292.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.292.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.05

method	result
pseudoelliptic	$-\frac{2\sqrt{b\,x^3+a}\,(-b\,x^3+2a)}{9b^2}$
risch	$\frac{2x^3(b\,x^3+a)}{7b\sqrt{x^2(b\,x^3+a)}}+\frac{8ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b} \cdots\right.}{\cdots}}}{\cdots}$
default	$-\frac{2x\left(3i(-a\,b^2)^{\frac{2}{3}}\sqrt{3}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}-2bx-(-a\,b^2)^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+(-a\,b^2)^{\frac{1}{3}}\right)}{(-a\,b^2)^{\frac{1}{3}}(i\sqrt{3}-3)}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}+2bx+(-a\,b^2)^{\frac{1}{3}}\right)\cdots}{(-a\,b^2)^{\frac{1}{3}}}} \cdots\right.}{\cdots}$

input
```
int(x^5/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b*x^3+a)^(1/2)*(-b*x^3+2*a)/b^2
```

3.292.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.09

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(\sqrt{bx^5+ax^2}bx+4\,a\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

input
```
integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(b*x^5 + a*x^2)*b*x + 4*a*sqrt(b)*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)))/b^2
```

3.292.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**5/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**5/sqrt(x**2*(a + b*x**3)), x)
```

3.292.7 Maxima [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

3.292.8 Giac [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^5/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^5/(a*x^2 + b*x^5)^(1/2), x)
```

3.293 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$

3.293.1 Optimal result

Integrand size = 19, antiderivative size = 484

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} + \frac{2\sqrt{2}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
2*x*(b*x^3+a)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)+
2/3*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b
^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticE((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.293.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\frac{x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^2/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(
2*Sqrt[x^2*(a + b*x^3)])
```

3.293.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1938, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{x\sqrt{a+bx^3}\int \frac{x}{\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{x\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{x\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{x\sqrt{a+bx^3}\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{\sqrt{ax^2+bx^5}}$$

input
```
Int[x^2/Sqrt[a*x^2 + b*x^5],x]
```

3.293. $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$

output
```
(x*Sqrt[a + b*x^3]*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])))/Sqrt[a*x^2 + b*x^5]
```

3.293.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.293.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.03

method	result
pseudoelliptic	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$
default	$-\frac{i x\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{2}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$

input
```
int(x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a)^(1/2)/b
```

3.293.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=-\frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{\sqrt{b}}$$

input
```
integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))/sqrt(b)
```

3.293.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^2}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**2/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(a + b*x**3)), x)
```

3.293.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

3.293.8 Giac [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^2}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^2/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^5)^(1/2), x)
```

3.294 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$

3.294.1 Optimal result

Integrand size = 19, antiderivative size = 510

$$
\begin{aligned}
\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx &= \frac{\sqrt[3]{b}x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2} \\
&\quad-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&\quad+\frac{\sqrt{2}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

output
```
b^(1/3)*x*(b*x^3+a)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)-
(b*x^5+a*x^2)^(1/2)/a/x^2+1/3*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(
1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/
(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

3.294.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.10

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/Sq
rt[x^2*(a + b*x^3)])
```

3.294.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1931, 1938, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1931}$$

$$\frac{b\int \frac{x^2}{\sqrt{bx^5+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 1938$$

$$\frac{bx\sqrt{a+bx^3}\int \frac{x}{\sqrt{bx^3+a}}dx}{2a\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 832$$

$$\frac{bx\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 759$$

$$\frac{bx\sqrt{a+bx^3}\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a\sqrt{ax^2+bx^5}}$$

$$\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

$$\Big\downarrow \; 2416$$

$$\frac{bx\sqrt{a+bx^3}\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{2a\sqrt{ax^2+bx^5}}$$

$$\frac{\sqrt{ax^2+bx^5}}{ax^2}$$

3.294. $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$

input
```
Int[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
-(Sqrt[a*x^2 + b*x^5]/(a*x^2)) + (b*x*Sqrt[a + b*x^3]*(((2*Sqrt[a + b*x^3]
)/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3
]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a*Sqrt[a*x^2 + b*x^5])
```

3.294.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.294.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.04

method	result
pseudoelliptic	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
risch	$-\frac{b\,x^{3}+a}{a\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right.}{2b}\right.}{}}}{}$
default	$\frac{3i\left(-a\,b^{2}\right)^{\frac{2}{3}}\sqrt{3}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,E\left(\right.}{}$

input
```
int(1/x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.294.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b}x^2\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\sqrt{bx^5+ax^2}}{ax^2}$$

input
```
integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)
) + sqrt(b*x^5 + a*x^2))/(a*x^2)
```

3.294.6 Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x**3))), x)
```

3.294.7 Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x}\,dx$$

input
```
integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

3.294.8 Giac [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x}\,dx$$

input
```
integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^5)^(1/2)), x)
```

3.295 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$

3.295.1 Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
1/5*x^(5/2)*(b*x^5+a*x^2)^(1/2)/b-7/20*a*(b*x^5+a*x^2)^(1/2)/b^2/x^(1/2)+7
/120*a^(5/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/b^2/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.295.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(-7a^2-3abx^3+4b^2x^6+7a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{20b^2\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[x^2*(a + b*x^3)])
```

3.295.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1930, 1930, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\int \frac{x^{7/2}}{\sqrt{bx^5+ax^2}}dx}{10b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{4b}\right)}{10b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{4b\sqrt{ax^2+bx^5}}\right)}{10b}$$

$$\Big\downarrow \text{851}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b\sqrt{ax^2+bx^5}}\right)}{10b}$$

$$\Big\downarrow \text{766}$$

$$\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}-\frac{7a\left(\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}\right)}{10b}$$

input
```
Int[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(5/2)*Sqrt[a*x^2 + b*x^5])/(5*b) - (7*a*(Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[
x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCo
s[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)]
, (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])))/(10*b)
```

3.295.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.295. $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.295.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{\left(-4b\,x^{3}+7a\right)x^{\frac{3}{2}}\left(b\,x^{3}+a\right)}{20b^{2}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{7a^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right.}{20b\left(\right.}$
default	Expression too large to display

input
```
int(x^(13/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-4*b*x^3+7*a)/b^2*x^(3/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)+7/20*a^2/
b*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^
2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))
^(1/2)
```

3.295.5 Fricas [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2 + bx^5}} \, dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5 + ax^2}} \, dx$$

input
```
integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*x^(9/2)/(b*x^3 + a), x)
```

3.295.6 Sympy [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2 + bx^5}} \, dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{x^2 (a + bx^3)}} \, dx$$

input
```
integrate(x**(13/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(13/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.295.7 Maxima [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

3.295.8 Giac [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{13/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(13/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(13/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.296 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

3.296.1 Optimal result

Integrand size = 21, antiderivative size = 525

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{5\left(1+\sqrt{3}\right)ax^{3/2}(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}$$
$$+\frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$+\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-5/8*a*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))/(b*x^5+a*x^2)^(1/2)+1/4*x^(3/2)*(b*x^5+a*x^2)^(1/2)/b+5/8*3^(1/4)*a^(4/
3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5
/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2)+5/48*a^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/
3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6
^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2
)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.296.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{x^{7/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(7/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11
/6, -((b*x^3)/a)]))/(4*b*Sqrt[x^2*(a + b*x^3)])
```

3.296.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1930, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5a\int \frac{x^{5/2}}{\sqrt{bx^5+ax^2}}dx}{8b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{8b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{851}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{837}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{766}$$

$$\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-$$

$$\frac{5ax\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{2420}$$

3.296. $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

$$\frac{\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{5ax\sqrt{a+bx^3}\left(\frac{\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\frac{\left(1-\right.}{}\right)}{4b\sqrt{ax^2+bx^5}}}{}$$

input
```
Int[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*Sqrt[a*x^2 + b*x^5])/(4*b) - (5*a*x*Sqrt[a + b*x^3]*((((1 + Sqrt[
3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4
)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^
(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)
*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (
1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3]
)/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(4*b*Sqrt[a*x^2 + b*x^5])
```

3.296. $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

3.296.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.296. $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.296.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.12

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2586

input
```
int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*x^(7/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)-5/8*a/b*(x*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1
/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*Ellipti
cF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a...
```

3.296.5 Fricas [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*x^(7/2)/(b*x^3 + a), x)
```

3.296.6 Sympy [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(11/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.296.7 Maxima [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

3.296.8 Giac [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{11/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(11/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(11/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.297 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx$

3.297.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

output
```
-1/3*a*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(3/2)+1/3*x^(1/2)*(b
*x^5+a*x^2)^(1/2)/b
```

3.297.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{b}x^{5/2}(a+bx^3)-ax\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + S
qrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

3.297.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\int \frac{x^{3/2}}{\sqrt{bx^5+ax^2}}dx}{2b}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\int \frac{1}{1-\frac{bx^5}{bx^5+ax^2}}d\frac{x^{5/2}}{\sqrt{bx^5+ax^2}}}{3b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

input
```
Int[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*
x^2 + b*x^5]])/(3*b^(3/2))
```

3.297.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.297.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{x^{\frac{3}{2}}\left(b\,x^{3}+a\right)\left(x\sqrt{b}\,\sqrt{x\left(b\,x^{3}+a\right)}-\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)a\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{x\left(b\,x^{3}+a\right)}\,b^{\frac{3}{2}}}$	79
risch	$\frac{x^{\frac{5}{2}}\left(b\,x^{3}+a\right)}{3b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)\sqrt{x}\,\sqrt{x\left(b\,x^{3}+a\right)}}{3b^{\frac{3}{2}}\,\sqrt{x^{2}\left(b\,x^{3}+a\right)}}$	82

input
```
int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3/2)*(b*x^3+a)*(x*b^(1/2)*(x*(b*x^3+a))^(1/2)-arctanh(1/x^2*(x*(b*x
^3+a))^(1/2)/b^(1/2))*a)/(b*x^5+a*x^2)^(1/2)/(x*(b*x^3+a))^(1/2)/b^(3/2)
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{a\sqrt{b}\log\left(-8\,b^2x^6-8\,abx^3+4\,\sqrt{bx^5+ax^2}(2\,bx^3+a)\sqrt{b}\sqrt{x}-a^2\right)+4\,\sqrt{bx^5+ax^2}b\sqrt{}}{12\,b^2}\right.$$

input
```
integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x
^3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6
*(a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a))
+ 2*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]
```

3.297.6 Sympy [F]

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{9}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(9/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.297.7 Maxima [F]

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{9}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt{bx^3+a}\,x^{\frac{3}{2}}}{3\,b\mathrm{sgn}\left(x\right)} + \frac{a\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^3 + a)*x^(3/2)/(b*sgn(x)) + 1/3*a*log(abs(-sqrt(b)*x^(3/2) +
sqrt(b*x^3 + a)))/(b^(3/2)*sgn(x))
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{9/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(9/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(9/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.298 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$

3.298.1 Optimal result

Integrand size = 21, antiderivative size = 237

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
1/2*(b*x^5+a*x^2)^(1/2)/b/x^(1/2)-1/12*a^(2/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)
*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipti
cF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^5+a*x^2)^(1/2)/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.298.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.30

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/
6, -((b*x^3)/a)]))/(2*b*Sqrt[x^2*(a + b*x^3)])
```

3.298.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1930, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{4b}$$

$$\Big\downarrow \text{1938}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{4b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{851}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{ax\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{766}$$

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*
x^2 + b*x^5])
```

3.298.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

3.298. $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.298.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.07

method	result
risch	$\frac{x^{\frac{3}{2}}\left(b\,x^{3}+a\right)}{2b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{a\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\ldots}{b\left(-\frac{\left(-a\,b\ldots}{2b\ldots}\ldots}}\ldots}{2\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\ldots\right.}$
default	Expression too large to display

input
```
int(x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*x^(3/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)-1/2*a*(1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*
b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/
(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*E
llipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

3.298.5 Fricas [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*x^(3/2)/(b*x^3 + a), x)
```

3.298.6 Sympy [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(7/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.298.7 Maxima [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

3.298.8 Giac [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{7/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(7/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(7/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.299 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$

3.299.1 Optimal result

Integrand size = 21, antiderivative size = 492

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\left(1+\sqrt{3}\right)x^{3/2}(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x
^5+a*x^2)^(1/2)-3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b
^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x^(3/2)*(a^
(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4
)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.299.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.12

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^{7/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^(7/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)
/a)])/(5*Sqrt[x^2*(a + b*x^3)])
```

3.299.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \ 1938$$

$$\frac{x\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 851$$

$$\frac{2x\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 837$$

$$\frac{2x\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 25$$

$$\frac{2x\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 766$$

$$\frac{2x\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{\ldots}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{\ldots}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \ 2420$$

3.299. $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$

$$\frac{2x\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\frac{(1-\cdots}{\cdots}\right)}{\sqrt{ax^2+bx^5}}$$

input
```
Int[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x*Sqrt[a + b*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3)
 + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2
*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*
x^3])))/Sqrt[a*x^2 + b*x^5]
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.299.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 2374, normalized size of antiderivative = 4.83

method	result	size
default	Expression too large to display	2374

input
```
int(x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2*(-I*(-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(
1/3)*3^(1/2)*b*x^2+2*I*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x-2*(-(I*3^(1
/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*(-a*b^2)^(1/3)*b*x^2+3*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-...
```

3.299.5 Fricas [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^3 + a), x)
```

3.299.6 Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.299.7 Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

3.299.8 Giac [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{5/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(5/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(5/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.300 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$

3.300.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x^2(a+bx^3)}}$$

input
```
Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sq
rt[x^2*(a + b*x^3)])
```

3.300.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$$
$$\Big\downarrow \text{ 1935}$$
$$\frac{2}{3}\int \frac{1}{1-\frac{bx^5}{bx^5+ax^2}}d\frac{x^{5/2}}{\sqrt{bx^5+ax^2}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

input
```
Int[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

3.300.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.300.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 1.78 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{2x^{\frac{3}{2}}\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{x\left(b\,x^{3}+a\right)}\,\sqrt{b}}$	59

input
```
int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/(x*(b*x^3+a))^(1/2)/b^(1/2)*arct
anh(1/x^2*(x*(b*x^3+a))^(1/2)/b^(1/2))
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-4\,\sqrt{bx^5+ax^2}(2\,bx^3+a)\sqrt{b}\sqrt{x}-a^2\right)}{6\,\sqrt{b}},\; -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^5+ax^2}\sqrt{-b}\sqrt{x}}{2\,bx^3+a}\right)}{3\,b}\right]$$

input
```
integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt
(b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqr
t(-b)*sqrt(x)/(2*b*x^3 + a))/b]
```

3.300.6 Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.300.7 Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\log\left(|a|\right)\mathrm{sgn}(x)}{3\sqrt{b}}-\frac{2\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\sqrt{b}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(a))*sgn(x)/sqrt(b) - 2/3*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3
 + a)))/(sqrt(b)*sgn(x))
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{3/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(3/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(3/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.301 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$

3.301.1 Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
1/3*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/a^(1/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.301.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^{3/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*x^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/
a)])/Sqrt[x^2*(a + b*x^3)]
```

3.301.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{x\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{2x\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{\sqrt{ax^2+bx^5}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

input
```
Int[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

output
```
(x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (
1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3]
)/4])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

3.301.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.301. $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$

3.301.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.15

method	result
default	$-\frac{4x^{\frac{3}{2}}\left(b\,x^{3}+a\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)xb}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\,\sqrt{\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\,\sqrt{\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\,F\left(\sqrt{-\frac{\left(i\sqrt{3}\cdots}{\left(i\sqrt{3}-1\right)\left(-\cdots\right.}}\right.}{\sqrt{b\,x^{5}+a\,x^{2}}\,b\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{x\left(b\,x^{3}+a\right)}\left(i\sqrt{3}-3\right)\sqrt{\frac{x\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\left(i\sqrt{3}\cdots\right.}{\cdots}}}$

input
```
int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b/(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
I*3^(1/2)*b^2*x^2-2*I*(-a*b^2)^(1/3)*3^(1/2)*b*x+I*3^(1/2)*(-a*b^2)^(2/3)-
b^2*x^2+2*(-a*b^2)^(1/3)*b*x-(-a*b^2)^(2/3))/(x*(b*x^3+a))^(1/2)/(I*3^(1/2
)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

3.301.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

input
```
integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

3.301.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(x**2*(a + b*x**3)), x)
```

3.301.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

3.301.8 Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(x^(1/2)/(a*x^2 + b*x^5)^(1/2),x)
```

output
```
int(x^(1/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.302 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$

3.302.1 Optimal result

Integrand size = 21, antiderivative size = 519

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$
$$- \frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$- \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
2*b^(1/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/
(b*x^5+a*x^2)^(1/2)-2*(b*x^5+a*x^2)^(1/2)/a/x^(3/2)-2*3^(1/4)*b^(1/3)*x^(3
/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*
x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1
/2)))^2)^(1/2)-1/3*b^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*
x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/
3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.302.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{x}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3
)/a)])/Sqrt[x^2*(a + b*x^3)]
```

3.302.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1931, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1931}$$

$$\frac{2b\int \frac{x^{5/2}}{\sqrt{bx^5+ax^2}}dx}{a}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{1938}$$

$$\frac{2bx\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{4bx\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{4bx\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4bx\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$\frac{4bx\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{a\sqrt{ax^2+bx^5}}$$

$$\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{2420}$$

3.302. $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}dx$

$$\frac{4bx\sqrt{a+bx^3}\left(\frac{\frac{(1+\sqrt{3})\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\frac{(1-\cdots}{\cdots}\right)}{a\sqrt{ax^2+bx^5}}$$

$$\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) + (4*b*x*Sqrt[a + b*x^3]*((((1 + Sqrt
[3])*Sqrt[x]*Sqrt[a + b*x^3])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/
4)*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a
^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3
)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) +
(1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3
])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(a*Sqrt[a*x^2 + b*x^5])
```

3.302. $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$

3.302.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.302.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.15

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2860

input
```
int(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^3+a)/a/(x^2*(b*x^3+a))^(1/2)*x^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*
(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(
((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^...
```

3.302.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

input
```
integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

3.302.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x**3))), x)
```

3.302.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

3.302.8 Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.303 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$

3.303.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

output
```
-2/3*(b*x^5+a*x^2)^(1/2)/a/x^(5/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{x^2(a+bx^3)}}{3ax^{5/2}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))
```

3.303.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{1920}$$

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

input
```
Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

3.303.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.303.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{2\left(b\,x^{3}+a\right)}{3\sqrt{x}\,a\sqrt{b\,x^{5}+a\,x^{2}}}$	29
default	$-\frac{2\left(b\,x^{3}+a\right)}{3\sqrt{x}\,a\sqrt{b\,x^{5}+a\,x^{2}}}$	29
risch	$-\frac{2\left(b\,x^{3}+a\right)}{3a\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,\sqrt{x}}$	29

input
```
int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(1/2)*(b*x^3+a)/a/(b*x^5+a*x^2)^(1/2)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{bx^5+ax^2}}{3\,ax^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^5 + a*x^2)/(a*x^(5/2))
```

3.303.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,(bx^4+ax)}{3\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(\frac{\sqrt{b+\frac{a}{x^3}}}{a}-\frac{\sqrt{b}}{a}\right)}{3\,\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(sqrt(b + a/x^3)/a - sqrt(b)/a)/sgn(x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.304 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$

3.304.1 Optimal result

Integrand size = 21, antiderivative size = 235

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-2/5*(b*x^5+a*x^2)^(1/2)/a/x^(7/2)-2/15*b*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^
(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1
-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^5+a*x^2)^(1/2)/(b^
(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.304.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5x^{3/2}\sqrt{x^2\,(a+bx^3)}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(
5*x^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

3.304.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1931, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \ 1931$$

$$-\frac{2b\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

$$\Big\downarrow \ 1938$$

$$-\frac{2bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

$$\Big\downarrow \ 851$$

$$-\frac{4bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

$$\Big\downarrow \ 766$$

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt
[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*
Sqrt[a*x^2 + b*x^5])
```

3.304.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.304.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.11

method	result
risch	$-\frac{2(b x^3+a)}{5a\, x^{\frac{3}{2}}\sqrt{x^2(b x^3+a)}}-\frac{4b^2\left(\frac{\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\, b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\, b^2\right)^{\frac{1}{3}}}{b}\right)^2\sqrt{\cdots}}{5a\left(-\frac{3(-\cdots}{\cdots}\cdots\right.}$
default	Expression too large to display

input
```
int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/a*(b*x^3+a)/x^(3/2)/(x^2*(b*x^3+a))^(1/2)-4/5*b^2/a*(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/
3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b
*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(
1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

3.304.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(2\sqrt{a}bx^4\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{5\,a^2x^4}$$

input
```
integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(2*sqrt(a)*b*x^4*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5 + a*
x^2)*a*sqrt(x))/(a^2*x^4)
```

3.304.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.304.7 Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

3.304.8 Giac [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.305 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

3.305.1 Optimal result

Integrand size = 21, antiderivative size = 555

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}$$
$$-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}+\frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}}$$
$$+\frac{8\sqrt[4]{3}b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$+\frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-8/7*b^(4/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))/(b*x^5+a*x^2)^(1/2)-2/7*(b*x^5+a*x^2)^(1/2)/a/x^(9/2)+8/7*b*(b*x^5+a*
x^2)^(1/2)/a^2/x^(3/2)+8/7*3^(1/4)*b^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((
1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+4/21*b^(4/3)
*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.305.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},-\frac{bx^3}{a}\right)}{7x^{5/2}\sqrt{x^2\,(a+bx^3)}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((b*x^3)/a)])/
(7*x^(5/2)*Sqrt[x^2*(a + b*x^3)])
```

3.305.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1931, 1931, 1938, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{4b\int \frac{1}{\sqrt{x}\sqrt{bx^5+ax^2}}dx}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{4b\left(\frac{2b\int \frac{x^{5/2}}{\sqrt{bx^5+ax^2}}dx}{a}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$$\Big\downarrow \text{ 1938}$$

$$-\frac{4b\left(\frac{2bx\sqrt{a+bx^3}\int \frac{x^{3/2}}{\sqrt{bx^3+a}}dx}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\int \frac{x^2}{\sqrt{bx^3+a}}d\sqrt{x}}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$$\Big\downarrow \text{ 837}$$

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}\right)}{a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}\right)}{7a}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\downarrow$ 766

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\left(\frac{\int \frac{2b^{2/3}x^2+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^3+a}}d\sqrt{x}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{a\sqrt{ax^2+bx^5}}\right.}{7a}$$

$$\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

$\downarrow$ 2420

3.305. $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

$$-\frac{4b\left(\frac{4bx\sqrt{a+bx^3}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{a+bx^3}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}-\frac{\sqrt[4]{3}\sqrt[3]{a}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2b^{2/3}}-\cdots\right)}{a\sqrt{ax^2+bx^5}}\cdots}{7a}$$

$$\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(7*a*x^(9/2)) - (4*b*((-2*Sqrt[a*x^2 + b*x^5])/(a
*x^(3/2)) + (4*b*x*Sqrt[a + b*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[a + b*x^3
])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x) - (3^(1/4)*a^(1/3)*Sqrt[x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1
/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*Sqrt[x]*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*S
qrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^
2]*Sqrt[a + b*x^3])))/(a*Sqrt[a*x^2 + b*x^5])))/(7*a)
```

3.305. $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

3.305.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.305.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.03

method	result	size
risch	Expression too large to display	1125
default	Expression too large to display	3048

input
```
int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^3+a)*(-4*b*x^3+a)/a^2/x^(5/2)/(x^2*(b*x^3+a))^(1/2)-8/7*b^2/a^2*
(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(
-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-
a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)...
```

3.305.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(4\sqrt{a}bx^5\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{7a^2x^5}$$

input
```
integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/7*(4*sqrt(a)*b*x^5*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4
*b/a, 1/x)) + sqrt(b*x^5 + a*x^2)*a*sqrt(x))/(a^2*x^5)
```

3.305.6 Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.305.7 Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

3.305.8 Giac [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{7/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.306 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$

3.306.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}$$

output
```
-2/9*(b*x^5+a*x^2)^(1/2)/a/x^(11/2)+4/9*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(5/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2(a-2bx^3)\sqrt{x^2(a+bx^3)}}{9a^2x^{11/2}}$$

input
```
Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*a^2*x^(11/2))
```

3.306.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2b\int \frac{1}{x^{3/2}\sqrt{bx^5+ax^2}}dx}{3a}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

input
```
Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2
*x^(5/2))
```

3.306.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.306.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2(b\,x^{3}+a)\,(-2b\,x^{3}+a)}{9x^{\frac{7}{2}}\,a^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
default	$-\frac{2(b\,x^{3}+a)\,(-2b\,x^{3}+a)}{9x^{\frac{7}{2}}\,a^{2}\sqrt{b\,x^{5}+a\,x^{2}}}$	37
risch	$-\frac{2(b\,x^{3}+a)\,(-2b\,x^{3}+a)}{9\sqrt{x^{2}(b\,x^{3}+a)}\,x^{\frac{7}{2}}\,a^{2}}$	37

input
```
int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{bx^5+ax^2}(2\,bx^3-a)}{9\,a^2x^{\frac{11}{2}}}$$

input
```
integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(2*b*x^3 - a)/(a^2*x^(11/2))
```

3.306.6 Sympy [F]

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

input
```
integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(9/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,(2\,b^2x^7+abx^4-a^2x)}{9\,\sqrt{bx^3+a}a^2x^{\frac{11}{2}}}$$

input
```
integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(\frac{\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{a^2}-\frac{3\sqrt{b+\frac{a}{x^3}}\,b}{a^2}+\frac{2\,b^{\frac{3}{2}}}{a^2}\right)}{9\,\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/9*((b + a/x^3)^(3/2)/a^2 - 3*sqrt(b + a/x^3)*b/a^2 + 2*b^(3/2)/a^2)/sgn
(x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{9/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.307 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$

3.307.1 Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

output
```
-2/11*(b*x^5+a*x^2)^(1/2)/a/x^(13/2)+16/55*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(7/
2)+16/165*b^2*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/a^(7/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

3.307.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{1}{2},-\frac{5}{6},-\frac{bx^3}{a}\right)}{11x^{9/2}\sqrt{x^2\,(a+bx^3)}}$$

input
```
Integrate[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)])
/(11*x^(9/2)*Sqrt[x^2*(a + b*x^3)])
```

3.307.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1931, 1931, 1938, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{8b\int \frac{1}{x^{5/2}\sqrt{bx^5+ax^2}}dx}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{8b\left(-\frac{2b\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}dx}{5a}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{ 1938}$$

$$-\frac{8b\left(-\frac{2bx\sqrt{a+bx^3}\int \frac{1}{\sqrt{x}\sqrt{bx^3+a}}dx}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{8b\left(-\frac{4bx\sqrt{a+bx^3}\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{x}}{5a\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{8b\left(-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\right)}{11a}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

input
```
Int[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

output
```
(-2*Sqrt[a*x^2 + b*x^5])/(11*a*x^(13/2)) - (8*b*((-2*Sqrt[a*x^2 + b*x^5])/
(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^
(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]))/(1
1*a)
```

3.307.3.1 Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.307.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{2(b\,x^3+a)(-8b\,x^3+5a)}{55a^2x^{\frac{9}{2}}\sqrt{x^2(b\,x^3+a)}}+\frac{32b^3\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)}{55a^2(}$
default	Expression too large to display

input
```
int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/55*(b*x^3+a)*(-8*b*x^3+5*a)/a^2/x^(9/2)/(x^2*(b*x^3+a))^(1/2)+32/55*b^3
/a^2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3
))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+
a))^(1/2)
```

3.307.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(16\sqrt{a}b^2x^7\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}(8abx^3-5a^2)\sqrt{x}\right)}{55a^3x^7}$$

input
```
integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(16*sqrt(a)*b^2*x^7*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5
 + a*x^2)*(8*a*b*x^3 - 5*a^2)*sqrt(x))/(a^3*x^7)
```

3.307.6 Sympy [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{\frac{11}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

input
```
integrate(1/x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(11/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.307.7 Maxima [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

3.307.8 Giac [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{11/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

input
```
int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

output
```
int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.308 $\int \frac{x}{ax^3+bx^4}\,dx$

3.308.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x}{ax^3+bx^4}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

3.308.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^3+bx^4}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

input
```
Integrate[x/(a*x^3 + b*x^4),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

3.308.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^3+bx^4}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^2(a+bx)}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

input
```
Int[x/(a*x^3 + b*x^4),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

3.308.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.308.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

input
```
int(x/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x}{ax^3+bx^4}\,dx = \frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

3.308.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{x}{ax^3+bx^4}\,dx = -\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

input
```
integrate(x/(b*x**4+a*x**3),x)
```

output
```
-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^3 + bx^4}\,dx = \frac{b\log(bx+a)}{a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

input
```
integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x}{ax^3 + bx^4}\,dx = \frac{b\log(|bx+a|)}{a^2} - \frac{b\log(|x|)}{a^2} - \frac{1}{ax}$$

input
```
integrate(x/(b*x^4+a*x^3),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

3.308.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{x}{ax^3 + bx^4}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2} - \frac{1}{a\,x}$$

input
```
int(x/(a*x^3 + b*x^4),x)
```

output
```
(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.309 $\int \frac{1}{ax^3+bx^4}\,dx$

3.309.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{1}{ax^3+bx^4}\,dx=-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

3.309.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^3+bx^4}\,dx=-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

input
```
Integrate[(a*x^3 + b*x^4)^(-1),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

3.309.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^3+bx^4}dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{1}{x^3(a+bx)}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

input
```
Int[(a*x^3 + b*x^4)^(-1),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

3.309.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.309.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a\,x^2}+\frac{b}{x\,a^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
norman	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(-x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	43
parallelrisch	$\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2}{2a^3x^2}$	44

input
```
int(1/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+b/x/a^2+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{ax^3+bx^4}\,dx=-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

input
```
integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

3.309.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{ax^3+bx^4}\,dx=\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

input
```
integrate(1/(b*x**4+a*x**3),x)
```

output
```
(-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^3+bx^4}\,dx = -\frac{b^2\log(bx+a)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

input
```
integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")
```

output
```
-b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{ax^3+bx^4}\,dx = -\frac{b^2\log(|bx+a|)}{a^3}+\frac{b^2\log(|x|)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/(b*x^4+a*x^3),x, algorithm="giac")
```

output
```
-b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^
3*x^2)
```

3.309.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{ax^3+bx^4}\,dx = -\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

input
```
int(1/(a*x^3 + b*x^4),x)
```

output
```
- (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.310 $\int \frac{x^4}{\sqrt{ax^3+bx^4}} \, dx$

3.310.1 Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}} \, dx = -\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}+\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}$$

output
```
-5/8*a^3*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(7/2)-5/12*a*(b*x^4+a*
x^3)^(1/2)/b^2+5/8*a^2*(b*x^4+a*x^3)^(1/2)/b^3/x+1/3*x*(b*x^4+a*x^3)^(1/2)
/b
```

3.310.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}} \, dx = \frac{\sqrt{b}x^2(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3x^{3/2}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{7/2}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x^4/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 30*a^3*x^(3/
2)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(24
*b^(7/2)*Sqrt[x^3*(a + b*x)])
```

3.310.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1930, 1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\int \frac{x^3}{\sqrt{bx^4+ax^3}}dx}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+ax^3}}dx}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{bx^4+ax^3}}dx}{2b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}}{b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a\left(\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}$$

input
```
Int[x^4/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a*(Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*(Sq
rt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/b
^(3/2)))/(4*b)))/(6*b)
```

3.310.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.310.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{8b^{\frac{5}{2}}x^2\sqrt{x^3(bx+a)}-10a\,b^{\frac{3}{2}}x\sqrt{x^3(bx+a)}-15\,\mathrm{arctanh}\left(\frac{\sqrt{x^3(bx+a)}}{x^2\sqrt{b}}\right)a^3x+15a^2\sqrt{x^3(bx+a)}\,\sqrt{b}}{24b^{\frac{7}{2}}x}$	91
risch	$\frac{(8b^2x^2-10abx+15a^2)x^2(bx+a)}{24b^3\sqrt{x^3(bx+a)}}-\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)x\sqrt{x(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x^3(bx+a)}}$	98
default	$\frac{x\sqrt{x(bx+a)}\left(16x^2\sqrt{b\,x^2+ax}\,b^{\frac{7}{2}}-20\sqrt{b\,x^2+ax}\,b^{\frac{5}{2}}ax+30\sqrt{b\,x^2+ax}\,b^{\frac{3}{2}}a^2-15\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^3b\right)}{48\sqrt{b\,x^4+a\,x^3}\,b^{\frac{9}{2}}}$	120

input
```
int(x^4/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^(5/2)*x^2*(x^3*(b*x+a))^(1/2)-10*a*b^(3/2)*x*(x^3*(b*x+a))^(1/2)
-15*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a^3*x+15*a^2*(x^3*(b*x+a))^(1
/2)*b^(1/2))/b^(7/2)/x
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.53

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$$
$$= \left[\frac{15\,a^3\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^4+ax^3}}{48\,b^4x}, \frac{15\,a^3\sqrt{-b}x\arctan\left(\frac{\sqrt{bx}\right.}{}\right.}{}\right.$$

input
```
integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b)
)/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x),
 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + (8
*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x)]
```

3.310.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(x**4/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**3*(a + b*x)), x)
```

3.310.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\,x\left(\frac{4\,x}{b\mathrm{sgn}\left(x\right)}-\frac{5\,a}{b^2\mathrm{sgn}\left(x\right)}\right)+\frac{15\,a^2}{b^3\mathrm{sgn}\left(x\right)}\right) - \frac{5\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{16\,b^{\frac{7}{2}}} + \frac{5\,a^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a*x)*(2*x*(4*x/(b*sgn(x)) - 5*a/(b^2*sgn(x))) + 15*a^2/(
b^3*sgn(x))) - 5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 5/16*a^3*log(abs(2*(s
qrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/(b^(7/2)*sgn(x))
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x^4/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x^4/(a*x^3 + b*x^4)^(1/2), x)
```

3.311 $\int \frac{x^3}{\sqrt{ax^3+bx^4}} dx$

3.311.1 Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}} dx = \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}$$

output
```
3/4*a^2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(5/2)+1/2*(b*x^4+a*x^3)
^(1/2)/b-3/4*a*(b*x^4+a*x^3)^(1/2)/b^2/x
```

3.311.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}} dx = \frac{\sqrt{b}x^2(-3a^2-abx+2b^2x^2)+6a^2x^{3/2}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x^3/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*x^(3/2)*Sqrt[a + b*x]*Ar
cTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2)*Sqrt[x^3*(
a + b*x)])
```

3.311.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1930, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+ax^3}}dx}{4b}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{bx^4+ax^3}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}}{b}\right)}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\left(\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}\right)}{4b}$$

input
```
Int[x^3/Sqrt[a*x^3 + b*x^4],x]
```

output
```
Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*(Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/b^(3/2)))/(4*b)
```

3.311.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.311.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right)a^{2}x+2b^{\frac{3}{2}}x\sqrt{x^{3}(bx+a)}-3a\sqrt{b}\,\sqrt{x^{3}(bx+a)}}{4b^{\frac{5}{2}}x}$	69
risch	$-\frac{(-2bx+3a)x^{2}(bx+a)}{4b^{2}\sqrt{x^{3}(bx+a)}}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)x\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x^{3}(bx+a)}}$	87
default	$\frac{x\sqrt{x(bx+a)}\,\left(4x\sqrt{b\,x^{2}+ax}\,b^{\frac{5}{2}}-6\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}a+3\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}b\right)}{8\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{7}{2}}}$	98

input
```
int(x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^(5/2)*(3*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a^2*x+2*b^(3/2)*x*
(x^3*(b*x+a))^(1/2)-3*a*b^(1/2)*(x^3*(b*x+a))^(1/2))/x
```

3.311.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{3\,a^2\sqrt{b}x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}(2\,b^2x-3\,ab)}{8\,b^3x}, -\frac{3\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)-\sqrt{bx^4+ax^3}(2\,b^2x-3\,ab)}{4\,b^3x}\right]$$

input
```
integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/
x) + 2*sqrt(b*x^4 + a*x^3)*(2*b^2*x - 3*a*b))/(b^3*x), -1/4*(3*a^2*sqrt(-b
)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) - sqrt(b*x^4 + a*x^3)*(2*
b^2*x - 3*a*b))/(b^3*x)]
```

3.311.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

3.311.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,x}{b\mathrm{sgn}\left(x\right)} - \frac{3\,a}{b^2\mathrm{sgn}\left(x\right)}\right) + \frac{3\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{8\,b^{\frac{5}{2}}} - \frac{3\,a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x/(b*sgn(x)) - 3*a/(b^2*sgn(x))) + 3/8*a^2*log(ab
s(a))*sgn(x)/b^(5/2) - 3/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*s
qrt(b) + a))/(b^(5/2)*sgn(x))
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x^3/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x^3/(a*x^3 + b*x^4)^(1/2), x)
```

3.312 $\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx$

3.312.1 Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx=\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

output
```
-a*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(3/2)+(b*x^4+a*x^3)^(1/2)/b/
x
```

3.312.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx=\frac{\sqrt{b}x^2(a+bx)+2ax^{3/2}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x^2/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(Sqrt[b]*x^2*(a + b*x) + 2*a*x^(3/2)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x
])/(Sqrt[a] - Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

3.312.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1930}$$

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{bx^4+ax^3}}dx}{2b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

input
```
Int[x^2/Sqrt[a*x^3 + b*x^4],x]
```

output
```
Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])
/b^(3/2)
```

3.312.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.312.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\operatorname{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right)ax+\sqrt{b}\,\sqrt{x^{3}(bx+a)}}{b^{\frac{3}{2}}x}$	47
risch	$\frac{x^{2}(bx+a)}{b\sqrt{x^{3}(bx+a)}}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)x\sqrt{x(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x^{3}(bx+a)}}$	76
default	$\frac{x\sqrt{x(bx+a)}\left(2\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}-a\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{5}{2}}}$	78

input
```
int(x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a*x+b^(1/2)*(x^3*(b*x+a))^(1/2)
)/b^(3/2)/x
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.18

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}b}{2\,b^2x}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)+\sqrt{bx^4+ax^3}b}{b^2x}\right]$$

input
```
integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) +
 2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3
)*sqrt(-b)/(b*x^2)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]
```

3.312.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(x**2/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**3*(a + b*x)), x)
```

3.312.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^4 + a*x^3), x)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = -\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{2\,b^{\frac{3}{2}}} + \frac{a\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{3}{2}}\operatorname{sgn}\left(x\right)} + \frac{\sqrt{bx^2+ax}}{b\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*log(abs(a))*sgn(x)/b^(3/2) + 1/2*a*log(abs(2*(sqrt(b)*x - sqrt(b*x^
2 + a*x))*sqrt(b) + a))/(b^(3/2)*sgn(x)) + sqrt(b*x^2 + a*x)/(b*sgn(x))
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x^2/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x^2/(a*x^3 + b*x^4)^(1/2), x)
```

3.313 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$

3.313.1 Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(1/2)
```

3.313.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2x^{3/2}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^3(a+bx)}}$$

input
```
Integrate[x/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(-2*x^(3/2)*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x^3*(a + b*x)])
```

3.313.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$$
$$\Big\downarrow \text{1935}$$
$$2\int \frac{1}{1-\frac{bx^4}{bx^4+ax^3}}d\frac{x^2}{\sqrt{bx^4+ax^3}}$$
$$\Big\downarrow \text{219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

input
```
Int[x/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

3.313.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.313.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right)}{\sqrt{b}}$	25
default	$\frac{x\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)}{\sqrt{b\,x^{4}+a\,x^{3}}\,\sqrt{b}}$	56

input
```
int(x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^(1/2)*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)}{b}\right]$$

input
```
integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-
b)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2))/b]
```

3.313.6 Sympy [F]

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x}{\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(x/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(x/sqrt(x**3*(a + b*x)), x)
```

3.313.7 Maxima [F]

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{x}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^4 + a*x^3), x)
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{\sqrt{b}}-\frac{\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{\sqrt{b}\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
log(abs(a))*sgn(x)/sqrt(b) - log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqr
t(b) + a))/(sqrt(b)*sgn(x))
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{x}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

input
```
int(x/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
int(x/(a*x^3 + b*x^4)^(1/2), x)
```

3.314 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$

3.314.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

output
```
-2*(b*x^4+a*x^3)^(1/2)/a/x^2
```

3.314.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

input
```
Integrate[1/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(-2*Sqrt[x^3*(a + b*x)])/(a*x^2)
```

3.314.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$$
$$\Big\downarrow \ 1906$$
$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

input
```
Int[1/Sqrt[a*x^3 + b*x^4],x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

3.314.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

3.314.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x^3(bx+a)}}{a\,x^2}$	20
trager	$-\frac{2\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$	22
risch	$-\frac{2x(bx+a)}{\sqrt{x^3(bx+a)}\,a}$	23
gosper	$-\frac{2x(bx+a)}{a\sqrt{b\,x^4+a\,x^3}}$	25
default	$-\frac{2\sqrt{x(bx+a)}\,\sqrt{b\,x^2+ax}}{\sqrt{b\,x^4+a\,x^3}\,a}$	39

input
```
int(1/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(x^3*(b*x+a))^(1/2)/a/x^2
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{bx^4+ax^3}}{ax^2}$$

input
```
integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^4 + a*x^3)/(a*x^2)
```

3.314.6 Sympy [F]

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**3 + b*x**4), x)
```

3.314.7 Maxima [F]

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}}\,dx$$

input
```
integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^4 + a*x^3), x)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = \frac{2}{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\operatorname{sgn}(x)}$$

input
```
integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))
```

3.314.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$$

input
```
int(1/(a*x^3 + b*x^4)^(1/2),x)
```

output
```
-(2*(a*x^3 + b*x^4)^(1/2))/(a*x^2)
```

3.315 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$

3.315.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}$$

output
```
-2/3*(b*x^4+a*x^3)^(1/2)/a/x^3+4/3*b*(b*x^4+a*x^3)^(1/2)/a^2/x^2
```

3.315.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = -\frac{2(a-2bx)(a+bx)}{3a^2\sqrt{x^3(a+bx)}}$$

input
```
Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*(a - 2*b*x)*(a + b*x))/(3*a^2*Sqrt[x^3*(a + b*x)])
```

3.315.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}dx$$
$$\Big\downarrow \text{ 1922}$$
$$-\frac{2b\int \frac{1}{\sqrt{bx^4+ax^3}}dx}{3a}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$
$$\Big\downarrow \text{ 1906}$$
$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

input
```
Int[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

3.315.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.315.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$-\frac{2(-2bx+a)\sqrt{x^3(bx+a)}}{3a^2x^3}$	26
trager	$-\frac{2(-2bx+a)\sqrt{b\,x^4+a\,x^3}}{3a^2x^3}$	28
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^3(bx+a)}\,a^2}$	28
gosper	$-\frac{2(bx+a)(-2bx+a)}{3a^2\sqrt{b\,x^4+a\,x^3}}$	30
default	$-\frac{2\sqrt{x(bx+a)}\,\sqrt{b\,x^2+ax}\,(-2bx+a)}{3x\sqrt{b\,x^4+a\,x^3}\,a^2}$	48

input
```
int(1/x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*b*x+a)/a^2/x^3*(x^3*(b*x+a))^(1/2)
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = \frac{2\,\sqrt{bx^4+ax^3}(2\,bx-a)}{3\,a^2x^3}$$

input
```
integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)
```

3.315.6 Sympy [F]

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(1/x/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**3*(a + b*x))), x)
```

3.315.7 Maxima [F]

$$\int \frac{1}{x\sqrt{ax^3 + bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4 + ax^3}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\sqrt{ax^3 + bx^4}}\,dx = \frac{2\left(3\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} + a\right)}{3\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)^3 \operatorname{sgn}(x)}$$

input
```
integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)/((sqrt(b)*x - sqrt(b*x
^2 + a*x))^3*sgn(x))
```

3.315.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{ax^3 + bx^4}}\,dx = -\frac{2\,a\,\sqrt{b\,x^4 + a\,x^3} - 4\,b\,x\,\sqrt{b\,x^4 + a\,x^3}}{3\,a^2\,x^3}$$

input
```
int(1/(x*(a*x^3 + b*x^4)^(1/2)),x)
```

output
```
-(2*a*(a*x^3 + b*x^4)^(1/2) - 4*b*x*(a*x^3 + b*x^4)^(1/2))/(3*a^2*x^3)
```

3.316 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$

3.316.1 Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}$$

output
```
-2/5*(b*x^4+a*x^3)^(1/2)/a/x^4+8/15*b*(b*x^4+a*x^3)^(1/2)/a^2/x^3-16/15*b^
2*(b*x^4+a*x^3)^(1/2)/a^3/x^2
```

3.316.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{x^3(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^4}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)
```

3.316.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\int \frac{1}{x\sqrt{bx^4+ax^3}}dx}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{bx^4+ax^3}}dx}{3a}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

$$\Big\downarrow \text{1906}$$

$$-\frac{4b\left(\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

input
```
Int[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) - (4*b*((-2*Sqrt[a*x^3 + b*x^4])/(3*a*x
^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)))/(5*a)
```

3.316.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.316.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{2\left(8b^{2}x^{2}-4abx+3a^{2}\right)\sqrt{x^{3}\left(bx+a\right)}}{15a^{3}x^{4}}$	39
trager	$-\frac{2\left(8b^{2}x^{2}-4abx+3a^{2}\right)\sqrt{b\,x^{4}+a\,x^{3}}}{15a^{3}x^{4}}$	41
risch	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x\sqrt{x^{3}\left(bx+a\right)}\,a^{3}}$	44
gosper	$-\frac{2\left(bx+a\right)\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x\,a^{3}\sqrt{b\,x^{4}+a\,x^{3}}}$	46
default	$-\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b\,x^{2}+ax}\,\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15x^{2}\sqrt{b\,x^{4}+a\,x^{3}}\,a^{3}}$	61

input
```
int(1/x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3/x^4*(x^3*(b*x+a))^(1/2)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{2\,\sqrt{bx^4+ax^3}(8\,b^2x^2-4\,abx+3\,a^2)}{15\,a^3x^4}$$

input
```
integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)
```

3.316.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x^2\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)
```

3.316.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)
```

3.316.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = \frac{2\left(20\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}+3\,a^2\right)}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5\operatorname{sgn}(x)}$$

input
```
integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/15*(20*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b + 15*(sqrt(b)*x - sqrt(b*x^2
+ a*x))*a*sqrt(b) + 3*a^2)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^5*sgn(x))
```

3.316.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x^3}\,\left(3\,a^2-4\,a\,b\,x+8\,b^2\,x^2\right)}{15\,a^3\,x^4}$$

input
```
int(1/(x^2*(a*x^3 + b*x^4)^(1/2)),x)
```

output
```
-(2*(a*x^3 + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^2 - 4*a*b*x))/(15*a^3*x^4)
```

3.317 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$

3.317.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}$$

output
```
-2/7*(b*x^4+a*x^3)^(1/2)/a/x^5+12/35*b*(b*x^4+a*x^3)^(1/2)/a^2/x^4-16/35*b
^2*(b*x^4+a*x^3)^(1/2)/a^3/x^3+32/35*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^2
```

3.317.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{2\sqrt{x^3(a+bx)}(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^5}$$

input
```
Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(3
5*a^4*x^5)
```

3.317.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1922, 1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\int \frac{1}{x^2\sqrt{bx^4+ax^3}}dx}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\int \frac{1}{x\sqrt{bx^4+ax^3}}dx}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{bx^4+ax^3}}dx}{3a}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

$$\Big\downarrow \text{1906}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

input
```
Int[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) - (6*b*((-2*Sqrt[a*x^3 + b*x^4])/(5*a*x
^4) - (4*b*((-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])
/(3*a^2*x^2)))/(5*a)))/(7*a)
```

3.317.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.317.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

method	result	size
pseudoelliptic	$-\frac{2\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)\sqrt{x^{3}\left(bx+a\right)}}{35a^{4}x^{5}}$	50
trager	$-\frac{2\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)\sqrt{b\,x^{4}+a\,x^{3}}}{35a^{4}x^{5}}$	52
risch	$-\frac{2\left(bx+a\right)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{2}\sqrt{x^{3}\left(bx+a\right)}\,a^{4}}$	55
gosper	$-\frac{2\left(bx+a\right)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{2}a^{4}\sqrt{b\,x^{4}+a\,x^{3}}}$	57
default	$-\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b\,x^{2}+ax}\,\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{3}\sqrt{b\,x^{4}+a\,x^{3}}\,a^{4}}$	72

input
```
int(1/x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/a^4/x^5*(x^3*(b*x+a))^(1/2
)
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx^4+ax^3}}{35\,a^4x^5}$$

input
```
integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a
^4*x^5)
```

3.317.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x^3\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)
```

3.317.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)
```

3.317.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{2\left(70\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3 b^{\frac{3}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2 ab+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}+5\,a^3\right)}{35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7 \operatorname{sgn}(x)}$$

input
```
integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/35*(70*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2) + 84*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^2*a*b + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b) + 5*a
^3)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^7*sgn(x))
```

3.317.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{12\,b\,\sqrt{b\,x^4+a\,x^3}}{35\,a^2\,x^4}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{7\,a\,x^5}-\frac{16\,b^2\,\sqrt{b\,x^4+a\,x^3}}{35\,a^3\,x^3}+\frac{32\,b^3\,\sqrt{b\,x^4+a\,x^3}}{35\,a^4\,x^2}$$

input
```
int(1/(x^3*(a*x^3 + b*x^4)^(1/2)),x)
```

output
```
(12*b*(a*x^3 + b*x^4)^(1/2))/(35*a^2*x^4) - (2*(a*x^3 + b*x^4)^(1/2))/(7*a
*x^5) - (16*b^2*(a*x^3 + b*x^4)^(1/2))/(35*a^3*x^3) + (32*b^3*(a*x^3 + b*x
^4)^(1/2))/(35*a^4*x^2)
```

3.318 $\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx$

3.318.1 Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}$$

output
```
-2/9*(b*x^4+a*x^3)^(1/2)/a/x^6+16/63*b*(b*x^4+a*x^3)^(1/2)/a^2/x^5-32/105*
b^2*(b*x^4+a*x^3)^(1/2)/a^3/x^4+128/315*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^3-25
6/315*b^4*(b*x^4+a*x^3)^(1/2)/a^5/x^2
```

3.318.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{x^3(a+bx)}(35a^4-40a^3bx+48a^2b^2x^2-64ab^3x^3+128b^4x^4)}{315a^5x^6}$$

input
```
Integrate[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[x^3*(a + b*x)]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x
^3 + 128*b^4*x^4))/(315*a^5*x^6)
```

3.318.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1922, 1922, 1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8b\int \frac{1}{x^3\sqrt{bx^4+ax^3}}dx}{9a}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8b\left(-\frac{6b\int \frac{1}{x^2\sqrt{bx^4+ax^3}}dx}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}\right)}{9a}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\int \frac{1}{x\sqrt{bx^4+ax^3}}dx}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}\right)}{9a}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{bx^4+ax^3}}dx}{3a}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}\right)}{9a}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

$$\Big\downarrow \text{1906}$$

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}\right)}{5a}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}\right)}{7a}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}\right)}{9a}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

input
```
Int[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^4])/(9*a*x^6) - (8*b*((-2*Sqrt[a*x^3 + b*x^4])/(7*a*x
^5) - (6*b*((-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) - (4*b*((-2*Sqrt[a*x^3 + b*
x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)))/(5*a)))/(7*a)))/
(9*a)
```

3.318.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.318.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

method	result	size
pseudoelliptic	$-\frac{2\left(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4\right)\sqrt{x^3\left(bx+a\right)}}{315a^5x^6}$	61
trager	$-\frac{2\left(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4\right)\sqrt{b\,x^4+a\,x^3}}{315a^5x^6}$	63
risch	$-\frac{2(bx+a)\left(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4\right)}{315x^3\sqrt{x^3\left(bx+a\right)}\,a^5}$	66
gosper	$-\frac{2(bx+a)\left(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4\right)}{315x^3a^5\sqrt{b\,x^4+a\,x^3}}$	68
default	$-\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b\,x^2+ax}\,\left(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4\right)}{315x^4\sqrt{b\,x^4+a\,x^3}\,a^5}$	83

input
```
int(1/x^4/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(128*b^4*x^4-64*a*b^3*x^3+48*a^2*b^2*x^2-40*a^3*b*x+35*a^4)/a^5/x^6
*(x^3*(b*x+a))^(1/2)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = -\frac{2\,(128\,b^4x^4-64\,ab^3x^3+48\,a^2b^2x^2-40\,a^3bx+35\,a^4)\sqrt{bx^4+ax^3}}{315\,a^5x^6}$$

input
```
integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(128*b^4*x^4 - 64*a*b^3*x^3 + 48*a^2*b^2*x^2 - 40*a^3*b*x + 35*a^4)
*sqrt(b*x^4 + a*x^3)/(a^5*x^6)
```

3.318.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x^4\sqrt{x^3\,(a+bx)}}\,dx$$

input
```
integrate(1/x**4/(b*x**4+a*x**3)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(x**3*(a + b*x))), x)
```

3.318.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a*x^3)*x^4), x)
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx$$
$$= \frac{2\left(1008\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4b^2+1680\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3ab^{\frac{3}{2}}+1080\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2b+315\right.}{315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^9\operatorname{sgn}(x)}$$

input
```
integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

output
```
2/315*(1008*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2 + 1680*(sqrt(b)*x - sqrt
(b*x^2 + a*x))^3*a*b^(3/2) + 1080*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b
+ 315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + 35*a^4)/((sqrt(b)*x -
sqrt(b*x^2 + a*x))^9*sgn(x))
```

3.318.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = \frac{16\,b\,\sqrt{b\,x^4+a\,x^3}}{63\,a^2\,x^5}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{9\,a\,x^6}-\frac{32\,b^2\,\sqrt{b\,x^4+a\,x^3}}{105\,a^3\,x^4}+\frac{128\,b^3\,\sqrt{b\,x^4+a\,x^3}}{315\,a^4\,x^3}-\frac{256\,b^4\,\sqrt{b\,x^4+a\,x^3}}{315\,a^5\,x^2}$$

input
```
int(1/(x^4*(a*x^3 + b*x^4)^(1/2)),x)
```

output
```
(16*b*(a*x^3 + b*x^4)^(1/2))/(63*a^2*x^5) - (2*(a*x^3 + b*x^4)^(1/2))/(9*a
*x^6) - (32*b^2*(a*x^3 + b*x^4)^(1/2))/(105*a^3*x^4) + (128*b^3*(a*x^3 + b
*x^4)^(1/2))/(315*a^4*x^3) - (256*b^4*(a*x^3 + b*x^4)^(1/2))/(315*a^5*x^2)
```

3.319 $\int \frac{1}{x^3+bx^5} \, dx$

3.319.1 Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{x^3+bx^5} \, dx = -\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1+bx^2\right)$$

output
```
-1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

3.319.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3+bx^5} \, dx = -\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1+bx^2\right)$$

input
```
Integrate[(x^3 + b*x^5)^(-1),x]
```

output
```
-1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

3.319.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2026, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^5+x^3}dx$$
$$\Big\downarrow \; 2026$$
$$\int \frac{1}{x^3(bx^2+1)}dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{1}{x^4(bx^2+1)}dx^2$$
$$\Big\downarrow \; 54$$
$$\frac{1}{2}\int \left(\frac{b^2}{bx^2+1}-\frac{b}{x^2}+\frac{1}{x^4}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-b\log\left(x^2\right)+b\log\left(bx^2+1\right)-\frac{1}{x^2}\right)$$

input
```
Int[(x^3 + b*x^5)^(-1),x]
```

output
```
(-x^(-2) - b*Log[x^2] + b*Log[1 + b*x^2])/2
```

3.319.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.319.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{2x^2}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^2+1\right)}{2}$	23
norman	$-\frac{1}{2x^2}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^2+1\right)}{2}$	23
risch	$-\frac{1}{2x^2}-b\ln\left(x\right)+\frac{b\ln\left(-b\,x^2-1\right)}{2}$	24
meijerg	$\frac{b\left(-\frac{1}{x^2b}-2\ln(x)-\ln(b)+\ln\left(b\,x^2+1\right)\right)}{2}$	29
parallelrisch	$-\frac{2b\ln(x)x^2-b\ln\left(b\,x^2+1\right)x^2+1}{2x^2}$	30

input
```
int(1/(b*x^5+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3+bx^5}\,dx=\frac{bx^2\log\left(bx^2+1\right)-2\,bx^2\log\left(x\right)-1}{2\,x^2}$$

input
```
integrate(1/(b*x^5+x^3),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

3.319.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 + bx^5}\,dx = -b\log(x) + \frac{b\log\left(x^2 + \frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

input
```
integrate(1/(b*x**5+x**3),x)
```

output
```
-b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 + bx^5}\,dx = \frac{1}{2}b\log\left(bx^2 + 1\right) - b\log(x) - \frac{1}{2x^2}$$

input
```
integrate(1/(b*x^5+x^3),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + 1) - b*log(x) - 1/2/x^2
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 + bx^5}\,dx = -\frac{1}{2}b\log\left(x^2\right) + \frac{1}{2}b\log\left(\left|bx^2 + 1\right|\right) + \frac{bx^2 - 1}{2x^2}$$

input
```
integrate(1/(b*x^5+x^3),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

3.319.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3+bx^5}\,dx=\frac{b\,\ln\left(b\,x^2+1\right)}{2}-b\,\ln\left(x\right)-\frac{1}{2\,x^2}$$

input
```
int(1/(b*x^5 + x^3),x)
```

output
```
(b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

3.320 $\int \frac{1}{-x^3+bx^5}\,dx$

3.320.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{-x^3+bx^5}\,dx=\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)$$

output
```
1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)
```

3.320.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+bx^5}\,dx=\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)$$

input
```
Integrate[(-x^3 + b*x^5)^(-1),x]
```

output
```
1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

3.320.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2026, 243, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^5-x^3}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{1}{x^3 (bx^2-1)}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int -\frac{1}{x^4 (1-bx^2)}dx^2$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{1}{2}\int \frac{1}{x^4 (1-bx^2)}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$-\frac{1}{2}\int \left(-\frac{b^2}{bx^2-1}+\frac{b}{x^2}+\frac{1}{x^4}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-b\log \left(x^2\right)+b\log \left(1-bx^2\right)+\frac{1}{x^2}\right)$$

input
```
Int[(-x^3 + b*x^5)^(-1),x]
```

output
```
(x^(-2) - b*Log[x^2] + b*Log[1 - b*x^2])/2
```

3.320.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.320.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^{2}-1\right)}{2}$	23
norman	$\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^{2}-1\right)}{2}$	23
risch	$\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(-b\,x^{2}+1\right)}{2}$	24
parallelrisch	$-\frac{2b\ln\left(x\right)x^{2}-b\ln\left(b\,x^{2}-1\right)x^{2}-1}{2x^{2}}$	30
meijerg	$\frac{b\left(\frac{1}{x^{2}b}-2\ln\left(x\right)-\ln\left(-b\right)+\ln\left(-b\,x^{2}+1\right)\right)}{2}$	31

input
```
int(1/(b*x^5-x^3),x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{bx^2\log(bx^2-1)-2\,bx^2\log(x)+1}{2\,x^2}$$

input
```
integrate(1/(b*x^5-x^3),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

3.320.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx = -b\log(x)+\frac{b\log\left(x^2-\frac{1}{b}\right)}{2}+\frac{1}{2x^2}$$

input
```
integrate(1/(b*x**5-x**3),x)
```

output
```
-b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{1}{2}\,b\log\left(bx^2-1\right)-b\log(x)+\frac{1}{2\,x^2}$$

input
```
integrate(1/(b*x^5-x^3),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2
```

3.320.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{-x^3+bx^5}\,dx = -\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2-1\right|\right)+\frac{bx^2+1}{2\,x^2}$$

input
```
integrate(1/(b*x^5-x^3),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

3.320.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{b\,\ln\left(b\,x^2-1\right)}{2}-b\,\ln\left(x\right)+\frac{1}{2\,x^2}$$

input
```
int(1/(b*x^5 - x^3),x)
```

output
```
(b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

3.321 $\int \frac{1}{ax+bx} \, dx$

3.321.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{1}{ax+bx} \, dx = \frac{\log(x)}{a+b}$$

output
```
ln(x)/(a+b)
```

3.321.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{ax+bx} \, dx = \frac{\log(ax+bx)}{a+b}$$

input
```
Integrate[(a*x + b*x)^(-1),x]
```

output
```
Log[a*x + b*x]/(a + b)
```

3.321.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx} \, dx$$
$$\Big\downarrow \; 6$$
$$\int \frac{1}{x(a+b)} dx$$
$$\Big\downarrow \; 14$$
$$\frac{\log(x)}{a+b}$$

input
```
Int[(a*x + b*x)^(-1),x]
```

output
```
Log[x]/(a + b)
```

3.321.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

3.321.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\ln(x)}{a+b}$	9
norman	$\frac{\ln(x)}{a+b}$	9
risch	$\frac{\ln(x)}{a+b}$	9
parallelrisch	$\frac{\ln(x)}{a+b}$	9

input
```
int(1/(a*x+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(a+b)
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx}\,dx = \frac{\log(x)}{a+b}$$

input
```
integrate(1/(a*x+b*x),x, algorithm="fricas")
```

output
```
log(x)/(a + b)
```

3.321.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{ax+bx}\,dx = \frac{\log(x)}{a+b}$$

input
```
integrate(1/(a*x+b*x),x)
```

output
```
log(x)/(a + b)
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{ax+bx}\,dx = \frac{\log{(ax+bx)}}{a+b}$$

input
```
integrate(1/(a*x+b*x),x, algorithm="maxima")
```

output
```
log(a*x + b*x)/(a + b)
```

3.321.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{ax+bx}\,dx = \frac{\log{(|ax+bx|)}}{a+b}$$

input
```
integrate(1/(a*x+b*x),x, algorithm="giac")
```

output
```
log(abs(a*x + b*x))/(a + b)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx}\,dx = \frac{\ln{(x)}}{a+b}$$

input
```
int(1/(a*x + b*x),x)
```

output
```
log(x)/(a + b)
```

3.322 $\int \frac{1}{(ax+bx)^2}\,dx$

3.322.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a+b)^2x}$$

output
```
-1/(a+b)^2/x
```

3.322.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a+b)^2x}$$

input
```
Integrate[(a*x + b*x)^(-2),x]
```

output
```
-(1/((a + b)^2*x))
```

3.322.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx)^2}dx$$

$$\Big\downarrow 6$$

$$\int \frac{1}{x^2(a+b)^2}dx$$

$$\Big\downarrow 15$$

$$-\frac{1}{x(a+b)^2}$$

input
```
Int[(a*x + b*x)^(-2),x]
```

output
```
-(1/((a + b)^2*x))
```

3.322.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.322.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{1}{(a+b)^2 x}$	11
default	$-\frac{1}{(a+b)^2 x}$	11
norman	$-\frac{1}{(a+b)^2 x}$	11
risch	$-\frac{1}{(a+b)^2 x}$	11
parallelrisch	$-\frac{1}{(a+b)^2 x}$	11

input
```
int(1/(a*x+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)^2/x
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a^2+2\,ab+b^2)x}$$

input
```
integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
```

output
```
-1/((a^2 + 2*a*b + b^2)*x)
```

3.322.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{x\,(a^2+2ab+b^2)}$$

input
```
integrate(1/(a*x+b*x)**2,x)
```

output
```
-1/(x*(a**2 + 2*a*b + b**2))
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(ax+bx)(a+b)}$$

input
```
integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
```

output
```
-1/((a*x + b*x)*(a + b))
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(ax+bx)(a+b)}$$

input
```
integrate(1/(a*x+b*x)^2,x, algorithm="giac")
```

output
```
-1/((a*x + b*x)*(a + b))
```

3.322.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{x\,(a+b)^2}$$

input
```
int(1/(a*x + b*x)^2,x)
```

output
```
-1/(x*(a + b)^2)
```

3.323 $\int \frac{1}{(ax+bx)^3} dx$

3.323.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{(ax+bx)^3} dx = -\frac{1}{2(a+b)^3x^2}$$

output
```
-1/2/(a+b)^3/x^2
```

3.323.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^3} dx = -\frac{1}{2(a+b)^3x^2}$$

input
```
Integrate[(a*x + b*x)^(-3),x]
```

output
```
-1/2*1/((a + b)^3*x^2)
```

3.323.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx)^3} \, dx$$
$$\Big\downarrow \text{6}$$
$$\int \frac{1}{x^3(a+b)^3}dx$$
$$\Big\downarrow \text{15}$$
$$-\frac{1}{2x^2(a+b)^3}$$

input
```
Int[(a*x + b*x)^(-3),x]
```

output
```
-1/2*1/((a + b)^3*x^2)
```

3.323.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.323.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1}{2(a+b)^3 x^2}$	11
default	$-\frac{1}{2(a+b)^3 x^2}$	11
norman	$-\frac{1}{2(a+b)^3 x^2}$	11
risch	$-\frac{1}{2(a+b)^3 x^2}$	11
parallelrisch	$-\frac{1}{2(a+b)^3 x^2}$	11

input
```
int(1/(a*x+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a+b)^3/x^2
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)x^2}$$

input
```
integrate(1/(a*x+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)
```

3.323.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2x^2\,(a^3+3a^2b+3ab^2+b^3)}$$

input
```
integrate(1/(a*x+b*x)**3,x)
```

output
```
-1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{(ax+bx)^3} \, dx = -\frac{1}{2\,(ax+bx)^2(a+b)}$$

input
```
integrate(1/(a*x+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2/((a*x + b*x)^2*(a + b))
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{(ax+bx)^3} \, dx = -\frac{1}{2\,(ax+bx)^2(a+b)}$$

input
```
integrate(1/(a*x+b*x)^3,x, algorithm="giac")
```

output
```
-1/2/((a*x + b*x)^2*(a + b))
```

3.323.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{(ax+bx)^3} \, dx = -\frac{1}{2\,x^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}$$

input
```
int(1/(a*x + b*x)^3,x)
```

output
```
-1/(2*x^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
```

3.324 $\int \frac{1}{ax^2+bx^2}\,dx$

3.324.1 Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{ax^2+bx^2}\,dx = -\frac{1}{(a+b)x}$$

output
```
-1/(a+b)/x
```

3.324.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx = -\frac{1}{(a+b)x}$$

input
```
Integrate[(a*x^2 + b*x^2)^(-1),x]
```

output
```
-(1/((a + b)*x))
```

3.324.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx^2}\,dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{1}{x^2(a+b)}dx$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{x(a+b)}$$

input
```
Int[(a*x^2 + b*x^2)^(-1),x]
```

output
```
-(1/((a + b)*x))
```

3.324.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.324.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{1}{(a+b)x}$	11
default	$-\frac{1}{(a+b)x}$	11
norman	$-\frac{1}{(a+b)x}$	11
risch	$-\frac{1}{(a+b)x}$	11
parallelrisch	$-\frac{1}{(a+b)x}$	11

input
```
int(1/(a*x^2+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)/x
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^2}\,dx = -\frac{1}{(a+b)x}$$

input
```
integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")
```

output
```
-1/((a + b)*x)
```

3.324.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{ax^2 + bx^2}\,dx = -\frac{1}{x\,(a+b)}$$

input
```
integrate(1/(a*x**2+b*x**2),x)
```

output
```
-1/(x*(a + b))
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^2} \, dx = -\frac{1}{(a+b)x}$$

input
```
integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")
```

output
```
-1/((a + b)*x)
```

3.324.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^2} \, dx = -\frac{1}{(a+b)x}$$

input
```
integrate(1/(a*x^2+b*x^2),x, algorithm="giac")
```

output
```
-1/((a + b)*x)
```

3.324.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^2} \, dx = -\frac{1}{x\ (a+b)}$$

input
```
int(1/(a*x^2 + b*x^2),x)
```

output
```
-1/(x*(a + b))
```

3.325 $\int \frac{1}{ax^n+bx^n}\,dx$

3.325.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{ax^n+bx^n}\,dx=\frac{x^{1-n}}{(a+b)(1-n)}$$

output
```
x^(1-n)/(a+b)/(1-n)
```

3.325.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^n+bx^n}\,dx=\frac{x^{1-n}}{(a+b)(1-n)}$$

input
```
Integrate[(a*x^n + b*x^n)^(-1),x]
```

output
```
x^(1 - n)/((a + b)*(1 - n))
```

3.325.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^n+bx^n}\,dx$$
$$\Big\downarrow \; 6$$
$$\int \frac{x^{-n}}{a+b}dx$$
$$\Big\downarrow \; 15$$
$$\frac{x^{1-n}}{(1-n)(a+b)}$$

input
```
Int[(a*x^n + b*x^n)^(-1),x]
```

output
```
x^(1 - n)/((a + b)*(1 - n))
```

3.325.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.325.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
risch	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
parallelrisch	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
norman	$-\frac{x\,\mathrm{e}^{-n\ln(x)}}{an+bn-a-b}$	26

input
```
int(1/(a*x^n+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-x/(-1+n)/(x^n)/(a+b)
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{ax^n+bx^n}\,dx=-\frac{x}{((a+b)n-a-b)x^n}$$

input
```
integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")
```

output
```
-x/(((a + b)*n - a - b)*x^n)
```

3.325.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{1}{ax^n+bx^n}\,dx=\begin{cases}-\frac{x}{anx^n-ax^n+bnx^n-bx^n} & \text{for } n\neq 1\\ \frac{\log(x)}{a+b} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x**n+b*x**n),x)
```

output
```
Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/
(a + b), True))
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{ax^n + bx^n} \, dx = -\frac{x}{(a(n-1) + b(n-1))x^n}$$

input
```
integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")
```

output
```
-x/((a*(n - 1) + b*(n - 1))*x^n)
```

3.325.8 Giac [F]

$$\int \frac{1}{ax^n + bx^n} \, dx = \int \frac{1}{ax^n + bx^n} \, dx$$

input
```
integrate(1/(a*x^n+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/(a*x^n + b*x^n), x)
```

3.325.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^n + bx^n} \, dx = -\frac{x^{1-n}}{(a+b)\,(n-1)}$$

input
```
int(1/(a*x^n + b*x^n),x)
```

output
```
-x^(1 - n)/((a + b)*(n - 1))
```

3.326 $\int \frac{1}{(ax^n+bx^n)^2}\,dx$

3.326.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

output
```
x^(1-2*n)/(a+b)^2/(1-2*n)
```

3.326.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

input
```
Integrate[(a*x^n + b*x^n)^(-2),x]
```

output
```
x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

3.326.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n+bx^n)^2} \, dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x^{-2n}}{(a+b)^2} dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

input
```
Int[(a*x^n + b*x^n)^(-2),x]
```

output
```
x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

3.326.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.326.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{x\,x^{-2n}}{(-1+2n)(a+b)^2}$	21
parallelrisch	$-\frac{x\,x^{-2n}}{(-1+2n)(a+b)^2}$	21
risch	$-\frac{x\,x^{-2n}}{(a^2+2ab+b^2)(-1+2n)}$	29
norman	$-\frac{x\,\mathrm{e}^{-2n\ln(x)}}{(2an+2bn-a-b)(a+b)}$	33

input
```
int(1/(a*x^n+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/(-1+2*n)/(x^n)^2/(a+b)^2
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \frac{x}{(a^2+2\,ab+b^2-2\,(a^2+2\,ab+b^2)n)x^{2\,n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")
```

output
```
x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))
```

3.326.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(15) = 30.

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \begin{cases} -\frac{x}{2a^2nx^{2n}-a^2x^{2n}+4abnx^{2n}-2abx^{2n}+2b^2nx^{2n}-b^2x^{2n}} & \text{for } n \neq \frac{1}{2} \\ \frac{\log(x)}{a^2+2ab+b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x**n+b*x**n)**2,x)
```

output
```
Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*
b*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**
2 + 2*a*b + b**2), True))
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{(ax^n + bx^n)^2} \, dx = -\frac{x}{(a^2(2\,n-1) + 2\,ab(2\,n-1) + b^2(2\,n-1))x^{2\,n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")
```

output
```
-x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))
```

3.326.8 Giac [F]

$$\int \frac{1}{(ax^n + bx^n)^2} \, dx = \int \frac{1}{(ax^n + bx^n)^2} \, dx$$

input
```
integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((a*x^n + b*x^n)^(-2), x)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^n + bx^n)^2} \, dx = -\frac{x^{1-2\,n}}{(a+b)^2\,(2\,n-1)}$$

input
```
int(1/(a*x^n + b*x^n)^2,x)
```

output
```
-x^(1 - 2*n)/((a + b)^2*(2*n - 1))
```

3.327 $\int \frac{1}{(ax^n+bx^n)^3}\,dx$

3.327.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

output
```
x^(1-3*n)/(a+b)^3/(1-3*n)
```

3.327.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

input
```
Integrate[(a*x^n + b*x^n)^(-3),x]
```

output
```
x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

3.327.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n+bx^n)^3} \, dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x^{-3n}}{(a+b)^3} dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

input
```
Int[(a*x^n + b*x^n)^(-3),x]
```

output
```
x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

3.327.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.327.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{x\,x^{-3n}}{(-1+3n)(a+b)^3}$	21
parallelrisch	$-\frac{x\,x^{-3n}}{(-1+3n)(a+b)^3}$	21
norman	$-\frac{x\,\mathrm{e}^{-3n\ln(x)}}{(3an+3bn-a-b)(a+b)^2}$	33
risch	$-\frac{x\,x^{-3n}}{(a^3+3a^2b+3a\,b^2+b^3)(-1+3n)}$	37

input
```
int(1/(a*x^n+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-x/(-1+3*n)/(x^n)^3/(a+b)^3
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x}{(a^3+3\,a^2b+3\,ab^2+b^3-3\,(a^3+3\,a^2b+3\,ab^2+b^3)n)x^{3\,n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")
```

output
```
x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x
^(3*n))
```

3.327.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(15) = 30.

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 5.95

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \begin{cases} -\frac{x}{3a^3nx^{3n}-a^3x^{3n}+9a^2bnx^{3n}-3a^2bx^{3n}+9ab^2nx^{3n}-3ab^2x^{3n}+3b^3nx^{3n}-b^3x^{3n}} & \text{for } n\neq\frac{1}{3} \\ \frac{\log(x)}{a^3+3a^2b+3ab^2+b^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x**n+b*x**n)**3,x)
```

output
```
Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3
*a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(
3*n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 +
b**3), True))
```

3.327.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(21) = 42$.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(ax^n + bx^n)^3} \, dx = -\frac{x}{(a^3(3\,n-1) + 3\,a^2b(3\,n-1) + 3\,ab^2(3\,n-1) + b^3(3\,n-1))x^{3\,n}}$$

input
```
integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")
```

output
```
-x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1)
)*x^(3*n))
```

3.327.8 Giac [F]

$$\int \frac{1}{(ax^n + bx^n)^3} \, dx = \int \frac{1}{(ax^n + bx^n)^3} \, dx$$

input
```
integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((a*x^n + b*x^n)^(-3), x)
```

3.327.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^n + bx^n)^3}\, dx = -\frac{x^{1-3\,n}}{(a+b)^3\,(3\,n-1)}$$

input
```
int(1/(a*x^n + b*x^n)^3,x)
```

output
```
-x^(1 - 3*n)/((a + b)^3*(3*n - 1))
```

3.328 $\int (ax + bx^{14})^{12} \, dx$

3.328.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int (ax + bx^{14})^{12} \, dx = \frac{(a + bx^{13})^{13}}{169b}$$

output
```
1/169*(b*x^13+a)^13/b
```

3.328.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int (ax + bx^{14})^{12} \, dx = {} & \frac{a^{12}x^{13}}{13} + \frac{6}{13}a^{11}bx^{26} + \frac{22}{13}a^{10}b^2x^{39} + \frac{55}{13}a^9b^3x^{52} + \frac{99}{13}a^8b^4x^{65} \\ & + \frac{132}{13}a^7b^5x^{78} + \frac{132}{13}a^6b^6x^{91} + \frac{99}{13}a^5b^7x^{104} + \frac{55}{13}a^4b^8x^{117} \\ & + \frac{22}{13}a^3b^9x^{130} + \frac{6}{13}a^2b^{10}x^{143} + \frac{1}{13}ab^{11}x^{156} + \frac{b^{12}x^{169}}{169}\end{aligned}$$

input
```
Integrate[(a*x + b*x^14)^12,x]
```

output
```
(a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*
x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^9
1)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)
/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

3.328.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^{14}\right)^{12} dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^{12}\left(a+bx^{13}\right)^{12} dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

input
```
Int[(a*x + b*x^14)^12,x]
```

output
```
(a + b*x^13)^13/(169*b)
```

3.328.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.328.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.79 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}$
parallelrisch	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}$
gosper	$\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4x}{169}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132a^7b^5x^{78}}{13}+99$

input
```
int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130
+55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^
5*x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*b*a^
11*x^26+1/13*a^12*x^13
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax+bx^{14})^{12}\,dx=\frac{1}{169}b^{12}x^{169}+\frac{1}{13}ab^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^{11}bx^{26}+\frac{1}{13}a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

3.328.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int (ax + bx^{14})^{12}\, dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

input
```
integrate((b*x**14+a*x)**12,x)
```

output
```
a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**
3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**
6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**
9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/16
9
```

3.328.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax + bx^{14})^{12}\, dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^2b^{10}x^{143} + \frac{22}{13}\,a^3b^9x^{130} + \frac{55}{13}\,a^4b^8x^{117} + \frac{99}{13}\,a^5b^7x^{104} + \frac{132}{13}\,a^6b^6x^{91} + \frac{132}{13}\,a^7b^5x^{78} + \frac{99}{13}\,a^8b^4x^{65} + \frac{55}{13}\,a^9b^3x^{52} + \frac{22}{13}\,a^{10}b^2x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

3.328.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{1}{169}b^{12}x^{169} + \frac{1}{13}ab^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91} + \frac{132}{13}a^7b^5x^{78} + \frac{99}{13}a^8b^4x^{65} + \frac{55}{13}a^9b^3x^{52} + \frac{22}{13}a^{10}b^2x^{39} + \frac{6}{13}a^{11}bx^{26} + \frac{1}{13}a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

3.328.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{a^{12}\,x^{13}}{13} + \frac{6\,a^{11}\,b\,x^{26}}{13} + \frac{22\,a^{10}\,b^2\,x^{39}}{13} + \frac{55\,a^9\,b^3\,x^{52}}{13} + \frac{99\,a^8\,b^4\,x^{65}}{13} + \frac{132\,a^7\,b^5\,x^{78}}{13} + \frac{132\,a^6\,b^6\,x^{91}}{13} + \frac{99\,a^5\,b^7\,x^{104}}{13} + \frac{55\,a^4\,b^8\,x^{117}}{13} + \frac{22\,a^3\,b^9\,x^{130}}{13} + \frac{6\,a^2\,b^{10}\,x^{143}}{13} + \frac{a\,b^{11}\,x^{156}}{13} + \frac{b^{12}\,x^{169}}{169}$$

input
```
int((a*x + b*x^14)^12,x)
```

output
```
(a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (
132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55
*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

3.329 $\int x^{12}(ax+bx^{26})^{12}\,dx$

3.329.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

3.329.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=&\ \frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}\\&+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}\\&+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}\end{aligned}$$

input
```
Integrate[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

3.329.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx$$

$$\Big\downarrow \ 9$$

$$\int x^{24}\left(a+bx^{25}\right)^{12}dx$$

$$\Big\downarrow \ 793$$

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

3.329.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.329.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.96 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}$
parallelrisch	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}$
gosper	$\frac{x^{25}(b^{12}x^{300}+13ab^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a^8b}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{ab^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$

input
```
int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
99/25*a^8*b^4*x^125+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x
^225+1/325*b^12*x^325+99/25*a^5*b^7*x^200+132/25*a^7*b^5*x^150+132/25*a^6*
b^6*x^175+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^50+1/25*a*b
^11*x^300+1/25*a^12*x^25
```

3.329.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{12}(ax+bx^{26})^{12}\,dx &= \frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}\\ &\quad+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}\\ &\quad+\frac{99}{25}a^8b^4x^{125}+\frac{11}{5}a^9b^3x^{100}+\frac{22}{25}a^{10}b^2x^{75}+\frac{6}{25}a^{11}bx^{50}+\frac{1}{25}a^{12}x^{25}\end{aligned}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.329.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(10) = 20.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}(ax+bx^{26})^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{22a^3b^9x^{250}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

input
```
integrate(x**12*(b*x**26+a*x)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

3.329.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}(ax+bx^{26})^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.329.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.329.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

input
```
int(x^12*(a*x + b*x^26)^12,x)
```

output
```
(a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 +
(132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 +
(11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.330 $\int x^{24}(ax+bx^{38})^{12}\,dx$

3.330.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

3.330.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

input
```
Integrate[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

3.330.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{24} \left(ax+bx^{38}\right)^{12} \, dx$$

$$\Big\downarrow 9$$

$$\int x^{36} \left(a+bx^{37}\right)^{12} dx$$

$$\Big\downarrow 793$$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

3.330.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.330.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 2.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a^8b}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

input
```
int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+
132/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^
37+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^1
1*x^444+6/37*a^2*b^10*x^407
```

3.330.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.330.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}(ax+bx^{38})^{12}\,dx = \frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**24*(b*x**38+a*x)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

3.330.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}(ax+bx^{38})^{12}\,dx = \frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.330.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}(ax+bx^{38})^{12}\,dx = \frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.330.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}(ax+bx^{38})^{12}\,dx = \frac{a^{12}\,x^{37}}{37} + \frac{6\,a^{11}\,b\,x^{74}}{37} + \frac{22\,a^{10}\,b^2\,x^{111}}{37} + \frac{55\,a^9\,b^3\,x^{148}}{37} + \frac{99\,a^8\,b^4\,x^{185}}{37} + \frac{132\,a^7\,b^5\,x^{222}}{37} + \frac{132\,a^6\,b^6\,x^{259}}{37} + \frac{99\,a^5\,b^7\,x^{296}}{37} + \frac{55\,a^4\,b^8\,x^{333}}{37} + \frac{22\,a^3\,b^9\,x^{370}}{37} + \frac{6\,a^2\,b^{10}\,x^{407}}{37} + \frac{a\,b^{11}\,x^{444}}{37} + \frac{b^{12}\,x^{481}}{481}$$

input
```
int(x^24*(a*x + b*x^38)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.331 $\int x^{12(-1+m)}(ax+bx^{2+12m})^{12}\,dx$

3.331.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

output
```
1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

3.331.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b+156bm}$$

input
```
Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

3.331.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {10, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12(m-1)}\left(ax+bx^{12m+2}\right)^{12}dx$$

$$\Big\downarrow \text{10}$$

$$\int x^{12m}\left(a+bx^{12m+1}\right)^{12}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

input
```
Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

3.331.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.331.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(25) = 50.

Time = 53.75 (sec) , antiderivative size = 338, normalized size of antiderivative = 12.52

method	result
parallelrisch	$\frac{13a^{12}x^{-12+12m}x^{13}+78b\,a^{11}x^{-12+12m}x^{2+12m}x^{12}+286a^{10}b^{2}x^{-12+12m}x^{4+24m}x^{11}+715a^{9}b^{3}x^{-12+12m}x^{6+36m}x^{10}+1287a^{8}b^{4}}{}$
risch	$\frac{b^{12}x^{26+156m}}{13(1+12m)x^{13}}+\frac{a\,b^{11}x^{24+144m}}{(1+12m)x^{12}}+\frac{6a^{2}b^{10}x^{22+132m}}{(1+12m)x^{11}}+\frac{22a^{3}b^{9}x^{20+120m}}{(1+12m)x^{10}}+\frac{55a^{4}b^{8}x^{18+108m}}{(1+12m)x^{9}}+\frac{99a^{5}b^{7}x^{16+96m}}{(1+12m)x^{8}}+ 1$

input
```
int(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x,method=_RETURNVERBOSE)
```

output
```
1/13*(13*a^12*x^(-12+12*m)*x^13+78*b*a^11*x^(-12+12*m)*x^(2+12*m)*x^12+286
*a^10*b^2*x^(-12+12*m)*(x^(2+12*m))^2*x^11+715*a^9*b^3*x^(-12+12*m)*(x^(2+
12*m))^3*x^10+1287*a^8*b^4*x^(-12+12*m)*(x^(2+12*m))^4*x^9+1716*a^7*b^5*x^
(-12+12*m)*(x^(2+12*m))^5*x^8+1716*a^6*b^6*x^(-12+12*m)*(x^(2+12*m))^6*x^7
+1287*a^5*b^7*x^(-12+12*m)*(x^(2+12*m))^7*x^6+715*a^4*b^8*x^(-12+12*m)*(x^
(2+12*m))^8*x^5+286*a^3*b^9*x^(-12+12*m)*(x^(2+12*m))^9*x^4+78*a^2*b^10*x^
(-12+12*m)*(x^(2+12*m))^10*x^3+13*a*b^11*x^(-12+12*m)*(x^(2+12*m))^11*x^2+
b^12*x^(-12+12*m)*(x^(2+12*m))^12*x)/(1+12*m)
```

3.331.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 8.56

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$
$$=\frac{13\,a^{12}x^{12}x^{12\,m+2}+78\,a^{11}bx^{11}x^{24\,m+4}+286\,a^{10}b^{2}x^{10}x^{36\,m+6}+715\,a^{9}b^{3}x^{9}x^{48\,m+8}+1287\,a^{8}b^{4}x^{8}x^{60\,m+10}+1}{}$$

input
```
integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="fricas")
```

output
```
1/13*(13*a^12*x^12*x^(12*m + 2) + 78*a^11*b*x^11*x^(24*m + 4) + 286*a^10*b
^2*x^10*x^(36*m + 6) + 715*a^9*b^3*x^9*x^(48*m + 8) + 1287*a^8*b^4*x^8*x^(
60*m + 10) + 1716*a^7*b^5*x^7*x^(72*m + 12) + 1716*a^6*b^6*x^6*x^(84*m + 1
4) + 1287*a^5*b^7*x^5*x^(96*m + 16) + 715*a^4*b^8*x^4*x^(108*m + 18) + 286
*a^3*b^9*x^3*x^(120*m + 20) + 78*a^2*b^10*x^2*x^(132*m + 22) + 13*a*b^11*x
*x^(144*m + 24) + b^12*x^(156*m + 26))/((12*m + 1)*x^13)
```

3.331.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(19) = 38.

Time = 6.30 (sec) , antiderivative size = 520, normalized size of antiderivative = 19.26

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$
$$=\begin{cases}\frac{13a^{12}x^{13}x^{12m-12}}{156m+13}+\frac{78a^{11}bx^{12}x^{12m-12}x^{12m+2}}{156m+13}+\frac{286a^{10}b^2x^{11}x^{12m-12}x^{24m+4}}{156m+13}+\frac{715a^9b^3x^{10}x^{12m-12}x^{36m+6}}{156m+13}+\frac{1287a^8b^4x^9x^{12m-12}}{156m+13}\\ a^{12}\log(x)+12a^{11}b\log(x)+66a^{10}b^2\log(x)+220a^9b^3\log(x)+495a^8b^4\log(x)+792a^7b^5\log(x)+924\end{cases}$$

input
```
integrate(x**(-12+12*m)*(a*x+b*x**(2+12*m))**12,x)
```

output
```
Piecewise((13*a**12*x**13*x**(12*m - 12)/(156*m + 13) + 78*a**11*b*x**12*x
**(12*m - 12)*x**(12*m + 2)/(156*m + 13) + 286*a**10*b**2*x**11*x**(12*m -
 12)*x**(24*m + 4)/(156*m + 13) + 715*a**9*b**3*x**10*x**(12*m - 12)*x**(3
6*m + 6)/(156*m + 13) + 1287*a**8*b**4*x**9*x**(12*m - 12)*x**(48*m + 8)/(
156*m + 13) + 1716*a**7*b**5*x**8*x**(12*m - 12)*x**(60*m + 10)/(156*m + 1
3) + 1716*a**6*b**6*x**7*x**(12*m - 12)*x**(72*m + 12)/(156*m + 13) + 1287
*a**5*b**7*x**6*x**(12*m - 12)*x**(84*m + 14)/(156*m + 13) + 715*a**4*b**8
*x**5*x**(12*m - 12)*x**(96*m + 16)/(156*m + 13) + 286*a**3*b**9*x**4*x**(
12*m - 12)*x**(108*m + 18)/(156*m + 13) + 78*a**2*b**10*x**3*x**(12*m - 12
)*x**(120*m + 20)/(156*m + 13) + 13*a*b**11*x**2*x**(12*m - 12)*x**(132*m
+ 22)/(156*m + 13) + b**12*x*x**(12*m - 12)*x**(144*m + 24)/(156*m + 13),
Ne(m, -1/12)), (a**12*log(x) + 12*a**11*b*log(x) + 66*a**10*b**2*log(x) +
220*a**9*b**3*log(x) + 495*a**8*b**4*log(x) + 792*a**7*b**5*log(x) + 924*a
**6*b**6*log(x) + 792*a**5*b**7*log(x) + 495*a**4*b**8*log(x) + 220*a**3*b
**9*log(x) + 66*a**2*b**10*log(x) + 12*a*b**11*log(x) + b**12*log(x), True
))
```

3.331.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(25) = 50$.

Time = 0.23 (sec) , antiderivative size = 275, normalized size of antiderivative = 10.19

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^2b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^3b^9x^{120\,m+10}}{12\,m+1}+\frac{55\,a^4b^8x^{108\,m+9}}{12\,m+1}+\frac{99\,a^5b^7x^{96\,m+8}}{12\,m+1}+\frac{132\,a^6b^6x^{84\,m+7}}{12\,m+1}+\frac{132\,a^7b^5x^{72\,m+6}}{12\,m+1}+\frac{99\,a^8b^4x^{60\,m+5}}{12\,m+1}+\frac{55\,a^9b^3x^{48\,m+4}}{12\,m+1}+\frac{22\,a^{10}b^2x^{36\,m+3}}{12\,m+1}+\frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1}+\frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

input
```
integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="maxima")
```

output
```
1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6
*a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1)
 + 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1
) + 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m +
 1) + 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m +
 1) + 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m +
1) + a^12*x^(12*m + 1)/(12*m + 1)
```

3.331.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(25) = 50$.

Time = 0.40 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$
$$=\frac{13\,a^{12}x^{12}e^{(12\,m\log(x)+2\,\log(x))}+78\,a^{11}bx^{11}e^{(24\,m\log(x)+4\,\log(x))}+286\,a^{10}b^2x^{10}e^{(36\,m\log(x)+6\,\log(x))}+715\,a^9b^3x^9e}{}$$

input
```
integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

output
```
1/13*(13*a^12*x^12*e^(12*m*log(x) + 2*log(x)) + 78*a^11*b*x^11*e^(24*m*log
(x) + 4*log(x)) + 286*a^10*b^2*x^10*e^(36*m*log(x) + 6*log(x)) + 715*a^9*b
^3*x^9*e^(48*m*log(x) + 8*log(x)) + 1287*a^8*b^4*x^8*e^(60*m*log(x) + 10*l
og(x)) + 1716*a^7*b^5*x^7*e^(72*m*log(x) + 12*log(x)) + 1716*a^6*b^6*x^6*e
^(84*m*log(x) + 14*log(x)) + 1287*a^5*b^7*x^5*e^(96*m*log(x) + 16*log(x))
+ 715*a^4*b^8*x^4*e^(108*m*log(x) + 18*log(x)) + 286*a^3*b^9*x^3*e^(120*m*
log(x) + 20*log(x)) + 78*a^2*b^10*x^2*e^(132*m*log(x) + 22*log(x)) + 13*a*
b^11*x*e^(144*m*log(x) + 24*log(x)) + b^12*e^(156*m*log(x) + 26*log(x)))/(
12*m*x^13 + x^13)
```

3.331.9 Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 287, normalized size of antiderivative = 10.63

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{13\,a^{12}\,x\,x^{12\,m}}{156\,m+13}+\frac{78\,a^{11}\,b\,x^{24\,m}\,x^2}{156\,m+13}+\frac{13\,a\,b^{11}\,x^{144\,m}\,x^{12}}{156\,m+13}+\frac{286\,a^{10}\,b^2\,x^{36\,m}\,x^3}{156\,m+13}+\frac{715\,a^9\,b^3\,x^{48\,m}\,x^4}{156\,m+13}+\frac{1287\,a^8\,b^4\,x^{60\,m}\,x^5}{156\,m+13}+\frac{1716\,a^7\,b^5\,x^{72\,m}\,x^6}{156\,m+13}+\frac{1716\,a^6\,b^6\,x^{84\,m}\,x^7}{156\,m+13}+\frac{1287\,a^5\,b^7\,x^{96\,m}\,x^8}{156\,m+13}+\frac{715\,a^4\,b^8\,x^{108\,m}\,x^9}{156\,m+13}+\frac{286\,a^3\,b^9\,x^{120\,m}\,x^{10}}{156\,m+13}+\frac{78\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{156\,m+13}$$

input
```
int(x^(12*m - 12)*(a*x + b*x^(12*m + 2))^12,x)
```

output
```
(b^12*x^(156*m)*x^13)/(156*m + 13) + (13*a^12*x*x^(12*m))/(156*m + 13) + (
78*a^11*b*x^(24*m)*x^2)/(156*m + 13) + (13*a*b^11*x^(144*m)*x^12)/(156*m +
 13) + (286*a^10*b^2*x^(36*m)*x^3)/(156*m + 13) + (715*a^9*b^3*x^(48*m)*x^
4)/(156*m + 13) + (1287*a^8*b^4*x^(60*m)*x^5)/(156*m + 13) + (1716*a^7*b^5
*x^(72*m)*x^6)/(156*m + 13) + (1716*a^6*b^6*x^(84*m)*x^7)/(156*m + 13) + (
1287*a^5*b^7*x^(96*m)*x^8)/(156*m + 13) + (715*a^4*b^8*x^(108*m)*x^9)/(156
*m + 13) + (286*a^3*b^9*x^(120*m)*x^10)/(156*m + 13) + (78*a^2*b^10*x^(132
*m)*x^11)/(156*m + 13)
```

3.332 $\int (ax + bx^{14})^{12} \, dx$

3.332.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int (ax + bx^{14})^{12} \, dx = \frac{(a + bx^{13})^{13}}{169b}$$

output
```
1/169*(b*x^13+a)^13/b
```

3.332.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int (ax + bx^{14})^{12} \, dx = \frac{a^{12}x^{13}}{13} + \frac{6}{13}a^{11}bx^{26} + \frac{22}{13}a^{10}b^2x^{39} + \frac{55}{13}a^9b^3x^{52} + \frac{99}{13}a^8b^4x^{65} + \frac{132}{13}a^7b^5x^{78} + \frac{132}{13}a^6b^6x^{91} + \frac{99}{13}a^5b^7x^{104} + \frac{55}{13}a^4b^8x^{117} + \frac{22}{13}a^3b^9x^{130} + \frac{6}{13}a^2b^{10}x^{143} + \frac{1}{13}ab^{11}x^{156} + \frac{b^{12}x^{169}}{169}$$

input
```
Integrate[(a*x + b*x^14)^12,x]
```

output
```
(a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*
x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^9
1)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)
/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

3.332.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^{14}\right)^{12} dx$$
$$\Big\downarrow \text{2027}$$
$$\int x^{12}\left(a+bx^{13}\right)^{12} dx$$
$$\Big\downarrow \text{793}$$
$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

input
```
Int[(a*x + b*x^14)^12,x]
```

output
```
(a + b*x^13)^13/(169*b)
```

3.332.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.332.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.82 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}$
parallelrisch	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}$
gosper	$\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4x}{169}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132a^7b^5x^{78}}{13}+99$

input
```
int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130
+55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^
5*x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*b*a^
11*x^26+1/13*a^12*x^13
```

3.332.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int (ax+bx^{14})^{12}\,dx = {} & \frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130} \\ & +\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78} \\ & +\frac{99}{13}\,a^8b^4x^{65}+\frac{55}{13}\,a^9b^3x^{52}+\frac{22}{13}\,a^{10}b^2x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}\end{aligned}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

3.332.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

input
```
integrate((b*x**14+a*x)**12,x)
```

output
```
a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**
3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**
6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**
9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/16
9
```

3.332.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12} dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^2b^{10}x^{143} + \frac{22}{13}\,a^3b^9x^{130} + \frac{55}{13}\,a^4b^8x^{117} + \frac{99}{13}\,a^5b^7x^{104} + \frac{132}{13}\,a^6b^6x^{91} + \frac{132}{13}\,a^7b^5x^{78} + \frac{99}{13}\,a^8b^4x^{65} + \frac{55}{13}\,a^9b^3x^{52} + \frac{22}{13}\,a^{10}b^2x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

3.332.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12}\,dx=\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130}+\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78}+\frac{99}{13}\,a^8b^4x^{65}+\frac{55}{13}\,a^9b^3x^{52}+\frac{22}{13}\,a^{10}b^2x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

3.332.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12}\,dx=\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a^4\,b^8\,x^{117}}{13}+\frac{22\,a^3\,b^9\,x^{130}}{13}+\frac{6\,a^2\,b^{10}\,x^{143}}{13}+\frac{a\,b^{11}\,x^{156}}{13}+\frac{b^{12}\,x^{169}}{169}$$

input
```
int((a*x + b*x^14)^12,x)
```

output
```
(a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (
132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55
*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

3.333 $\int (ax^2 + bx^{27})^{12}\, dx$

3.333.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \left(ax^2 + bx^{27}\right)^{12}\, dx = \frac{(a + bx^{25})^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

3.333.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int \left(ax^2 + bx^{27}\right)^{12}\, dx = {} & \frac{a^{12}x^{25}}{25} + \frac{6}{25}a^{11}bx^{50} + \frac{22}{25}a^{10}b^2x^{75} + \frac{11}{5}a^9b^3x^{100} + \frac{99}{25}a^8b^4x^{125} \\ & + \frac{132}{25}a^7b^5x^{150} + \frac{132}{25}a^6b^6x^{175} + \frac{99}{25}a^5b^7x^{200} + \frac{11}{5}a^4b^8x^{225} \\ & + \frac{22}{25}a^3b^9x^{250} + \frac{6}{25}a^2b^{10}x^{275} + \frac{1}{25}ab^{11}x^{300} + \frac{b^{12}x^{325}}{325}\end{aligned}$$

input
```
Integrate[(a*x^2 + b*x^27)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

3.333.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^{27}\right)^{12} dx$$

$$\Big\downarrow \; 2027$$

$$\int x^{24}\left(a+bx^{25}\right)^{12} dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[(a*x^2 + b*x^27)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

3.333.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.333.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.99 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}$
parallelrisch	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}$
gosper	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a^8b}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$

input
```
int((b*x^27+a*x^2)^12,x,method=_RETURNVERBOSE)
```

output
```
99/25*a^8*b^4*x^125+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x
^225+1/325*b^12*x^325+99/25*a^5*b^7*x^200+132/25*a^7*b^5*x^150+132/25*a^6*
b^6*x^175+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^50+1/25*a*b
^11*x^300+1/25*a^12*x^25
```

3.333.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax^2+bx^{27})^{12}\,dx = \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250} + \frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150} + \frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.333.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int \left(ax^2+bx^{27}\right)^{12} dx = \frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{22a^3b^9x^{250}}{25} + \frac{6a^2b^{10}x^{275}}{25} + \frac{ab^{11}x^{300}}{25} + \frac{b^{12}x^{325}}{325}$$

input
```
integrate((b*x**27+a*x**2)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

3.333.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^2+bx^{27}\right)^{12} dx = \frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$$

input
```
integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.333.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax^2 + bx^{27})^{12} dx = \frac{1}{325} b^{12}x^{325} + \frac{1}{25} ab^{11}x^{300} + \frac{6}{25} a^2b^{10}x^{275} + \frac{22}{25} a^3b^9x^{250} + \frac{11}{5} a^4b^8x^{225} + \frac{99}{25} a^5b^7x^{200} + \frac{132}{25} a^6b^6x^{175} + \frac{132}{25} a^7b^5x^{150} + \frac{99}{25} a^8b^4x^{125} + \frac{11}{5} a^9b^3x^{100} + \frac{22}{25} a^{10}b^2x^{75} + \frac{6}{25} a^{11}bx^{50} + \frac{1}{25} a^{12}x^{25}$$

input
```
integrate((b*x^27+a*x^2)^12,x, algorithm="giac")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.333.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax^2 + bx^{27})^{12} dx = \frac{a^{12} x^{25}}{25} + \frac{6 a^{11} b x^{50}}{25} + \frac{22 a^{10} b^2 x^{75}}{25} + \frac{11 a^9 b^3 x^{100}}{5} + \frac{99 a^8 b^4 x^{125}}{25} + \frac{132 a^7 b^5 x^{150}}{25} + \frac{132 a^6 b^6 x^{175}}{25} + \frac{99 a^5 b^7 x^{200}}{25} + \frac{11 a^4 b^8 x^{225}}{5} + \frac{22 a^3 b^9 x^{250}}{25} + \frac{6 a^2 b^{10} x^{275}}{25} + \frac{a b^{11} x^{300}}{25} + \frac{b^{12} x^{325}}{325}$$

input
```
int((a*x^2 + b*x^27)^12,x)
```

output
```
(a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 +
(132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 +
(11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.334 $\int (ax^3 + bx^{40})^{12} \, dx$

3.334.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (ax^3 + bx^{40})^{12} \, dx = \frac{(a + bx^{37})^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

3.334.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int (ax^3 + bx^{40})^{12} \, dx = \frac{a^{12}x^{37}}{37} + \frac{6}{37}a^{11}bx^{74} + \frac{22}{37}a^{10}b^2x^{111} + \frac{55}{37}a^9b^3x^{148} + \frac{99}{37}a^8b^4x^{185} + \frac{132}{37}a^7b^5x^{222} + \frac{132}{37}a^6b^6x^{259} + \frac{99}{37}a^5b^7x^{296} + \frac{55}{37}a^4b^8x^{333} + \frac{22}{37}a^3b^9x^{370} + \frac{6}{37}a^2b^{10}x^{407} + \frac{1}{37}ab^{11}x^{444} + \frac{b^{12}x^{481}}{481}$$

input
```
Integrate[(a*x^3 + b*x^40)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

3.334.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^{40}\right)^{12} \, dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{36}\left(a+bx^{37}\right)^{12}dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[(a*x^3 + b*x^40)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

3.334.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.334.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 2.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{22}{37}a^{3}b^{9}x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{22}{37}a^{3}b^{9}x^{370}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^{2}b^{10}x^{370}+286a^{3}b^{9}x^{333}+715a^{4}b^{8}x^{296}+1287a^{5}b^{7}x^{259}+1716a^{6}b^{6}x^{222}+1716a^{7}b^{5}x^{185}+1287a^{8}b}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+$

input
```
int((b*x^40+a*x^3)^12,x,method=_RETURNVERBOSE)
```

output
```
6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+
132/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^
37+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^1
1*x^444+6/37*a^2*b^10*x^407
```

3.334.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax^3+bx^{40})^{12}\,dx = \frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

input
```
integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.334.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int (ax^3 + bx^{40})^{12}\,dx = \frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

input
```
integrate((b*x**40+a*x**3)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

3.334.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int (ax^3 + bx^{40})^{12}\,dx = \frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

input
```
integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.334.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^3+bx^{40}\right)^{12} dx = \frac{1}{481} b^{12}x^{481} + \frac{1}{37} ab^{11}x^{444} + \frac{6}{37} a^2b^{10}x^{407} + \frac{22}{37} a^3b^9x^{370} + \frac{55}{37} a^4b^8x^{333} + \frac{99}{37} a^5b^7x^{296} + \frac{132}{37} a^6b^6x^{259} + \frac{132}{37} a^7b^5x^{222} + \frac{99}{37} a^8b^4x^{185} + \frac{55}{37} a^9b^3x^{148} + \frac{22}{37} a^{10}b^2x^{111} + \frac{6}{37} a^{11}bx^{74} + \frac{1}{37} a^{12}x^{37}$$

input
```
integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.334.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^3+bx^{40}\right)^{12} dx = \frac{a^{12}\,x^{37}}{37} + \frac{6\,a^{11}\,b\,x^{74}}{37} + \frac{22\,a^{10}\,b^2\,x^{111}}{37} + \frac{55\,a^9\,b^3\,x^{148}}{37} + \frac{99\,a^8\,b^4\,x^{185}}{37} + \frac{132\,a^7\,b^5\,x^{222}}{37} + \frac{132\,a^6\,b^6\,x^{259}}{37} + \frac{99\,a^5\,b^7\,x^{296}}{37} + \frac{55\,a^4\,b^8\,x^{333}}{37} + \frac{22\,a^3\,b^9\,x^{370}}{37} + \frac{6\,a^2\,b^{10}\,x^{407}}{37} + \frac{a\,b^{11}\,x^{444}}{37} + \frac{b^{12}\,x^{481}}{481}$$

input
```
int((a*x^3 + b*x^40)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.335 $\int (ax^m + bx^{1+13m})^{12} \, dx$

3.335.1 Optimal result

Integrand size = 17, antiderivative size = 27

$$\int (ax^m + bx^{1+13m})^{12} \, dx = \frac{(a + bx^{1+12m})^{13}}{13b(1 + 12m)}$$

output
```
1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

3.335.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 7.15

$$\int (ax^m + bx^{1+13m})^{12} \, dx = \frac{x^{1+12m}(13a^{12} + 78a^{11}bx^{1+12m} + 286a^{10}b^2x^{2+24m} + 715a^9b^3x^{3+36m} + 1287a^8b^4x^{4+48m} + 1716a^7b^5x^{5+60m} + \cdots}{}$$

input
```
Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]
```

output
```
(x^(1 + 12*m)*(13*a^12 + 78*a^11*b*x^(1 + 12*m) + 286*a^10*b^2*x^(2 + 24*m
) + 715*a^9*b^3*x^(3 + 36*m) + 1287*a^8*b^4*x^(4 + 48*m) + 1716*a^7*b^5*x^
(5 + 60*m) + 1716*a^6*b^6*x^(6 + 72*m) + 1287*a^5*b^7*x^(7 + 84*m) + 715*a
^4*b^8*x^(8 + 96*m) + 286*a^3*b^9*x^(9 + 108*m) + 78*a^2*b^10*x^(10 + 120*
m) + 13*a*b^11*x^(11 + 132*m) + b^12*x^(12 + 144*m)))/(13 + 156*m)
```

3.335.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^m+bx^{13m+1}\right)^{12}dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^{12m}\left(a+bx^{12m+1}\right)^{12}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

input
```
Int[(a*x^m + b*x^(1 + 13*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

3.335.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.335.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(25) = 50$.

Time = 46.02 (sec) , antiderivative size = 281, normalized size of antiderivative = 10.41

method	result
parallelrisch	$13a^{12}x^{12m}x+78b\,a^{11}x^{1+13m}x^{11m}x+286a^{10}b^2x^{2+26m}x^{10m}x+715a^9b^3x^{3+39m}x^{9m}x+1287a^8b^4x^{4+52m}x^{8m}x+1716a^7b^5x^{5+6}$
risch	$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{1+12m}+\frac{6a^2b^{10}x^{11}x^{132m}}{1+12m}+\frac{22a^3b^9x^{10}x^{120m}}{1+12m}+\frac{55a^4b^8x^9x^{108m}}{1+12m}+\frac{99a^5b^7x^8x^{96m}}{1+12m}+\frac{132a^6}{1\text{-}}$

input
```
int((x^m*a+b*x^(1+13*m))^12,x,method=_RETURNVERBOSE)
```

output
```
1/13*(13*a^12*(x^m)^12*x+78*b*a^11*x^(1+13*m)*(x^m)^11*x+286*a^10*b^2*(x^(
1+13*m))^2*(x^m)^10*x+715*a^9*b^3*(x^(1+13*m))^3*(x^m)^9*x+1287*a^8*b^4*(x
^(1+13*m))^4*(x^m)^8*x+1716*a^7*b^5*(x^(1+13*m))^5*(x^m)^7*x+1716*a^6*b^6*
(x^(1+13*m))^6*(x^m)^6*x+1287*a^5*b^7*(x^(1+13*m))^7*(x^m)^5*x+715*a^4*b^8
*(x^(1+13*m))^8*(x^m)^4*x+286*a^3*b^9*(x^(1+13*m))^9*(x^m)^3*x+78*a^2*b^10
*(x^(1+13*m))^10*(x^m)^2*x+13*a*b^11*(x^(1+13*m))^11*x^m*x+b^12*(x^(1+13*m
))^12*x)/(1+12*m)
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 7.59

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx$$
$$= \frac{b^{12}x^{13}x^{156\,m} + 13\,ab^{11}x^{12}x^{144\,m} + 78\,a^2b^{10}x^{11}x^{132\,m} + 286\,a^3b^9x^{10}x^{120\,m} + 715\,a^4b^8x^9x^{108\,m} + 1287\,a^5b^7x^8x}{}$$

input
```
integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")
```

output
```
1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^
(132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^
5*b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m)
 + 1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3
*x^(36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

3.335.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(19) = 38.

Time = 5.03 (sec) , antiderivative size = 471, normalized size of antiderivative = 17.44

$$\int \left(ax^m + bx^{1+13m}\right)^{12} \, dx$$
$$= \begin{cases} \frac{13a^{12}xx^{12m}}{156m+13} + \frac{78a^{11}bxx^{11m}x^{13m+1}}{156m+13} + \frac{286a^{10}b^2xx^{10m}x^{26m+2}}{156m+13} + \frac{715a^9b^3xx^{9m}x^{39m+3}}{156m+13} + \frac{1287a^8b^4xx^{8m}x^{52m+4}}{156m+13} + \frac{1716a^7b^5xx^{7m}x^{6}}{156m+13} \\ a^{12}\log(x) + 12a^{11}b\log(x) + 66a^{10}b^2\log(x) + 220a^9b^3\log(x) + 495a^8b^4\log(x) + 792a^7b^5\log(x) + 924 \end{cases}$$

input
```
integrate((a*x**m+b*x**(1+13*m))**12,x)
```

output
```
Piecewise((13*a**12*x*x**(12*m)/(156*m + 13) + 78*a**11*b*x*x**(11*m)*x**(
13*m + 1)/(156*m + 13) + 286*a**10*b**2*x*x**(10*m)*x**(26*m + 2)/(156*m +
 13) + 715*a**9*b**3*x*x**(9*m)*x**(39*m + 3)/(156*m + 13) + 1287*a**8*b**
4*x*x**(8*m)*x**(52*m + 4)/(156*m + 13) + 1716*a**7*b**5*x*x**(7*m)*x**(65
*m + 5)/(156*m + 13) + 1716*a**6*b**6*x*x**(6*m)*x**(78*m + 6)/(156*m + 13
) + 1287*a**5*b**7*x*x**(5*m)*x**(91*m + 7)/(156*m + 13) + 715*a**4*b**8*x
*x**(4*m)*x**(104*m + 8)/(156*m + 13) + 286*a**3*b**9*x*x**(3*m)*x**(117*m
 + 9)/(156*m + 13) + 78*a**2*b**10*x*x**(2*m)*x**(130*m + 10)/(156*m + 13)
 + 13*a*b**11*x*x**m*x**(143*m + 11)/(156*m + 13) + b**12*x*x**(156*m + 12
)/(156*m + 13), Ne(m, -1/12)), (a**12*log(x) + 12*a**11*b*log(x) + 66*a**1
0*b**2*log(x) + 220*a**9*b**3*log(x) + 495*a**8*b**4*log(x) + 792*a**7*b**
5*log(x) + 924*a**6*b**6*log(x) + 792*a**5*b**7*log(x) + 495*a**4*b**8*log
(x) + 220*a**3*b**9*log(x) + 66*a**2*b**10*log(x) + 12*a*b**11*log(x) + b*
*12*log(x), True))
```

3.335.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 10.19

$$\int \left(ax^m + bx^{1+13m}\right)^{12} \, dx = \frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)} + \frac{ab^{11}x^{144\,m+12}}{12\,m+1} + \frac{6\,a^2b^{10}x^{132\,m+11}}{12\,m+1} + \frac{22\,a^3b^9x^{120\,m+10}}{12\,m+1} + \frac{55\,a^4b^8x^{108\,m+9}}{12\,m+1} + \frac{99\,a^5b^7x^{96\,m+8}}{12\,m+1} + \frac{132\,a^6b^6x^{84\,m+7}}{12\,m+1} + \frac{132\,a^7b^5x^{72\,m+6}}{12\,m+1} + \frac{99\,a^8b^4x^{60\,m+5}}{12\,m+1} + \frac{55\,a^9b^3x^{48\,m+4}}{12\,m+1} + \frac{22\,a^{10}b^2x^{36\,m+3}}{12\,m+1} + \frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1} + \frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

input
```
integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")
```

output
```
1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6
*a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1)
 + 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1
) + 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m +
 1) + 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m +
 1) + 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m +
1) + a^12*x^(12*m + 1)/(12*m + 1)
```

3.335.8 Giac [F(-1)]

Timed out.

$$\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx=\text{Timed out}$$

input
```
integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")
```

output
```
Timed out
```

3.335.9 Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx=\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1}+\frac{132\,a^7\,b^5\,x^{72\,m}\,x^6}{12\,m+1}+\frac{132\,a^6\,b^6\,x^{84\,m}\,x^7}{12\,m+1}+\frac{99\,a^5\,b^7\,x^{96\,m}\,x^8}{12\,m+1}+\frac{55\,a^4\,b^8\,x^{108\,m}\,x^9}{12\,m+1}+\frac{22\,a^3\,b^9\,x^{120\,m}\,x^{10}}{12\,m+1}+\frac{6\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{12\,m+1}$$

input
```
int((a*x^m + b*x^(13*m + 1))^12,x)
```

output
```
(b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^1
1*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^
10*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (
99*a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1
) + (132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*
m + 1) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^1
0)/(12*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

3.335. $\int \left(ax^m + bx^{1+13m}\right)^{12} \, dx$

3.336 $\int (ax^m + bx^{1+6m})^5 \, dx$

3.336.1 Optimal result

Integrand size = 17, antiderivative size = 27

$$\int (ax^m + bx^{1+6m})^5 \, dx = \frac{(a + bx^{1+5m})^6}{6b(1 + 5m)}$$

output
```
1/6*(a+b*x^(1+5*m))^6/b/(1+5*m)
```

3.336.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(27) = 54.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int (ax^m + bx^{1+6m})^5 \, dx = \frac{x^{1+5m}(6a^5 + 15a^4bx^{1+5m} + 20a^3b^2x^{2+10m} + 15a^2b^3x^{3+15m} + 6ab^4x^{4+20m} + b^5x^{5+25m})}{6 + 30m}$$

input
```
Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]
```

output
```
(x^(1 + 5*m)*(6*a^5 + 15*a^4*b*x^(1 + 5*m) + 20*a^3*b^2*x^(2 + 10*m) + 15*
a^2*b^3*x^(3 + 15*m) + 6*a*b^4*x^(4 + 20*m) + b^5*x^(5 + 25*m)))/(6 + 30*m
)
```

3.336.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^m+bx^{6m+1}\right)^5 dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^{5m}\left(a+bx^{5m+1}\right)^5 dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

input
```
Int[(a*x^m + b*x^(1 + 6*m))^5,x]
```

output
```
(a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

3.336.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.336.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(25) = 50.

Time = 2.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.44

method	result	size
parallelrisch	$\frac{6x\,x^{5m}a^{5}+15x\,x^{4m}x^{1+6m}a^{4}b+20x\,x^{3m}x^{2+12m}a^{3}b^{2}+15x\,x^{2m}x^{3+18m}a^{2}b^{3}+6x\,x^{m}x^{4+24m}a\,b^{4}+x\,x^{5+30m}b^{5}}{6+30m}$	120
risch	$\frac{b^{5}x^{6}x^{30m}}{6+30m}+\frac{a\,b^{4}x^{5}x^{25m}}{1+5m}+\frac{5a^{2}b^{3}x^{4}x^{20m}}{2(1+5m)}+\frac{10a^{3}b^{2}x^{3}x^{15m}}{3(1+5m)}+\frac{5a^{4}b\,x^{2}x^{10m}}{2(1+5m)}+\frac{a^{5}x\,x^{5m}}{1+5m}$	126

input
```
int((x^m*a+b*x^(1+6*m))^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*x*(x^m)^5*a^5+15*x*(x^m)^4*x^(1+6*m)*a^4*b+20*x*(x^m)^3*(x^(1+6*m))
^2*a^3*b^2+15*x*(x^m)^2*(x^(1+6*m))^3*a^2*b^3+6*x*x^m*(x^(1+6*m))^4*a*b^4+
x*(x^(1+6*m))^5*b^5)/(1+5*m)
```

3.336.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \left(ax^{m}+bx^{1+6m}\right)^{5}\,dx$$
$$=\frac{b^{5}x^{6}x^{30\,m}+6\,ab^{4}x^{5}x^{25\,m}+15\,a^{2}b^{3}x^{4}x^{20\,m}+20\,a^{3}b^{2}x^{3}x^{15\,m}+15\,a^{4}bx^{2}x^{10\,m}+6\,a^{5}xx^{5\,m}}{6\,(5\,m+1)}$$

input
```
integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")
```

output
```
1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 2
0*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1
)
```

3.336.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(19) = 38$.

Time = 0.63 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.30

$$\int \left(ax^m + bx^{1+6m}\right)^5 \, dx$$
$$= \begin{cases} \frac{6a^5xx^{5m}}{30m+6} + \frac{15a^4bxx^{4m}x^{6m+1}}{30m+6} + \frac{20a^3b^2xx^{3m}x^{12m+2}}{30m+6} + \frac{15a^2b^3xx^{2m}x^{18m+3}}{30m+6} + \frac{6ab^4xx^mx^{24m+4}}{30m+6} + \frac{b^5xx^{30m+5}}{30m+6} & \text{for } m \neq -\frac{1}{5} \\ a^5\log(x) + 5a^4b\log(x) + 10a^3b^2\log(x) + 10a^2b^3\log(x) + 5ab^4\log(x) + b^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a*x**m+b*x**(1+6*m))**5,x)
```

output
```
Piecewise((6*a**5*x*x**(5*m)/(30*m + 6) + 15*a**4*b*x*x**(4*m)*x**(6*m + 1
)/(30*m + 6) + 20*a**3*b**2*x*x**(3*m)*x**(12*m + 2)/(30*m + 6) + 15*a**2*
b**3*x*x**(2*m)*x**(18*m + 3)/(30*m + 6) + 6*a*b**4*x*x**m*x**(24*m + 4)/(
30*m + 6) + b**5*x*x**(30*m + 5)/(30*m + 6), Ne(m, -1/5)), (a**5*log(x) +
5*a**4*b*log(x) + 10*a**3*b**2*log(x) + 10*a**2*b**3*log(x) + 5*a*b**4*log
(x) + b**5*log(x), True))
```

3.336.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.48

$$\int \left(ax^m + bx^{1+6m}\right)^5 \, dx = \frac{b^5x^{30\,m+6}}{6\,(5\,m+1)} + \frac{ab^4x^{25\,m+5}}{5\,m+1} + \frac{5\,a^2b^3x^{20\,m+4}}{2\,(5\,m+1)} + \frac{10\,a^3b^2x^{15\,m+3}}{3\,(5\,m+1)} + \frac{5\,a^4bx^{10\,m+2}}{2\,(5\,m+1)} + \frac{a^5x^{5\,m+1}}{5\,m+1}$$

input
```
integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")
```

output
```
1/6*b^5*x^(30*m + 6)/(5*m + 1) + a*b^4*x^(25*m + 5)/(5*m + 1) + 5/2*a^2*b^
3*x^(20*m + 4)/(5*m + 1) + 10/3*a^3*b^2*x^(15*m + 3)/(5*m + 1) + 5/2*a^4*b
*x^(10*m + 2)/(5*m + 1) + a^5*x^(5*m + 1)/(5*m + 1)
```

3.336.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int (ax^m + bx^{1+6m})^5 \, dx = \frac{b^5x^6x^{30\,m} + 6\,ab^4x^5x^{25\,m} + 15\,a^2b^3x^4x^{20\,m} + 20\,a^3b^2x^3x^{15\,m} + 15\,a^4bx^2x^{10\,m} + 6\,a^5xx^{5\,m}}{6\,(5\,m + 1)}$$

input
```
integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")
```

output
```
1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 2
0*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1
)
```

3.336.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.59

$$\int (ax^m + bx^{1+6m})^5 \, dx = \frac{b^5\,x^{30\,m}\,x^6}{30\,m+6} + \frac{a^5\,x\,x^{5\,m}}{5\,m+1} + \frac{5\,a^4\,b\,x^{10\,m}\,x^2}{10\,m+2} + \frac{a\,b^4\,x^{25\,m}\,x^5}{5\,m+1} + \frac{5\,a^2\,b^3\,x^{20\,m}\,x^4}{10\,m+2} + \frac{10\,a^3\,b^2\,x^{15\,m}\,x^3}{15\,m+3}$$

input
```
int((a*x^m + b*x^(6*m + 1))^5,x)
```

output
```
(b^5*x^(30*m)*x^6)/(30*m + 6) + (a^5*x*x^(5*m))/(5*m + 1) + (5*a^4*b*x^(10
*m)*x^2)/(10*m + 2) + (a*b^4*x^(25*m)*x^5)/(5*m + 1) + (5*a^2*b^3*x^(20*m)
*x^4)/(10*m + 2) + (10*a^3*b^2*x^(15*m)*x^3)/(15*m + 3)
```

3.337 $\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx$

3.337.1 Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx=-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

output
```
-1/2/b/(1-3*m)/(a+b*x^(1-3*m))^2
```

3.337.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx=-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

input
```
Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

output
```
-1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

3.337.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2027, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^m+bx^{1-2m})^3}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x^{-3m}}{(a+bx^{1-3m})^3}dx$$
$$\Big\downarrow \text{793}$$
$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

input
```
Int[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

output
```
-1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

3.337.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.337.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{x\left(2a\,x^{3m}+bx\right)}{2(3m-1)a^{2}\left(a\,x^{3m}+bx\right)^{2}}$	39

input
```
int(1/(b*x^(1-2*m)+x^m*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(2*a*(x^m)^3+b*x)/(3*m-1)/a^2/(a*(x^m)^3+b*x)^2
```

3.337.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx = -\frac{2\,axx^{3\,m}+bx^2}{2\,(2\,(3\,a^3bm-a^3b)xx^{3\,m}+(3\,a^2b^2m-a^2b^2)x^2+(3\,a^4m-a^4)x^{6\,m})}$$

input
```
integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2
*m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))
```

3.337.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.337.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = -\frac{2\,axx^{3\,m}+bx^2}{2\,(2\,a^3b(3\,m-1)xx^{3\,m}+a^2b^2(3\,m-1)x^2+a^4(3\,m-1)x^{6\,m})}$$

input
```
integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m -
 1)*x^2 + a^4*(3*m - 1)*x^(6*m))
```

3.337.8 Giac [F]

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = \int \frac{1}{(ax^m+bx^{-2\,m+1})^3}\,dx$$

input
```
integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")
```

output
```
integrate((a*x^m + b*x^(-2*m + 1))^(-3), x)
```

3.337.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = -\frac{x\,(b\,x+2\,a\,x^{3\,m})}{2\,a^2\,(3\,m-1)\,(b\,x+a\,x^{3\,m})^2}$$

input
```
int(1/(a*x^m + b*x^(1 - 2*m))^3,x)
```

output
```
-(x*(b*x + 2*a*x^(3*m)))/(2*a^2*(3*m - 1)*(b*x + a*x^(3*m))^2)
```

3.338 $\int \frac{1}{\frac{b}{x}+ax}\,dx$

3.338.1 Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(b+ax^2\right)}{2a}$$

output
```
1/2*ln(a*x^2+b)/a
```

3.338.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(b+ax^2\right)}{2a}$$

input
```
Integrate[(b/x + a*x)^(-1),x]
```

output
```
Log[b + a*x^2]/(2*a)
```

3.338.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{b}{x}}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x}{ax^2+b}dx$$
$$\Big\downarrow \text{ 240}$$
$$\frac{\log(ax^2+b)}{2a}$$

input
```
Int[(b/x + a*x)^(-1),x]
```

output
```
Log[b + a*x^2]/(2*a)
```

3.338.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.338.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^2+b)}{2a}$	14
norman	$\frac{\ln(a\,x^2+b)}{2a}$	14
risch	$\frac{\ln(a\,x^2+b)}{2a}$	14
parallelrisch	$\frac{\ln(a\,x^2+b)}{2a}$	14

input
```
int(1/(b/x+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(a*x^2+b)/a
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx = \frac{\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(1/(b/x+a*x),x, algorithm="fricas")
```

output
```
1/2*log(a*x^2 + b)/a
```

3.338.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x}+ax}\,dx = \frac{\log\left(ax^2+b\right)}{2a}$$

input
```
integrate(1/(b/x+a*x),x)
```

output
```
log(a*x**2 + b)/(2*a)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(1/(b/x+a*x),x, algorithm="maxima")
```

output
```
1/2*log(a*x^2 + b)/a
```

3.338.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(|ax^2+b|\right)}{2\,a}$$

input
```
integrate(1/(b/x+a*x),x, algorithm="giac")
```

output
```
1/2*log(abs(a*x^2 + b))/a
```

3.338.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\ln\left(a\,x^2+b\right)}{2\,a}$$

input
```
int(1/(a*x + b/x),x)
```

output
```
log(b + a*x^2)/(2*a)
```

3.339 $\int \frac{1}{\frac{b}{x^2}+ax}\,dx$

3.339.1 Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log(b+ax^3)}{3a}$$

output
```
1/3*ln(a*x^3+b)/a
```

3.339.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log(b+ax^3)}{3a}$$

input
```
Integrate[(b/x^2 + a*x)^(-1),x]
```

output
```
Log[b + a*x^3]/(3*a)
```

3.339.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^2}{ax^3+b}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{\log(ax^3+b)}{3a}$$

input
```
Int[(b/x^2 + a*x)^(-1),x]
```

output
```
Log[b + a*x^3]/(3*a)
```

3.339.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.339.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^3+b)}{3a}$	14
norman	$\frac{\ln(a\,x^3+b)}{3a}$	14
risch	$\frac{\ln(a\,x^3+b)}{3a}$	14
parallelrisch	$\frac{\ln(a\,x^3+b)}{3a}$	14

input
```
int(1/(b/x^2+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a*x^3+b)/a
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\log(ax^3+b)}{3\,a}$$

input
```
integrate(1/(b/x^2+a*x),x, algorithm="fricas")
```

output
```
1/3*log(a*x^3 + b)/a
```

3.339.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\log(ax^3+b)}{3a}$$

input
```
integrate(1/(b/x**2+a*x),x)
```

output
```
log(a*x**3 + b)/(3*a)
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log\left(ax^3+b\right)}{3\,a}$$

input
```
integrate(1/(b/x^2+a*x),x, algorithm="maxima")
```

output
```
1/3*log(a*x^3 + b)/a
```

3.339.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log\left(|ax^3+b|\right)}{3\,a}$$

input
```
integrate(1/(b/x^2+a*x),x, algorithm="giac")
```

output
```
1/3*log(abs(a*x^3 + b))/a
```

3.339.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\ln\left(a\,x^3+b\right)}{3\,a}$$

input
```
int(1/(a*x + b/x^2),x)
```

output
```
log(b + a*x^3)/(3*a)
```

3.340 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$

3.340.1 Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx = \frac{\log(b+ax^4)}{4a}$$

output
```
1/4*ln(a*x^4+b)/a
```

3.340.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx = \frac{\log(b+ax^4)}{4a}$$

input
```
Integrate[(b/x^3 + a*x)^(-1),x]
```

output
```
Log[b + a*x^4]/(4*a)
```

3.340.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^3}{ax^4+b}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{\log \left(ax^4+b\right)}{4a}$$

input
```
Int[(b/x^3 + a*x)^(-1),x]
```

output
```
Log[b + a*x^4]/(4*a)
```

3.340.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.340.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^4+b)}{4a}$	14
norman	$\frac{\ln(a\,x^4+b)}{4a}$	14
risch	$\frac{\ln(a\,x^4+b)}{4a}$	14
parallelrisch	$\frac{\ln(a\,x^4+b)}{4a}$	14

input
```
int(1/(b/x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a*x^4+b)/a
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx = \frac{\log(ax^4+b)}{4\,a}$$

input
```
integrate(1/(b/x^3+a*x),x, algorithm="fricas")
```

output
```
1/4*log(a*x^4 + b)/a
```

3.340.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx = \frac{\log(ax^4+b)}{4a}$$

input
```
integrate(1/(b/x**3+a*x),x)
```

output
```
log(a*x**4 + b)/(4*a)
```

3.340.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3} + ax}\,dx = \frac{\log(ax^4 + b)}{4\,a}$$

input
```
integrate(1/(b/x^3+a*x),x, algorithm="maxima")
```

output
```
1/4*log(a*x^4 + b)/a
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x^3} + ax}\,dx = \frac{\log(|ax^4 + b|)}{4\,a}$$

input
```
integrate(1/(b/x^3+a*x),x, algorithm="giac")
```

output
```
1/4*log(abs(a*x^4 + b))/a
```

3.340.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3} + ax}\,dx = \frac{\ln(a\,x^4 + b)}{4\,a}$$

input
```
int(1/(a*x + b/x^3),x)
```

output
```
log(b + a*x^4)/(4*a)
```

3.341 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$

3.341.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx=\frac{x^4}{4b\left(b+ax^2\right)^2}$$

output
```
1/4*x^4/b/(a*x^2+b)^2
```

3.341.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx=-\frac{b+2ax^2}{4a^2\left(b+ax^2\right)^2}$$

input
```
Integrate[(b/x + a*x)^(-3),x]
```

output
```
-1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

3.341.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^3}{\left(ax^2+b\right)^3}dx$$

$$\Big\downarrow \text{242}$$

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

input
```
Int[(b/x + a*x)^(-3),x]
```

output
```
x^4/(4*b*(b + a*x^2)^2)
```

3.341.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.341.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2a\,x^2+b}{4(a\,x^2+b)^2 a^2}$	23
parallelrisch	$\frac{-2a\,x^2-b}{4a^2(a\,x^2+b)^2}$	25
norman	$\frac{-\frac{x^2}{2a}-\frac{b}{4a^2}}{(a\,x^2+b)^2}$	26
risch	$\frac{-\frac{x^2}{2a}-\frac{b}{4a^2}}{(a\,x^2+b)^2}$	26
default	$-\frac{1}{2a^2(a\,x^2+b)}+\frac{b}{4a^2(a\,x^2+b)^2}$	31

input
```
int(1/(b/x+a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*a*x^2+b)/(a*x^2+b)^2/a^2
```

3.341.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\,(a^4x^4+2\,a^3bx^2+a^2b^2)}$$

input
```
integrate(1/(b/x+a*x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

3.341.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = \frac{-2ax^2-b}{4a^4x^4+8a^3bx^2+4a^2b^2}$$

input
```
integrate(1/(b/x+a*x)**3,x)
```

output
```
(-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

3.341.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\,(a^4x^4+2\,a^3bx^2+a^2b^2)}$$

input
```
integrate(1/(b/x+a*x)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\,(ax^2+b)^2a^2}$$

input
```
integrate(1/(b/x+a*x)^3,x, algorithm="giac")
```

output
```
-1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx=-\frac{\frac{b}{4\,a^2}+\frac{x^2}{2\,a}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

input
```
int(1/(a*x + b/x)^3,x)
```

output
```
-(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

3.342 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} dx$

3.342.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} dx = \frac{x^{10}}{10b\,(b+ax^5)^2}$$

output
```
1/10*x^10/b/(a*x^5+b)^2
```

3.342.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} dx = -\frac{b+2ax^5}{10a^2\,(b+ax^5)^2}$$

input
```
Integrate[(b/x^3 + a*x^2)^(-3),x]
```

output
```
-1/10*(b + 2*a*x^5)/(a^2*(b + a*x^5)^2)
```

3.342.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2027, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^2+\frac{b}{x^3}\right)^3}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x^9}{(ax^5+b)^3}dx$$
$$\Big\downarrow \text{ 796}$$
$$\frac{x^{10}}{10b\,(ax^5+b)^2}$$

input
```
Int[(b/x^3 + a*x^2)^(-3),x]
```

output
```
x^10/(10*b*(b + a*x^5)^2)
```

3.342.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.342.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2x^5a+b}{10(x^5a+b)^2a^2}$	23
parallelrisch	$\frac{-2x^5a-b}{10a^2(x^5a+b)^2}$	25
norman	$\frac{-\frac{x^5}{5a}-\frac{b}{10a^2}}{(x^5a+b)^2}$	26
risch	$\frac{-\frac{x^5}{5a}-\frac{b}{10a^2}}{(x^5a+b)^2}$	26
default	$-\frac{1}{5a^2(x^5a+b)}+\frac{b}{10a^2(x^5a+b)^2}$	31

input
```
int(1/(b/x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(2*a*x^5+b)/(a*x^5+b)^2/a^2
```

3.342.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,(a^4x^{10}+2\,a^3bx^5+a^2b^2)}$$

input
```
integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")
```

output
```
-1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

3.342.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = \frac{-2ax^5-b}{10a^4x^{10}+20a^3bx^5+10a^2b^2}$$

input
```
integrate(1/(b/x**3+a*x**2)**3,x)
```

output
```
(-2*a*x**5 - b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)
```

3.342.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,(a^4x^{10}+2\,a^3bx^5+a^2b^2)}$$

input
```
integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")
```

output
```
-1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

3.342.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,(ax^5+b)^2a^2}$$

input
```
integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")
```

output
```
-1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{\frac{b}{10\,a^2}+\frac{x^5}{5\,a}}{a^2\,x^{10}+2\,a\,b\,x^5+b^2}$$

input
```
int(1/(a*x^2 + b/x^3)^3,x)
```

output
```
-(b/(10*a^2) + x^5/(5*a))/(b^2 + a^2*x^10 + 2*a*b*x^5)
```

3.343 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3} dx$

3.343.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3} dx = \frac{x^{16}}{16b\left(b+ax^8\right)^2}$$

output
```
1/16*x^16/b/(a*x^8+b)^2
```

3.343.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3} dx = -\frac{b+2ax^8}{16a^2\left(b+ax^8\right)^2}$$

input
```
Integrate[(b/x^5 + a*x^3)^(-3),x]
```

output
```
-1/16*(b + 2*a*x^8)/(a^2*(b + a*x^8)^2)
```

3.343.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2027, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^3+\frac{b}{x^5}\right)^3}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^{15}}{(ax^8+b)^3}dx$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x^{16}}{16b\,(ax^8+b)^2}$$

input
```
Int[(b/x^5 + a*x^3)^(-3),x]
```

output
```
x^16/(16*b*(b + a*x^8)^2)
```

3.343.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.343.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2a\,x^8+b}{16(a\,x^8+b)^2a^2}$	23
parallelrisch	$\frac{-2a\,x^8-b}{16a^2(a\,x^8+b)^2}$	25
norman	$\frac{-\frac{b}{16a^2}-\frac{x^8}{8a}}{(a\,x^8+b)^2}$	26
risch	$\frac{-\frac{b}{16a^2}-\frac{x^8}{8a}}{(a\,x^8+b)^2}$	26
default	$-\frac{1}{8a^2(a\,x^8+b)}+\frac{b}{16a^2(a\,x^8+b)^2}$	31

input
```
int(1/(b/x^5+a*x^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(2*a*x^8+b)/(a*x^8+b)^2/a^2
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{2\,ax^8+b}{16\,(a^4x^{16}+2\,a^3bx^8+a^2b^2)}$$

input
```
integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

3.343.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = \frac{-2ax^8-b}{16a^4x^{16}+32a^3bx^8+16a^2b^2}$$

input
```
integrate(1/(b/x**5+a*x**3)**3,x)
```

output
```
(-2*a*x**8 - b)/(16*a**4*x**16 + 32*a**3*b*x**8 + 16*a**2*b**2)
```

3.343.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{2\,ax^8+b}{16\,(a^4x^{16}+2\,a^3bx^8+a^2b^2)}$$

input
```
integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

3.343.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{2\,ax^8+b}{16\,(ax^8+b)^2a^2}$$

input
```
integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")
```

output
```
-1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{\frac{b}{16\,a^2}+\frac{x^8}{8\,a}}{a^2\,x^{16}+2\,a\,b\,x^8+b^2}$$

input
```
int(1/(a*x^3 + b/x^5)^3,x)
```

output
```
-(b/(16*a^2) + x^8/(8*a))/(b^2 + a^2*x^16 + 2*a*b*x^8)
```

3.344 $\int \left(\frac{a}{x}+bx\right)^2 dx$

3.344.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \left(\frac{a}{x}+bx\right)^2 dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

3.344.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^2 dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Integrate[(a/x + b*x)^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

3.344.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x}+bx\right)^2 dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{(a+bx^2)^2}{x^2}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^2}+2ab+b^2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Int[(a/x + b*x)^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

3.344.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.344.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
parallelrisch	$\frac{b^2x^4+6ab\,x^2-3a^2}{3x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27

input
```
int((a/x+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

input
```
integrate((a/x+b*x)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

3.344.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
integrate((a/x+b*x)**2,x)
```

output
```
-a**2/x + 2*a*b*x + b**2*x**3/3
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2 dx = \frac{1}{3}b^2x^3 + 2abx - \frac{a^2}{x}$$

input
```
integrate((a/x+b*x)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2 dx = \frac{1}{3}b^2x^3 + 2abx - \frac{a^2}{x}$$

input
```
integrate((a/x+b*x)^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

3.344.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2 dx = \frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

input
```
int((b*x + a/x)^2,x)
```

output
```
(b^2*x^3)/3 - a^2/x + 2*a*b*x
```

3.345 $\int \left(\frac{a}{x}+bx\right)^3 dx$

3.345.1 Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \left(\frac{a}{x}+bx\right)^3 dx = -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

output
```
-1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

3.345.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^3 dx = -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

input
```
Integrate[(a/x + b*x)^3,x]
```

output
```
-1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

3.345.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2027, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x}+bx\right)^3 dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{(a+bx^2)^3}{x^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^4}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^2}+3b^2a+b^3x^2\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{x^2}+3a^2b\log\left(x^2\right)+3ab^2x^2+\frac{b^3x^4}{2}\right)$$

input
```
Int[(a/x + b*x)^3,x]
```

output
```
(-(a^3/x^2) + 3*a*b^2*x^2 + (b^3*x^4)/2 + 3*a^2*b*Log[x^2])/2
```

3.345.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.345.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{2 x^{2}}+\frac{3 a \,b^{2} x^{2}}{2}+\frac{b^{3} x^{4}}{4}+3 a^{2} b \ln \left(x \right)$	35
norman	$\frac{-\frac{1}{2} a^{3}+\frac{1}{4} b^{3} x^{6}+\frac{3}{2} a \,b^{2} x^{4}}{x^{2}}+3 a^{2} b \ln \left(x \right)$	37
parallelrisch	$\frac{b^{3} x^{6}+6 a \,b^{2} x^{4}+12 a^{2} b \ln \left(x \right) x^{2}-2 a^{3}}{4 x^{2}}$	39
risch	$\frac{b^{3} x^{4}}{4}+\frac{3 a \,b^{2} x^{2}}{2}+\frac{9 a^{2} b}{4}-\frac{a^{3}}{2 x^{2}}+3 a^{2} b \ln \left(x \right)$	41

input
```
int((a/x+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log\left(x\right)-2\,a^3}{4\,x^2}$$

input
```
integrate((a/x+b*x)^3,x, algorithm="fricas")
```

output
```
1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

3.345.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=-\frac{a^3}{2x^2}+3a^2b\log{(x)}+\frac{3ab^2x^2}{2}+\frac{b^3x^4}{4}$$

input
```
integrate((a/x+b*x)**3,x)
```

output
```
-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+3\,a^2b\log{(x)}-\frac{a^3}{2\,x^2}$$

input
```
integrate((a/x+b*x)^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+\frac{3}{2}\,a^2b\log{(x^2)}-\frac{3\,a^2bx^2+a^3}{2\,x^2}$$

input
```
integrate((a/x+b*x)^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)
/x^2
```

3.345.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(\frac{a}{x}+bx\right)^3 dx = \frac{b^3\,x^4}{4}-\frac{a^3}{2\,x^2}+\frac{3\,a\,b^2\,x^2}{2}+3\,a^2\,b\,\ln\left(x\right)$$

input
```
int((b*x + a/x)^3,x)
```

output
```
(b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

3.346 $\int \left(\frac{a}{x}+bx\right)^4 dx$

3.346.1 Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \left(\frac{a}{x}+bx\right)^4 dx = -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

output
```
-1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

3.346.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^4 dx = -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

input
```
Integrate[(a/x + b*x)^4,x]
```

output
```
-1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

3.346.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2027, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a}{x}+bx\right)^4 dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{(a+bx^2)^4}{x^4}dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+6a^2b^2+4ab^3x^2+b^4x^4\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

input
```
Int[(a/x + b*x)^4,x]
```

output
```
-1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

3.346.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.346.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4a\,b^3x^3}{3}+\frac{b^4x^5}{5}$	45
risch	$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x+\frac{-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	47
norman	$\frac{\frac{1}{5}x^8b^4+\frac{4}{3}a\,b^3x^6+6a^2x^4b^2-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	48
gosper	$-\frac{-3x^8b^4-20a\,b^3x^6-90a^2x^4b^2+60a^3b\,x^2+5a^4}{15x^3}$	49
parallelrisch	$\frac{3x^8b^4+20a\,b^3x^6+90a^2x^4b^2-60a^3b\,x^2-5a^4}{15x^3}$	49

input
```
int((a/x+b*x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(\frac{a}{x}+bx\right)^4\,dx=\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

input
```
integrate((a/x+b*x)^4,x, algorithm="fricas")
```

output
```
1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^
3
```

3.346.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \left(\frac{a}{x}+bx\right)^4\,dx=6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}+\frac{-a^4-12a^3bx^2}{3x^3}$$

input
```
integrate((a/x+b*x)**4,x)
```

output
```
6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(
3*x**3)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(\frac{a}{x}+bx\right)^4\,dx=\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{4\,a^3b}{x}-\frac{a^4}{3\,x^3}$$

input
```
integrate((a/x+b*x)^4,x, algorithm="maxima")
```

output
```
1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \left(\frac{a}{x}+bx\right)^4\,dx=\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

input
```
integrate((a/x+b*x)^4,x, algorithm="giac")
```

output
```
1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

3.346.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \left(\frac{a}{x}+bx\right)^4\,dx=\frac{b^4\,x^5}{5}-\frac{\frac{a^4}{3}+4\,b\,a^3\,x^2}{x^3}+6\,a^2\,b^2\,x+\frac{4\,a\,b^3\,x^3}{3}$$

input
```
int((b*x + a/x)^4,x)
```

output
```
(b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

3.347 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$

3.347.1 Optimal result

Integrand size = 9, antiderivative size = 185

$$\begin{aligned}\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = &-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+2\sqrt{\frac{2}{5+\sqrt{5}}}x\right)\\ &-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}x\right)\\ &+\frac{1}{5}\log(1+x)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2\right)\\ &-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right)\end{aligned}$$

output
```
1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(1+x^2-1/
2*x*(-5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*x*2^
(1/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/10*arctan(1/5*x*(50+10*5^(
1/2))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.78

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{1}{20}\left(-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1+\sqrt{5}-4x}{\sqrt{10-2\sqrt{5}}}\right) - 2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-1+\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + 4\log(1+x) - \left(1+\sqrt{5}\right)\log\left(1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2\right) + \left(-1+\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right)\right)$$

input
```
Integrate[(x^(-2) + x^3)^(-1),x]
```

output
```
(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])
]] + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-
1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

3.347.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2027, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3+\frac{1}{x^2}}\,dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2}{x^5+1}dx$$

$$\Big\downarrow \text{822}$$

$$\frac{2}{5}\int -\frac{\left(1+\sqrt{5}\right)x+\sqrt{5}+1}{2\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}dx+\frac{2}{5}\int -\frac{\left(1-\sqrt{5}\right)x-\sqrt{5}+1}{2\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}dx+\frac{1}{5}\int \frac{1}{x+1}dx$$

$$\big\downarrow 16$$

$$\frac{2}{5}\int -\frac{\left(1+\sqrt{5}\right)x+\sqrt{5}+1}{2\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}dx+\frac{2}{5}\int -\frac{\left(1-\sqrt{5}\right)x-\sqrt{5}+1}{2\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}dx+\frac{1}{5}\log(x+1)$$

$$\big\downarrow 27$$

$$-\frac{1}{5}\int \frac{\left(1+\sqrt{5}\right)x+\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx-\frac{1}{5}\int \frac{\left(1-\sqrt{5}\right)x-\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx+\frac{1}{5}\log(x+1)$$

$$\big\downarrow 1142$$

$$\frac{1}{5}\left(-\sqrt{5}\int \frac{1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{-4x-\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx\right)+$$
$$\frac{1}{5}\left(\sqrt{5}\int \frac{1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{-4x+\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx\right)+\frac{1}{5}\log(x+1)$$

$$\big\downarrow 25$$

$$\frac{1}{5}\left(\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{-4x-\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx-\sqrt{5}\int \frac{1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx\right)+$$
$$\frac{1}{5}\left(\sqrt{5}\int \frac{1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{-4x+\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx\right)+\frac{1}{5}\log(x+1)$$

$$\big\downarrow 1083$$

$$\frac{1}{5}\left(\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{-4x+\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx-2\sqrt{5}\int \frac{1}{-\left(4x-\sqrt{5}-1\right)^2-2\left(5-\sqrt{5}\right)}d\left(4x-\sqrt{5}-1\right)\right)+$$
$$\frac{1}{5}\left(\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{-4x-\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx+2\sqrt{5}\int \frac{1}{-\left(4x+\sqrt{5}-1\right)^2-2\left(5+\sqrt{5}\right)}d\left(4x+\sqrt{5}-1\right)\right)+$$
$$\frac{1}{5}\log(x+1)$$

$$\big\downarrow 217$$

$$\frac{1}{5}\left(\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{-4x-\sqrt{5}+1}{2x^2-\left(1-\sqrt{5}\right)x+2}dx-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)\right)+$$
$$\frac{1}{5}\left(\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{-4x+\sqrt{5}+1}{2x^2-\left(1+\sqrt{5}\right)x+2}dx+\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)\right)+\frac{1}{5}\log(x+1)$$

$$\big\downarrow 1103$$

3.347. $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$

$$\frac{1}{5}\left(-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)\right)+\frac{1}{5}\left(\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)\right)+\frac{1}{5}\log(x+1)$$

input
```
Int[(x^(-2) + x^3)^(-1),x]
```

output
```
Log[1 + x]/5 + (-(Sqrt[10/(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[
2*(5 + Sqrt[5])]]) - ((1 + Sqrt[5])*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/4)/5
 + (Sqrt[10/(5 - Sqrt[5])]*ArcTan[(-1 - Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5]
)]] - ((1 - Sqrt[5])*Log[2 - (1 + Sqrt[5])*x + 2*x^2])/4)/5
```

3.347.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.347.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+\_Z^{3}+\_Z^{2}+\_Z+1\right)}\_R\ln\left(\_R^{2}+x\right)\right)}{5}+\frac{\ln(1+x)}{5}$
default	$\frac{\ln(1+x)}{5}-\frac{\left(-\sqrt{5}+1\right)\ln\left(-x\sqrt{5}+2x^{2}-x+2\right)}{20}-\frac{2\left(-\sqrt{5}+1-\frac{\left(-\sqrt{5}+1\right)\left(-\sqrt{5}-1\right)}{4}\right)\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\left(-\sqrt{5}-1\right)\ln\left(x\sqrt{5}+\right.}{20}$

input
```
int(1/(1/x^2+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum(_R*ln(_R^2+x),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))+1/5*ln(1+x)
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(122) = 244$.

Time = 0.90 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.44

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$$

$$= -\frac{1}{20}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+x\right)$$

$$+\frac{1}{20}\left(\sqrt{5}+2\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}\right.}\right.$$

$$-\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2$$

$$+\frac{1}{2}\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right.}$$

$$\left.+2x-1\right)$$

$$+\frac{1}{20}\left(\sqrt{5}-2\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}\right.}\right.$$

$$-\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2$$

$$-\frac{1}{2}\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right.}$$

$$\left.+2x-1\right)$$

$$+\frac{1}{20}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+x\right)$$

$$+\frac{1}{5}\log(x+1)$$

input
```
integrate(1/(1/x^2+x^3),x, algorithm="fricas")
```

output
```
-1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/16*(2*sqrt(1/2)*
sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + x) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*
sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/1
6*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5)
 - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) +
 sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1
/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqr
t(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - s
qrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(
1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x - 1) + 1/2
0*(sqrt(5) - 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt
(5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*
sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt
(5) - 5) - sqrt(5) - 1)^2 - 1/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5)
+ sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5)
- 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2...
```

3.347.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.19

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{\log{(x+1)}}{5} + \operatorname{RootSum}{\left(625t^4+125t^3+25t^2+5t+1, \left(t\mapsto t\log{\left(25t^2+x\right)}\right)\right)}$$

input
```
integrate(1/(1/x**2+x**3),x)
```

output
```
log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda
(_t, _t*log(25*_t**2 + x)))
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}} + \frac{\log\left(2x^2-x(\sqrt{5}+1)+2\right)}{5(\sqrt{5}+1)} - \frac{\log\left(2x^2+x(\sqrt{5}-1)+2\right)}{5(\sqrt{5}-1)} + \frac{1}{5}\log(x+1)$$

input
```
integrate(1/(1/x^2+x^3),x, algorithm="maxima")
```

output
```
-2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(
5) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/s
qrt(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1)
- 1/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

3.347.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) - \frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right) - \frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{1}{5}\log\left(|x+1|\right)$$

input
```
integrate(1/(1/x^2+x^3),x, algorithm="giac")
```

output
```
1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)
*log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4
*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan
((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

3.347.9 Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{\ln(x+1)}{5} - \ln\left(1-\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right) + \ln\left(\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}+\sqrt{5}-1\right)^3}{64}+1\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}+\frac{\sqrt{5}}{20}-\frac{1}{20}\right) - \ln\left(1-\frac{x\left(\sqrt{5}+\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2}\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right) - \ln\left(1-\frac{x\left(\sqrt{5}-\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}-\frac{\sqrt{2}\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right)$$

input
```
int(1/(1/x^2 + x^3),x)
```

output
```
log(x + 1)/5 - log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)
/64)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log((x*(2^
(1/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(1 - (x*(5^(1/2) + 2^(1/2)*(5^(1
/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20
+ 1/20) - log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5
^(1/2)/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

3.348 $\int x^p(ax^n + bx^{1+13n+p})^{12}\, dx$

3.348.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int x^p\left(ax^n + bx^{1+13n+p}\right)^{12}\, dx = \frac{\left(a+bx^{1+12n+p}\right)^{13}}{13b(1+12n+p)}$$

output
```
1/13*(a+b*x^(1+12*n+p))^13/b/(1+12*n+p)
```

3.348.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(29) = 58.

Time = 0.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 8.00

$$\int x^p\left(ax^n + bx^{1+13n+p}\right)^{12}\, dx = \frac{x^{1+12n+p}(13a^{12} + 78a^{11}bx^{1+12n+p} + 286a^{10}b^2x^{2+24n+2p} + 715a^9b^3x^{3+36n+3p} + 1287a^8b^4x^{4+48n+4p} + 1716a^7b}{}$$

input
```
Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

output
```
(x^(1 + 12*n + p)*(13*a^12 + 78*a^11*b*x^(1 + 12*n + p) + 286*a^10*b^2*x^(
2 + 24*n + 2*p) + 715*a^9*b^3*x^(3 + 36*n + 3*p) + 1287*a^8*b^4*x^(4 + 48*
n + 4*p) + 1716*a^7*b^5*x^(5 + 60*n + 5*p) + 1716*a^6*b^6*x^(6 + 72*n + 6*
p) + 1287*a^5*b^7*x^(7 + 84*n + 7*p) + 715*a^4*b^8*x^(8 + 96*n + 8*p) + 28
6*a^3*b^9*x^(9 + 108*n + 9*p) + 78*a^2*b^10*x^(10 + 120*n + 10*p) + 13*a*b
^11*x^(11 + 132*n + 11*p) + b^12*x^(12 + 144*n + 12*p)))/(13*(1 + 12*n + p
))
```

3.348.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {10, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p \left(ax^n + bx^{13n+p+1}\right)^{12} dx$$

$$\Big\downarrow \; 10$$

$$\int x^{12n+p} \left(a + bx^{12n+p+1}\right)^{12} dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n+p+1)}$$

input
```
Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

output
```
(a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

3.348.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.348.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 363, normalized size of antiderivative = 12.52

$$\frac{b^{12}x^{13}x^{156n}x^{13p}}{13+156n+13p}+\frac{a\,b^{11}x^{12}x^{144n}x^{12p}}{1+12n+p}+\frac{6a^2b^{10}x^{11}x^{132n}x^{11p}}{1+12n+p}+\frac{22a^3b^9x^{10}x^{120n}x^{10p}}{1+12n+p}+\frac{55a^4b^8x^9x^{108n}x^{9p}}{1+12n+p}+\frac{99a^5b^7x^8}{1+12}$$

input
```
int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)
```

output
```
1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n
+p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10
*(x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9
+99*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12
*n+p)*(x^p)^7+132*a^7*b^5*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(
x^n)^60/(1+12*n+p)*(x^p)^5+55*a^9*b^3*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a
^10*b^2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*b*a^11*x^2*(x^n)^24/(1+12*n+p)*(
x^p)^2+a^12/(1+12*n+p)*x*(x^n)^12*x^p
```

3.348.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 10.24

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx$$
$$=\frac{78\,a^2b^{10}x^{2\,n}x^{143\,n+11\,p+11}+286\,a^3b^9x^{3\,n}x^{130\,n+10\,p+10}+715\,a^4b^8x^{4\,n}x^{117\,n+9\,p+9}+1287\,a^5b^7x^{5\,n}x^{104\,n+8\,p+8}+\cdots}{\cdots}$$

input
```
integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")
```

output
```
1/13*(78*a^2*b^10*x^(2*n)*x^(143*n + 11*p + 11) + 286*a^3*b^9*x^(3*n)*x^(1
30*n + 10*p + 10) + 715*a^4*b^8*x^(4*n)*x^(117*n + 9*p + 9) + 1287*a^5*b^7
*x^(5*n)*x^(104*n + 8*p + 8) + 1716*a^6*b^6*x^(6*n)*x^(91*n + 7*p + 7) + 1
716*a^7*b^5*x^(7*n)*x^(78*n + 6*p + 6) + 1287*a^8*b^4*x^(8*n)*x^(65*n + 5*
p + 5) + 715*a^9*b^3*x^(9*n)*x^(52*n + 4*p + 4) + 286*a^10*b^2*x^(10*n)*x^
(39*n + 3*p + 3) + 78*a^11*b*x^(11*n)*x^(26*n + 2*p + 2) + 13*a^12*x^(12*n
)*x^(13*n + p + 1) + 13*a*b^11*x^(156*n + 12*p + 12)*x^n + b^12*x^(169*n +
 13*p + 13))/((12*n + p + 1)*x^(13*n))
```

3.348.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(22) = 44.

Time = 149.34 (sec) , antiderivative size = 690, normalized size of antiderivative = 23.79

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=\text{Too large to display}$$

input
```
integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)
```

output
```
Piecewise((13*a**12*x*x**(12*n)*x**p/(156*n + 13*p + 13) + 78*a**11*b*x*x*
*(11*n)*x**p*x**(13*n + p + 1)/(156*n + 13*p + 13) + 286*a**10*b**2*x*x**(
10*n)*x**p*x**(26*n + 2*p + 2)/(156*n + 13*p + 13) + 715*a**9*b**3*x*x**(9
*n)*x**p*x**(39*n + 3*p + 3)/(156*n + 13*p + 13) + 1287*a**8*b**4*x*x**(8*
n)*x**p*x**(52*n + 4*p + 4)/(156*n + 13*p + 13) + 1716*a**7*b**5*x*x**(7*n
)*x**p*x**(65*n + 5*p + 5)/(156*n + 13*p + 13) + 1716*a**6*b**6*x*x**(6*n)
*x**p*x**(78*n + 6*p + 6)/(156*n + 13*p + 13) + 1287*a**5*b**7*x*x**(5*n)*
x**p*x**(91*n + 7*p + 7)/(156*n + 13*p + 13) + 715*a**4*b**8*x*x**(4*n)*x*
*p*x**(104*n + 8*p + 8)/(156*n + 13*p + 13) + 286*a**3*b**9*x*x**(3*n)*x**
p*x**(117*n + 9*p + 9)/(156*n + 13*p + 13) + 78*a**2*b**10*x*x**(2*n)*x**p
*x**(130*n + 10*p + 10)/(156*n + 13*p + 13) + 13*a*b**11*x*x**n*x**p*x**(1
43*n + 11*p + 11)/(156*n + 13*p + 13) + b**12*x*x**p*x**(156*n + 12*p + 12
)/(156*n + 13*p + 13), Ne(n, -p/12 - 1/12)), (a**12*Piecewise((log(x), Eq(
p, 0)), (log(x**p)/p, True)) + 12*a**11*b*Piecewise((log(x), Eq(p, 0)), (l
og(x**p)/p, True)) + 66*a**10*b**2*Piecewise((log(x), Eq(p, 0)), (log(x**p
)/p, True)) + 220*a**9*b**3*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, Tr
ue)) + 495*a**8*b**4*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) +
792*a**7*b**5*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 924*a**
6*b**6*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 792*a**5*b**7*
Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 495*a**4*b**8*Piec...
```

3.348.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(27) = 54$.

Time = 0.22 (sec) , antiderivative size = 325, normalized size of antiderivative = 11.21

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx = \frac{b^{12}x^{156\,n+13\,p+13}}{13\,(12\,n+p+1)} + \frac{ab^{11}x^{144\,n+12\,p+12}}{12\,n+p+1} + \frac{6\,a^2b^{10}x^{132\,n+11\,p+11}}{12\,n+p+1} + \frac{22\,a^3b^9x^{120\,n+10\,p+10}}{12\,n+p+1} + \frac{55\,a^4b^8x^{108\,n+9\,p+9}}{12\,n+p+1} + \frac{99\,a^5b^7x^{96\,n+8\,p+8}}{12\,n+p+1} + \frac{132\,a^6b^6x^{84\,n+7\,p+7}}{12\,n+p+1} + \frac{132\,a^7b^5x^{72\,n+6\,p+6}}{12\,n+p+1} + \frac{99\,a^8b^4x^{60\,n+5\,p+5}}{12\,n+p+1} + \frac{55\,a^9b^3x^{48\,n+4\,p+4}}{12\,n+p+1} + \frac{22\,a^{10}b^2x^{36\,n+3\,p+3}}{12\,n+p+1} + \frac{6\,a^{11}bx^{24\,n+2\,p+2}}{12\,n+p+1} + \frac{a^{12}x^{12\,n+p+1}}{12\,n+p+1}$$

input
```
integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")
```

output
```
1/13*b^12*x^(156*n + 13*p + 13)/(12*n + p + 1) + a*b^11*x^(144*n + 12*p +
12)/(12*n + p + 1) + 6*a^2*b^10*x^(132*n + 11*p + 11)/(12*n + p + 1) + 22*
a^3*b^9*x^(120*n + 10*p + 10)/(12*n + p + 1) + 55*a^4*b^8*x^(108*n + 9*p +
 9)/(12*n + p + 1) + 99*a^5*b^7*x^(96*n + 8*p + 8)/(12*n + p + 1) + 132*a^
6*b^6*x^(84*n + 7*p + 7)/(12*n + p + 1) + 132*a^7*b^5*x^(72*n + 6*p + 6)/(
12*n + p + 1) + 99*a^8*b^4*x^(60*n + 5*p + 5)/(12*n + p + 1) + 55*a^9*b^3*
x^(48*n + 4*p + 4)/(12*n + p + 1) + 22*a^10*b^2*x^(36*n + 3*p + 3)/(12*n +
 p + 1) + 6*a^11*b*x^(24*n + 2*p + 2)/(12*n + p + 1) + a^12*x^(12*n + p +
1)/(12*n + p + 1)
```

3.348.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50971 vs. $2(27) = 54$.

Time = 1.46 (sec) , antiderivative size = 50971, normalized size of antiderivative = 1757.62

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} \, dx = \text{Too large to display}$$

input
```
integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="giac")
```

output
```
(31408819200*a^2*b^10*n^10*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) +
 35*log(x)) + 331405966080*a^2*b^10*n^9*p^2*x*x^(2*n)*x^p*e^(455*n*log(x)
+ 35*p*log(x) + 35*log(x)) + 1230778965888*a^2*b^10*n^8*p^3*x*x^(2*n)*x^p*
e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 2139674600448*a^2*b^10*n^7*p^
4*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 1890383812992
*a^2*b^10*n^6*p^5*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x))
 + 874552702464*a^2*b^10*n^5*p^6*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(
x) + 35*log(x)) + 222844093056*a^2*b^10*n^4*p^7*x*x^(2*n)*x^p*e^(455*n*log
(x) + 35*p*log(x) + 35*log(x)) + 32330382336*a^2*b^10*n^3*p^8*x*x^(2*n)*x^
p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 2661766272*a^2*b^10*n^2*p^9
*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 115879680*a^2*
b^10*n*p^10*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 207
3600*a^2*b^10*p^11*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)
) + 3156586329600*a^5*b^7*n^10*p*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(
x) + 34*log(x)) + 33302016570240*a^5*b^7*n^9*p^2*x*x^(5*n)*x^p*e^(442*n*lo
g(x) + 34*p*log(x) + 34*log(x)) + 123648483714624*a^5*b^7*n^8*p^3*x*x^(5*n
)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 214873536791232*a^5*b^7
*n^7*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 189706
686719616*a^5*b^7*n^6*p^5*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34
*log(x)) + 87659938485504*a^5*b^7*n^5*p^6*x*x^(5*n)*x^p*e^(442*n*log(x)...
```

3.348.9 Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 363, normalized size of antiderivative = 12.52

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=\frac{a^{12}\,x\,x^p\,x^{12\,n}}{12\,n+p+1}+\frac{b^{12}\,x^{156\,n}\,x^{13\,p}\,x^{13}}{156\,n+13\,p+13}+\frac{22\,a^{10}\,b^2\,x^{36\,n}\,x^{3\,p}\,x^3}{12\,n+p+1}+\frac{55\,a^9\,b^3\,x^{48\,n}\,x^{4\,p}\,x^4}{12\,n+p+1}+\frac{99\,a^8\,b^4\,x^{60\,n}\,x^{5\,p}\,x^5}{12\,n+p+1}+\frac{132\,a^7\,b^5\,x^{72\,n}\,x^{6\,p}\,x^6}{12\,n+p+1}+\frac{132\,a^6\,b^6\,x^{84\,n}\,x^{7\,p}\,x^7}{12\,n+p+1}+\frac{99\,a^5\,b^7\,x^{96\,n}\,x^{8\,p}\,x^8}{12\,n+p+1}+\frac{55\,a^4\,b^8\,x^{108\,n}\,x^{9\,p}\,x^9}{12\,n+p+1}+\frac{22\,a^3\,b^9\,x^{120\,n}\,x^{10\,p}\,x^{10}}{12\,n+p+1}+\frac{6\,a^2\,b^{10}\,x^{132\,n}\,x^{11\,p}\,x^{11}}{12\,n+p+1}+\frac{6\,a^{11}\,b\,x^{24\,n}\,x^{2\,p}\,x^2}{12\,n+p+1}+\frac{a\,b^{11}\,x^{144\,n}\,x^{12\,p}\,x^{12}}{12\,n+p+1}$$

input
```
int(x^p*(a*x^n + b*x^(13*n + p + 1))^12,x)
```

output
```
(a^12*x*x^p*x^(12*n))/(12*n + p + 1) + (b^12*x^(156*n)*x^(13*p)*x^13)/(156
*n + 13*p + 13) + (22*a^10*b^2*x^(36*n)*x^(3*p)*x^3)/(12*n + p + 1) + (55*
a^9*b^3*x^(48*n)*x^(4*p)*x^4)/(12*n + p + 1) + (99*a^8*b^4*x^(60*n)*x^(5*p
)*x^5)/(12*n + p + 1) + (132*a^7*b^5*x^(72*n)*x^(6*p)*x^6)/(12*n + p + 1)
+ (132*a^6*b^6*x^(84*n)*x^(7*p)*x^7)/(12*n + p + 1) + (99*a^5*b^7*x^(96*n)
*x^(8*p)*x^8)/(12*n + p + 1) + (55*a^4*b^8*x^(108*n)*x^(9*p)*x^9)/(12*n +
p + 1) + (22*a^3*b^9*x^(120*n)*x^(10*p)*x^10)/(12*n + p + 1) + (6*a^2*b^10
*x^(132*n)*x^(11*p)*x^11)/(12*n + p + 1) + (6*a^11*b*x^(24*n)*x^(2*p)*x^2)
/(12*n + p + 1) + (a*b^11*x^(144*n)*x^(12*p)*x^12)/(12*n + p + 1)
```

3.349 $\int x^{12}(a+bx^{13})^{12}\,dx$

3.349.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{12}(a+bx^{13})^{12}\,dx=\frac{(a+bx^{13})^{13}}{169b}$$

output
```
1/169*(b*x^13+a)^13/b
```

3.349.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{12}(a+bx^{13})^{12}\,dx&=\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}\\&+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}\\&+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}\end{aligned}$$

input
```
Integrate[x^12*(a + b*x^13)^12,x]
```

output
```
(a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*
x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^9
1)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)
/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

3.349.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

input
```
Int[x^12*(a + b*x^13)^12,x]
```

output
```
(a + b*x^13)^13/(169*b)
```

3.349.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.349.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{13}+a\right)^{13}}{169b}$
gosper	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}$
parallelrisch	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^{2}b^{10}x^{143}}{13}+\frac{22a^{3}b^{9}x^{130}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{132a^{7}b^{5}x^{78}}{13}+99$

input
```
int(x^12*(b*x^13+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/169*(b*x^13+a)^13/b
```

3.349.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}(a+bx^{13})^{12}\,dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^2b^{10}x^{143} + \frac{22}{13}\,a^3b^9x^{130} + \frac{55}{13}\,a^4b^8x^{117} + \frac{99}{13}\,a^5b^7x^{104} + \frac{132}{13}\,a^6b^6x^{91} + \frac{132}{13}\,a^7b^5x^{78} + \frac{99}{13}\,a^8b^4x^{65} + \frac{55}{13}\,a^9b^3x^{52} + \frac{22}{13}\,a^{10}b^2x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

3.349.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}(a+bx^{13})^{12}\,dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

input
```
integrate(x**12*(b*x**13+a)**12,x)
```

output
```
a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**
3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**
6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**
9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/16
9
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}(a+bx^{13})^{12}\,dx=\frac{(bx^{13}+a)^{13}}{169\,b}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
```

output
```
1/169*(b*x^13 + a)^13/b
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}(a+bx^{13})^{12}\,dx=\frac{(bx^{13}+a)^{13}}{169\,b}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

output
```
1/169*(b*x^13 + a)^13/b
```

3.349.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}(a+bx^{13})^{12}\,dx=\frac{(b\,x^{13}+a)^{13}}{169\,b}$$

input
```
int(x^12*(a + b*x^13)^12,x)
```

output
```
(a + b*x^13)^13/(169*b)
```

3.350 $\int x^{12}(ax+bx^{26})^{12}\,dx$

3.350.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

3.350.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

input
```
Integrate[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

3.350.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx$$

$$\Big\downarrow 9$$

$$\int x^{24}\left(a+bx^{25}\right)^{12}dx$$

$$\Big\downarrow 793$$

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[x^12*(a*x + b*x^26)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

3.350.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.350.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 2.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{25}a^8b^4x^{125} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{1}{325}b^{12}x^{325} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^7b^5x^{150}$
parallelrisch	$\frac{99}{25}a^8b^4x^{125} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{1}{325}b^{12}x^{325} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^7b^5x^{150}$
gosper	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a^8b}{325}$
risch	$\frac{b^{12}x^{325}}{325} + \frac{a\,b^{11}x^{300}}{25} + \frac{6a^2b^{10}x^{275}}{25} + \frac{22a^3b^9x^{250}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{99a^5b^7x^{200}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{132a^7b^5x^{150}}{25} +$

input
```
int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
99/25*a^8*b^4*x^125+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x
^225+1/325*b^12*x^325+99/25*a^5*b^7*x^200+132/25*a^7*b^5*x^150+132/25*a^6*
b^6*x^175+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^50+1/25*a*b
^11*x^300+1/25*a^12*x^25
```

3.350.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}(ax+bx^{26})^{12}\,dx = \frac{1}{325}\,b^{12}x^{325} + \frac{1}{25}\,ab^{11}x^{300} + \frac{6}{25}\,a^2b^{10}x^{275} + \frac{22}{25}\,a^3b^9x^{250} + \frac{11}{5}\,a^4b^8x^{225} + \frac{99}{25}\,a^5b^7x^{200} + \frac{132}{25}\,a^6b^6x^{175} + \frac{132}{25}\,a^7b^5x^{150} + \frac{99}{25}\,a^8b^4x^{125} + \frac{11}{5}\,a^9b^3x^{100} + \frac{22}{25}\,a^{10}b^2x^{75} + \frac{6}{25}\,a^{11}bx^{50} + \frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.350.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(10) = 20.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

input
```
integrate(x**12*(b*x**26+a*x)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

3.350.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.350.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.350.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

input
```
int(x^12*(a*x + b*x^26)^12,x)
```

output
```
(a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 +
(132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 +
(11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.351 $\int x^{12}(ax^2+bx^{39})^{12}\,dx$

3.351.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

3.351.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=&\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}\\&+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}\\&+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}\end{aligned}$$

input
```
Integrate[x^12*(a*x^2 + b*x^39)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

3.351.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12} dx$$

$$\Big\downarrow \; 9$$

$$\int x^{36}\left(a+bx^{37}\right)^{12} dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[x^12*(a*x^2 + b*x^39)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

3.351.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.351.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 2.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a^8b}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

input
```
int(x^12*(b*x^39+a*x^2)^12,x,method=_RETURNVERBOSE)
```

output
```
6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+
132/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^
37+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^1
1*x^444+6/37*a^2*b^10*x^407
```

3.351.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.351.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**12*(b*x**39+a*x**2)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

3.351.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.351.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.351.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx = \frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

input
```
int(x^12*(a*x^2 + b*x^39)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.352 $\int x^{24}(a+bx^{25})^{12}\,dx$

3.352.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{24}(a+bx^{25})^{12}\,dx=\frac{(a+bx^{25})^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

3.352.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}(a+bx^{25})^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

input
```
Integrate[x^24*(a + b*x^25)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

3.352.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

input
```
Int[x^24*(a + b*x^25)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

3.352.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.352.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{25}+a\right)^{13}}{325b}$
gosper	$\frac{99}{25}a^{8}b^{4}x^{125}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{7}b^{5}x^{150}$
parallelrisch	$\frac{99}{25}a^{8}b^{4}x^{125}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{7}b^{5}x^{150}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+$

input
```
int(x^24*(b*x^25+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/325*(b*x^25+a)^13/b
```

3.352.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx = \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

3.352.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx = \frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{22a^3b^9x^{250}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

input
```
integrate(x**24*(b*x**25+a)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}(a+bx^{25})^{12}\,dx=\frac{(bx^{25}+a)^{13}}{325\,b}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

output
```
1/325*(b*x^25 + a)^13/b
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}(a+bx^{25})^{12}\,dx=\frac{(bx^{25}+a)^{13}}{325\,b}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

output
```
1/325*(b*x^25 + a)^13/b
```

3.352.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}(a+bx^{25})^{12}\,dx=\frac{(b\,x^{25}+a)^{13}}{325\,b}$$

input
```
int(x^24*(a + b*x^25)^12,x)
```

output
```
(a + b*x^25)^13/(325*b)
```

3.353 $\int x^{24}(ax+bx^{38})^{12}\,dx$

3.353.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

3.353.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

input
```
Integrate[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

3.353.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{36}\left(a+bx^{37}\right)^{12}dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[x^24*(a*x + b*x^38)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

3.353.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.353.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 2.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{22}{37}a^{3}b^{9}x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{22}{37}a^{3}b^{9}x^{370}$
gosper	$\frac{x^{37}\left(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^{2}b^{10}x^{370}+286a^{3}b^{9}x^{333}+715a^{4}b^{8}x^{296}+1287a^{5}b^{7}x^{259}+1716a^{6}b^{6}x^{222}+1716a^{7}b^{5}x^{185}+1287a^{8}b\right.}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+$

input
```
int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

output
```
6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+
132/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^
37+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^1
1*x^444+6/37*a^2*b^10*x^407
```

3.353.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.353.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**24*(b*x**38+a*x)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

3.353.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.353.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

input
```
integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^{2}\,x^{111}}{37}+\frac{55\,a^{9}\,b^{3}\,x^{148}}{37}+\frac{99\,a^{8}\,b^{4}\,x^{185}}{37}+\frac{132\,a^{7}\,b^{5}\,x^{222}}{37}+\frac{132\,a^{6}\,b^{6}\,x^{259}}{37}+\frac{99\,a^{5}\,b^{7}\,x^{296}}{37}+\frac{55\,a^{4}\,b^{8}\,x^{333}}{37}+\frac{22\,a^{3}\,b^{9}\,x^{370}}{37}+\frac{6\,a^{2}\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

input
```
int(x^24*(a*x + b*x^38)^12,x)
```

output
```
(a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37
+ (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37
+ (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.354 $\int x^{36}(a+bx^{37})^{12}\,dx$

3.354.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

3.354.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

input
```
Integrate[x^36*(a + b*x^37)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

3.354.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{36} \left(a+b x^{37}\right)^{12} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+b x^{37}\right)^{13}}{481 b}$$

input
```
Int[x^36*(a + b*x^37)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

3.354.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.354.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{37}+a\right)^{13}}{481 b}$
gosper	$\frac{6}{37} b\,a^{11} x^{74}+\frac{1}{481} b^{12} x^{481}+\frac{55}{37} a^{9} b^{3} x^{148}+\frac{99}{37} a^{5} b^{7} x^{296}+\frac{132}{37} a^{7} b^{5} x^{222}+\frac{132}{37} a^{6} b^{6} x^{259}+\frac{22}{37} a^{3} b^{9} x^{370}$
parallelrisch	$\frac{6}{37} b\,a^{11} x^{74}+\frac{1}{481} b^{12} x^{481}+\frac{55}{37} a^{9} b^{3} x^{148}+\frac{99}{37} a^{5} b^{7} x^{296}+\frac{132}{37} a^{7} b^{5} x^{222}+\frac{132}{37} a^{6} b^{6} x^{259}+\frac{22}{37} a^{3} b^{9} x^{370}$
risch	$\frac{b^{12} x^{481}}{481}+\frac{a\,b^{11} x^{444}}{37}+\frac{6 a^{2} b^{10} x^{407}}{37}+\frac{22 a^{3} b^{9} x^{370}}{37}+\frac{55 a^{4} b^{8} x^{333}}{37}+\frac{99 a^{5} b^{7} x^{296}}{37}+\frac{132 a^{6} b^{6} x^{259}}{37}+\frac{132 a^{7} b^{5} x^{222}}{37}+$

input
```
int(x^36*(b*x^37+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/481*(b*x^37+a)^13/b
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx = \frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

3.354.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**36*(b*x**37+a)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

output
```
1/481*(b*x^37 + a)^13/b
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

output
```
1/481*(b*x^37 + a)^13/b
```

3.354.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(b\,x^{37}+a\right)^{13}}{481\,b}$$

input
```
int(x^36*(a + b*x^37)^12,x)
```

output
```
(a + b*x^37)^13/(481*b)
```

3.355 $\int \frac{1}{ax+bx^n} dx$

3.355.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{1}{ax+bx^n} dx = \frac{\log(b+ax^{1-n})}{a(1-n)}$$

output
```
ln(b+a*x^(1-n))/a/(1-n)
```

3.355.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^n} dx = \frac{\log(b+ax^{1-n})}{a(1-n)}$$

input
```
Integrate[(a*x + b*x^n)^(-1),x]
```

output
```
Log[b + a*x^(1 - n)]/(a*(1 - n))
```

3.355.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^n}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^{-n}}{ax^{1-n}+b}dx$$

$$\Big\downarrow \text{792}$$

$$\frac{\log \left(ax^{1-n}+b\right)}{a(1-n)}$$

input
```
Int[(a*x + b*x^n)^(-1),x]
```

output
```
Log[b + a*x^(1 - n)]/(a*(1 - n))
```

3.355.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.355.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(ax+b\,x^n)}{a(-1+n)}$	27
risch	$\frac{n\ln(x)}{a(-1+n)}-\frac{\ln\left(x^n+\frac{ax}{b}\right)}{a(-1+n)}$	35
norman	$\frac{n\ln(x)}{a(-1+n)}-\frac{\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)}{a(-1+n)}$	36

input
```
int(1/(a*x+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(a*x+b*x^n))/a/(-1+n)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax+bx^n}\,dx=\frac{n\log(x)-\log(ax+bx^n)}{an-a}$$

input
```
integrate(1/(a*x+b*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

3.355.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{ax+bx^n}\,dx=\begin{cases}\tilde{\infty}\log(x) & \text{for } a=0\wedge b=0\wedge n=1\\ \frac{\log(x)}{a} & \text{for } b=0\\ -\frac{x}{b(nx^n-x^n)} & \text{for } a=0\\ \frac{\log(x)}{a+b} & \text{for } n=1\\ \frac{n\log(x)}{an-a}-\frac{\log\left(\frac{ax}{b}+x^n\right)}{an-a} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 1)), (log(x)/a, Eq(b, 0
)), (-x/(b*(n*x**n - x**n)), Eq(a, 0)), (log(x)/(a + b), Eq(n, 1)), (n*log
(x)/(a*n - a) - log(a*x/b + x**n)/(a*n - a), True))
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{1}{ax+bx^n}\,dx = \frac{n\log(x)}{a(n-1)} - \frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}$$

input
```
integrate(1/(a*x+b*x^n),x, algorithm="maxima")
```

output
```
n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))
```

3.355.8 Giac [F]

$$\int \frac{1}{ax+bx^n}\,dx = \int \frac{1}{ax+bx^n}\,dx$$

input
```
integrate(1/(a*x+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/(a*x + b*x^n), x)
```

3.355.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{ax+bx^n}\,dx = -\frac{\ln(b\,x^n + a\,x) - n\,\ln(x)}{a\,(n-1)}$$

input
```
int(1/(b*x^n + a*x),x)
```

output
```
-(log(b*x^n + a*x) - n*log(x))/(a*(n - 1))
```

3.356 $\int \frac{1}{ax+bx^{1+n}}\,dx$

3.356.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log(x)}{a}-\frac{\log(a+bx^n)}{an}$$

output
```
ln(x)/a-ln(a+b*x^n)/a/n
```

3.356.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log(x^n)-\log(an(a+bx^n))}{an}$$

input
```
Integrate[(a*x + b*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

3.356.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2027, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^{n+1}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{x(a+bx^n)}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{-n}}{bx^n+a}dx^n}{n}$$
$$\Big\downarrow \text{47}$$
$$\frac{\frac{\int x^{-n}dx^n}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$
$$\Big\downarrow \text{14}$$
$$\frac{\frac{\log(x^n)}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$
$$\Big\downarrow \text{16}$$
$$\frac{\frac{\log(x^n)}{a}-\frac{\log(a+bx^n)}{a}}{n}$$

input
```
Int[(a*x + b*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n]/a - Log[a + b*x^n]/a)/n
```

3.356.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.356.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
parallelrisch	$\frac{n\ln(x)+\ln(x)-\ln(ax+b\,x^{1+n})}{an}$	29
norman	$\frac{(1+n)\ln(x)}{an}-\frac{\ln(ax+b\,\mathrm{e}^{(1+n)\ln(x)})}{an}$	36
risch	$\frac{\ln(x)}{an}+\frac{\ln(x)}{a}-\frac{\ln(x^{1+n}+\frac{ax}{b})}{an}$	38

input
```
int(1/(a*x+b*x^(1+n)),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)+ln(x)-ln(a*x+b*x^(1+n)))/a/n
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{ax + bx^{1+n}}\,dx = \frac{(n+1)\log(x) - \log(ax + bx^{n+1})}{an}$$

input
```
integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")
```

output
```
((n + 1)*log(x) - log(a*x + b*x^(n + 1)))/(a*n)
```

3.356.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(15) = 30.

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \frac{1}{ax + bx^{1+n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for } a = 0 \wedge n = 0 \\ -\frac{xx^{-n-1}}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} + \frac{\log(x)}{an} - \frac{\log\left(x + \frac{bx^{n+1}}{a}\right)}{an} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b*x**(1+n)),x)
```

output
```
Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(b*n), Eq(a, 0)
), (log(x)/(a + b), Eq(n, 0)), (log(x)/a + log(x)/(a*n) - log(x + b*x**(n
+ 1)/a)/(a*n), True))
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax + bx^{1+n}}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n + a}{b}\right)}{an}$$

input
```
integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")
```

output
```
log(x)/a - log((b*x^n + a)/b)/(a*n)
```

3.356.8 Giac [F]

$$\int \frac{1}{ax + bx^{1+n}} dx = \int \frac{1}{ax + bx^{n+1}} dx$$

input
```
integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
```

output
```
integrate(1/(a*x + b*x^(n + 1)), x)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{ax + bx^{1+n}} dx = \frac{\ln(x)\ (n+1)}{a\, n} - \frac{\ln(x\,(a + b\, x^n))}{a\, n}$$

input
```
int(1/(a*x + b*x^(n + 1)),x)
```

output
```
(log(x)*(n + 1))/(a*n) - log(x*(a + b*x^n))/(a*n)
```

3.357 $\int \frac{1}{ax+bx^{1-n}}\,dx$

3.357.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\log\left(b+ax^n\right)}{an}$$

output
```
ln(b+a*x^n)/a/n
```

3.357.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\log\left(b+ax^n\right)}{an}$$

input
```
Integrate[(a*x + b*x^(1 - n))^(-1),x]
```

output
```
Log[b + a*x^n]/(a*n)
```

3.357.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^{1-n}}dx$$

$$\Big\downarrow \; 2027$$

$$\int \frac{x^{n-1}}{ax^n+b}dx$$

$$\Big\downarrow \; 792$$

$$\frac{\log(ax^n+b)}{an}$$

input
```
Int[(a*x + b*x^(1 - n))^(-1),x]
```

output
```
Log[b + a*x^n]/(a*n)
```

3.357.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.357.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(x)+\ln(ax+b\,x^{1-n})}{an}$	31
norman	$\frac{(-1+n)\ln(x)}{an}+\frac{\ln(ax+b\,\mathrm{e}^{(1-n)\ln(x)})}{an}$	37
risch	$-\frac{\ln(x)}{an}+\frac{\ln(x)}{a}+\frac{\ln(x^{1-n}+\frac{ax}{b})}{an}$	40

input
```
int(1/(a*x+b*x^(1-n)),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(x)+ln(a*x+b*x^(1-n)))/a/n
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{(n-1)\log(x)+\log(ax+bx^{-n+1})}{an}$$

input
```
integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")
```

output
```
((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

3.357.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(10) = 20.

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.40

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\begin{cases}\tilde{\infty}\log(x) & \text{for } a=0\wedge b=0\wedge n=0\\ \frac{xx^{n-1}}{bn} & \text{for } a=0\\ \frac{\log(x)}{a} & \text{for } b=0\\ \frac{\log(x)}{a+b} & \text{for } n=0\\ \frac{\log(x)}{a}-\frac{\log(x)}{an}+\frac{\log\left(\frac{ax}{b}+x^{1-n}\right)}{an} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x+b*x**(1-n)),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(b*n
), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a
- log(x)/(a*n) + log(a*x/b + x**(1 - n))/(a*n), True))
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1}{ax + bx^{1-n}}\,dx = \frac{\log\left(\frac{ax^n+b}{a}\right)}{an}$$

input
```
integrate(1/(a*x+b*x^(1-n)),x, algorithm="maxima")
```

output
```
log((a*x^n + b)/a)/(a*n)
```

3.357.8 Giac [F]

$$\int \frac{1}{ax + bx^{1-n}}\,dx = \int \frac{1}{ax + bx^{-n+1}}\,dx$$

input
```
integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
```

output
```
integrate(1/(a*x + b*x^(-n + 1)), x)
```

3.357.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{1}{ax + bx^{1-n}}\,dx = \frac{\ln\left(a\,x + b\,x^{1-n}\right)}{a\,n} + \frac{\ln(x)\,(n-1)}{a\,n}$$

input
```
int(1/(a*x + b*x^(1 - n)),x)
```

output
```
log(a*x + b*x^(1 - n))/(a*n) + (log(x)*(n - 1))/(a*n)
```

3.358 $\int \frac{1}{2x+3x^{1+n}}\,dx$

3.358.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{\log(x)}{2}-\frac{\log(2+3x^n)}{2n}$$

output
```
1/2*ln(x)-1/2*ln(2+3*x^n)/n
```

3.358.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{\log(x^n)-\log(n(2+3x^n))}{2n}$$

input
```
Integrate[(2*x + 3*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n] - Log[n*(2 + 3*x^n)])/(2*n)
```

3.358.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2027, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^{n+1}+2x}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{1}{x(3x^n+2)}dx$$
$$\Big\downarrow \text{ 798}$$
$$\frac{\int \frac{x^{-n}}{3x^n+2}dx^n}{n}$$
$$\Big\downarrow \text{ 47}$$
$$\frac{\frac{\int x^{-n}dx^n}{2}-\frac{3}{2}\int \frac{1}{3x^n+2}dx^n}{n}$$
$$\Big\downarrow \text{ 14}$$
$$\frac{\frac{\log(x^n)}{2}-\frac{3}{2}\int \frac{1}{3x^n+2}dx^n}{n}$$
$$\Big\downarrow \text{ 16}$$
$$\frac{\frac{\log(x^n)}{2}-\frac{1}{2}\log(3x^n+2)}{n}$$

input
```
Int[(2*x + 3*x^(1 + n))^(-1),x]
```

output
```
(Log[x^n]/2 - Log[2 + 3*x^n]/2)/n
```

3.358.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.358.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result	size
parallelrisch	$\frac{n\ln(x)+\ln(x)-\ln\left(x+\frac{3x^{1+n}}{2}\right)}{2n}$	25
meijerg	$\frac{n\ln(x)+\ln(3)-\ln(2)-\ln\left(1+\frac{3x^{n}}{2}\right)}{2n}$	27
risch	$\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}-\frac{\ln\left(\frac{2x}{3}+x^{1+n}\right)}{2n}$	28
norman	$\frac{(1+n)\ln(x)}{2n}-\frac{\ln\left(2x+3\,\mathrm{e}^{(1+n)\ln(x)}\right)}{2n}$	31

input
```
int(1/(2*x+3*x^(1+n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(n*ln(x)+ln(x)-ln(x+3/2*x^(1+n)))/n
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{(n+1)\log(x)-\log\left(3\,x^{n+1}+2\,x\right)}{2\,n}$$

input
```
integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")
```

output
```
1/2*((n + 1)*log(x) - log(3*x^(n + 1) + 2*x))/n
```

3.358.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\begin{cases}\frac{\log(x)}{2}+\frac{\log(x)}{2n}-\frac{\log(2x+3x^{n+1})}{2n} & \text{for } n\neq 0\\ \frac{\log(x)}{5} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(2*x+3*x**(1+n)),x)
```

output
```
Piecewise((log(x)/2 + log(x)/(2*n) - log(2*x + 3*x**(n + 1))/(2*n), Ne(n,
0)), (log(x)/5, True))
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{2x+3x^{1+n}}\,dx=-\frac{\log\left(x^n+\frac{2}{3}\right)}{2\,n}+\frac{1}{2}\log(x)$$

input
```
integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")
```

output
```
-1/2*log(x^n + 2/3)/n + 1/2*log(x)
```

3.358.8 Giac [F]

$$\int \frac{1}{2x + 3x^{1+n}}\,dx = \int \frac{1}{3\,x^{n+1} + 2\,x}\,dx$$

input
```
integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")
```

output
```
integrate(1/(3*x^(n + 1) + 2*x), x)
```

3.358.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{2x + 3x^{1+n}}\,dx = \frac{\ln(x)\,(n+1)}{2\,n} - \frac{\ln\left(\frac{2\,x}{3} + x^{n+1}\right)}{2\,n}$$

input
```
int(1/(2*x + 3*x^(n + 1)),x)
```

output
```
(log(x)*(n + 1))/(2*n) - log((2*x)/3 + x^(n + 1))/(2*n)
```

3.359 $\int \frac{1}{2x+3x^{1-n}}\,dx$

3.359.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{2x+3x^{1-n}}\,dx=\frac{\log(3+2x^n)}{2n}$$

output
```
1/2*ln(3+2*x^n)/n
```

3.359.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{2x+3x^{1-n}}\,dx=\frac{\log(3+2x^n)}{2n}$$

input
```
Integrate[(2*x + 3*x^(1 - n))^(-1),x]
```

output
```
Log[3 + 2*x^n]/(2*n)
```

3.359.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^{1-n}+2x} \, dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x^{n-1}}{2x^n+3}dx$$
$$\Big\downarrow \text{792}$$
$$\frac{\log\left(2x^n+3\right)}{2n}$$

input
```
Int[(2*x + 3*x^(1 - n))^(-1),x]
```

output
```
Log[3 + 2*x^n]/(2*n)
```

3.359.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.359.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(x)+\ln\left(x+\frac{3x^{1-n}}{2}\right)}{2n}$	27
meijerg	$-\frac{-n\ln(x)+\ln(3)-\ln(2)-\ln\left(1+\frac{3x^{-n}}{2}\right)}{2n}$	30
risch	$-\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}+\frac{\ln\left(\frac{2x}{3}+x^{1-n}\right)}{2n}$	30
norman	$\frac{(-1+n)\ln(x)}{2n}+\frac{\ln\left(2x+3\,\mathrm{e}^{(1-n)\ln(x)}\right)}{2n}$	33

input
```
int(1/(2*x+3*x^(1-n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(n*ln(x)-ln(x)+ln(x+3/2*x^(1-n)))/n
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \frac{(n-1)\log(x)+\log\left(3\,x^{-n+1}+2\,x\right)}{2\,n}$$

input
```
integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")
```

output
```
1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n
```

3.359.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \begin{cases} \frac{\log(x)}{2}-\frac{\log(x)}{2n}+\frac{\log(2x+3x^{1-n})}{2n} & \text{for}\ n\neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*x+3*x**(1-n)),x)
```

output
```
Piecewise((log(x)/2 - log(x)/(2*n) + log(2*x + 3*x**(1 - n))/(2*n), Ne(n,
0)), (log(x)/5, True))
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \frac{\log\left(x^n+\frac{3}{2}\right)}{2\,n}$$

input
```
integrate(1/(2*x+3*x^(1-n)),x, algorithm="maxima")
```

output
```
1/2*log(x^n + 3/2)/n
```

3.359.8 Giac [F]

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \int \frac{1}{3\,x^{-n+1}+2\,x}\,dx$$

input
```
integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")
```

output
```
integrate(1/(3*x^(-n + 1) + 2*x), x)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \frac{\ln\left(\frac{2\,x}{3}+x^{1-n}\right)}{2\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{2\,n}$$

input
```
int(1/(2*x + 3*x^(1 - n)),x)
```

output
```
log((2*x)/3 + x^(1 - n))/(2*n) + (log(x)*(n - 1))/(2*n)
```

3.360 $\int \frac{1}{-\sqrt{x}+x} \, dx$

3.360.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{-\sqrt{x}+x} \, dx = 2\log\left(1-\sqrt{x}\right)$$

output
```
2*ln(1-x^(1/2))
```

3.360.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{-\sqrt{x}+x} \, dx = 2\log\left(-1+\sqrt{x}\right)$$

input
```
Integrate[(-Sqrt[x] + x)^(-1),x]
```

output
```
2*Log[-1 + Sqrt[x]]
```

3.360.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{x}}dx$$
$$\Big\downarrow \ 2027$$
$$\int \frac{1}{\left(\sqrt{x}-1\right)\sqrt{x}}dx$$
$$\Big\downarrow \ 792$$
$$2\log\left(1-\sqrt{x}\right)$$

input
```
Int[(-Sqrt[x] + x)^(-1),x]
```

output
```
2*Log[1 - Sqrt[x]]
```

3.360.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.360.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$2\ln\left(\sqrt{x}-1\right)$	9
meijerg	$2\ln\left(1-\sqrt{x}\right)$	11
default	$\ln\left(-1+x\right)-2\,\operatorname{arctanh}\left(\sqrt{x}\right)$	12
trager	$\ln\left(2\sqrt{x}-1-x\right)$	12

input
```
int(1/(x-x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x^(1/2)-1)
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(x-x^(1/2)),x, algorithm="fricas")
```

output
```
2*log(sqrt(x) - 1)
```

3.360.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(x-x**(1/2)),x)
```

output
```
2*log(sqrt(x) - 1)
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(x-x^(1/2)),x, algorithm="maxima")
```

output
```
2*log(sqrt(x) - 1)
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate(1/(x-x^(1/2)),x, algorithm="giac")
```

output
```
2*log(abs(sqrt(x) - 1))
```

3.360.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\ln\left(\sqrt{x}-1\right)$$

input
```
int(1/(x - x^(1/2)),x)
```

output
```
2*log(x^(1/2) - 1)
```

3.361 $\int \frac{1}{-x^{3/5}+x} dx$

3.361.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{-x^{3/5}+x} dx = \frac{5}{2}\log\left(1-x^{2/5}\right)$$

output
```
5/2*ln(1-x^(2/5))
```

3.361.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{-x^{3/5}+x} dx = \frac{5}{2}\log\left(-1+\sqrt[5]{x}\right)+\frac{5}{2}\log\left(1+\sqrt[5]{x}\right)$$

input
```
Integrate[(-x^(3/5) + x)^(-1),x]
```

output
```
(5*Log[-1 + x^(1/5)])/2 + (5*Log[1 + x^(1/5)])/2
```

3.361.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-x^{3/5}}\,dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{\left(x^{2/5}-1\right)x^{3/5}}dx$$
$$\Big\downarrow \text{792}$$
$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

input
```
Int[(-x^(3/5) + x)^(-1),x]
```

output
```
(5*Log[1 - x^(2/5)])/2
```

3.361.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.361.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result
meijerg	$\frac{5 \ln\left(1-x^{\frac{2}{5}}\right)}{2}$
derivativedivides	$\frac{5 \ln\left(x^{\frac{1}{5}}-1\right)}{2}+\frac{5 \ln\left(1+x^{\frac{1}{5}}\right)}{2}$
trager	$\frac{\ln\left(-10 x^{\frac{4}{5}}-5 x^{\frac{8}{5}}+5 x^{\frac{2}{5}}+10 x^{\frac{6}{5}}+x^{2}-1\right)}{2}$
default	$2 \ln\left(1+x^{\frac{1}{5}}\right)+\frac{\left(-\sqrt{5}-1\right) \ln\left(\sqrt{5}\, x^{\frac{1}{5}}+2 x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{4}-\frac{\left(-\sqrt{5}+1\right) \ln\left(-\sqrt{5}\, x^{\frac{1}{5}}+2 x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{4}-\frac{\left(-\sqrt{5}+1\right)}{}$

input
```
int(1/(-x^(3/5)+x),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(1-x^(2/5))
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\log\left(x^{\frac{2}{5}}-1\right)$$

input
```
integrate(1/(-x^(3/5)+x),x, algorithm="fricas")
```

output
```
5/2*log(x^(2/5) - 1)
```

3.361.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5\log\left(\sqrt[5]{x}-1\right)}{2}+\frac{5\log\left(\sqrt[5]{x}+1\right)}{2}$$

input
```
integrate(1/(-x**(3/5)+x),x)
```

output
```
5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\,\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\,\log\left(x^{\frac{1}{5}}-1\right)$$

input
```
integrate(1/(-x^(3/5)+x),x, algorithm="maxima")
```

output
```
5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\,\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\,\log\left(\left|x^{\frac{1}{5}}-1\right|\right)$$

input
```
integrate(1/(-x^(3/5)+x),x, algorithm="giac")
```

output
```
5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))
```

3.361.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5\,\ln\left(x^{2/5}-1\right)}{2}$$

input
```
int(1/(x - x^(3/5)),x)
```

output
```
(5*log(x^(2/5) - 1))/2
```

3.362 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx$

3.362.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx=\frac{3}{4} \log \left(1+x^{4/3}\right)$$

output
```
3/4*ln(1+x^(4/3))
```

3.362.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx=\frac{3}{4} \log \left(1+x^{4/3}\right)$$

input
```
Integrate[(x^(-1/3) + x)^(-1),x]
```

output
```
(3*Log[1 + x^(4/3)])/4
```

3.362.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x+\frac{1}{\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{\sqrt[3]{x}}{x^{4/3}+1}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

input
```
Int[(x^(-1/3) + x)^(-1),x]
```

output
```
(3*Log[1 + x^(4/3)])/4
```

3.362.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.362.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
default	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
trager	$-\frac{\ln\left(\frac{3x^{\frac{20}{3}}-x^{8}-6x^{\frac{16}{3}}-6x^{\frac{8}{3}}+7x^{4}+3x^{\frac{4}{3}}-1}{\left(x^{4}+1\right)^{3}}\right)}{4}$	44

input
```
int(1/(1/x^(1/3)+x),x,method=_RETURNVERBOSE)
```

output
```
3/4*ln(1+x^(4/3))
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

input
```
integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
```

output
```
3/4*log(x^(4/3) + 1)
```

3.362.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

input
```
integrate(1/(1/x**(1/3)+x),x)
```

output
```
3*log(x**(4/3) + 1)/4
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

input
```
integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
```

output
```
3/4*log(x^(4/3) + 1)
```

3.362.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(8) = 16.

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3}{4}\log\left(\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{3}{4}\log\left(-\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

output
```
3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3
) + 1)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx=\frac{3\,\ln\left(x^{4/3}+1\right)}{4}$$

input
```
int(1/(x + 1/x^(1/3)),x)
```

output
```
(3*log(x^(4/3) + 1))/4
```

3.363 $\int \frac{1}{x+x^{\sqrt{2}}}\,dx$

3.363.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)$$

output
```
ln(x)-ln(1+x^(2^(1/2)-1))*(1+2^(1/2))
```

3.363.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)$$

input
```
Integrate[(x + x^Sqrt[2])^(-1),x]
```

output
```
Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

3.363.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2027, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{\sqrt{2}}+x}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{1}{x\left(x^{\sqrt{2}-1}+1\right)}dx$$
$$\Big\downarrow \text{ 798}$$
$$\left(1+\sqrt{2}\right)\int \frac{x^{1-\sqrt{2}}}{x^{-1+\sqrt{2}}+1}dx^{-1+\sqrt{2}}$$
$$\Big\downarrow \text{ 47}$$
$$\left(1+\sqrt{2}\right)\left(\int x^{1-\sqrt{2}}dx^{-1+\sqrt{2}}-\int \frac{1}{x^{-1+\sqrt{2}}+1}dx^{-1+\sqrt{2}}\right)$$
$$\Big\downarrow \text{ 14}$$
$$\left(1+\sqrt{2}\right)\left(\log\left(x^{\sqrt{2}-1}\right)-\int \frac{1}{x^{-1+\sqrt{2}}+1}dx^{-1+\sqrt{2}}\right)$$
$$\Big\downarrow \text{ 16}$$
$$\left(1+\sqrt{2}\right)\left(\log\left(x^{\sqrt{2}-1}\right)-\log\left(x^{\sqrt{2}-1}+1\right)\right)$$

input
```
Int[(x + x^Sqrt[2])^(-1),x]
```

output
```
(1 + Sqrt[2])*(Log[x^(-1 + Sqrt[2])] - Log[1 + x^(-1 + Sqrt[2])])
```

3.363.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.363.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
norman	$\left(2+\sqrt{2}\right) \ln \left(x \right)+\left(-\sqrt{2}-1\right) \ln \left(x +{\mathrm e}^{\sqrt{2}\, \ln \left(x \right)}\right)$	28
meijerg	$\frac{\left(\sqrt{2}-1\right) \ln \left(x \right)-\ln \left(1+x^{\sqrt{2}-1}\right)}{\sqrt{2}-1}$	30
risch	$2 \ln \left(x \right)+\sqrt{2}\, \ln \left(x \right)-\ln \left(x +x^{\sqrt{2}}\right) \sqrt{2}-\ln \left(x +x^{\sqrt{2}}\right)$	35
parallelrisch	$2 \ln \left(x \right)+\sqrt{2}\, \ln \left(x \right)-\ln \left(x +x^{\sqrt{2}}\right) \sqrt{2}-\ln \left(x +x^{\sqrt{2}}\right)$	35

input
```
int(1/(x+x^(2^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
(2+2^(1/2))*ln(x)+(-2^(1/2)-1)*ln(x+exp(2^(1/2)*ln(x)))
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=-\left(\sqrt{2}+1\right)\log\left(x+x^{\left(\sqrt{2}\right)}\right)+\left(\sqrt{2}+2\right)\log\left(x\right)$$

input
```
integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")
```

output
```
-(sqrt(2) + 1)*log(x + x^sqrt(2)) + (sqrt(2) + 2)*log(x)
```

3.363.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\frac{\sqrt{2}\log\left(x\right)}{-1+\sqrt{2}}-\frac{\log\left(x+x^{\sqrt{2}}\right)}{-1+\sqrt{2}}$$

input
```
integrate(1/(x+x**(2**(1/2))),x)
```

output
```
sqrt(2)*log(x)/(-1 + sqrt(2)) - log(x + x**(sqrt(2)))/(-1 + sqrt(2))
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\frac{\sqrt{2}\log\left(x\right)}{\sqrt{2}-1}-\frac{\log\left(x+x^{\left(\sqrt{2}\right)}\right)}{\sqrt{2}-1}$$

input
```
integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")
```

output
```
sqrt(2)*log(x)/(sqrt(2) - 1) - log(x + x^sqrt(2))/(sqrt(2) - 1)
```

3.363.8 Giac [F]

$$\int \frac{1}{x + x^{\sqrt{2}}}\,dx = \int \frac{1}{x + x^{\left(\sqrt{2}\right)}}\,dx$$

input
```
integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")
```

output
```
integrate(1/(x + x^sqrt(2)), x)
```

3.363.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x + x^{\sqrt{2}}}\,dx = \ln(x)\,\left(\sqrt{2} + 2\right) - \frac{\ln\left(x + x^{\sqrt{2}}\right)}{\sqrt{2} - 1}$$

input
```
int(1/(x + x^(2^(1/2))),x)
```

output
```
log(x)*(2^(1/2) + 2) - log(x + x^(2^(1/2)))/(2^(1/2) - 1)
```

3.364 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

3.364.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}$$

output
```
2*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/(j-n)-2*(a*x^j+b*
x^n)^(1/2)/(j-n)/(x^(1/2*j))
```

3.364.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=-\frac{2x^{-j/2}\left(ax^j+bx^n-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,\mathrm{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/
b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/((j - n)*x^(j/2)*Sqrt[a*x^j
+ b*x^n])
```

3.364.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

$$\Big\downarrow \ 1934$$

$$a\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

$$\Big\downarrow \ 1935$$

$$\frac{2a\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

input
```
Int[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x
^(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

3.364.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.364.4 Maple [F]

$$\int x^{-1-\frac{j}{2}} \sqrt{a\,x^{j}+b\,x^{n}}dx$$

input
```
int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

3.364.5 Fricas [F(-2)]

Exception generated.

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.364.6 Sympy [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

input
```
integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral(x**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

3.364.7 Maxima [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}x^{-\frac{1}{2}j-1}\,dx$$

input
```
integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

3.364.8 Giac [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}x^{-\frac{1}{2}j-1}\,dx$$

input
```
integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \frac{\sqrt{a\,x^j+b\,x^n}}{x^{\frac{j}{2}+1}}\,dx$$

input
```
int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1),x)
```

output
```
int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1), x)
```

3.365 $\int (cx)^{-1-\frac{j}{2}} \sqrt{ax^j+bx^n} \, dx$

3.365.1 Optimal result

Integrand size = 27, antiderivative size = 99

$$\int (cx)^{-1-\frac{j}{2}} \sqrt{ax^j+bx^n} \, dx = -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)} + \frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

output
```
2*x^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/c/(j-n)
/((c*x)^(1/2*j))-2*(a*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(1/2*j))
```

3.365.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int (cx)^{-1-\frac{j}{2}} \sqrt{ax^j+bx^n} \, dx = -\frac{2(cx)^{-j/2}\left(ax^j+bx^n-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/
b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(c*(j - n)*(c*x)^(j/2)*Sqrt[
a*x^j + b*x^n])
```

3.365.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1937, 1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

$$\Big\downarrow \; 1937$$

$$\frac{x^{j/2}(cx)^{-j/2}\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}dx}{c}$$

$$\Big\downarrow \; 1934$$

$$\frac{x^{j/2}(cx)^{-j/2}\left(a\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)}{c}$$

$$\Big\downarrow \; 1935$$

$$\frac{x^{j/2}(cx)^{-j/2}\left(\frac{2a\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)}{c}$$

$$\Big\downarrow \; 219$$

$$\frac{x^{j/2}(cx)^{-j/2}\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)}{c}$$

input
```
Int[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

output
```
(x^(j/2)*((-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)))/(c*(c*x)^(j/2))
```

3.365.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.365.4 Maple [F]

$$\int (cx)^{-1-\frac{j}{2}} \sqrt{a\,x^{j}+b\,x^{n}}dx$$

input
```
int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

3.365.5 Fricas [F(-2)]

Exception generated.

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.365.6 Sympy [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

input
```
integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

3.365.7 Maxima [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int \sqrt{ax^j+bx^n}(cx)^{-\frac{1}{2}j-1}\,dx$$

input
```
integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

3.365.8 Giac [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^{-\frac{1}{2}j-1}\,dx$$

input
```
integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \frac{\sqrt{a\,x^j+b\,x^n}}{(c\,x)^{\frac{j}{2}+1}}\,dx$$

input
```
int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1),x)
```

output
```
int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1), x)
```

3.366 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$

3.366.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\sqrt{cx}\,\mathrm{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$

output
```
2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*a^(1/2)*(c*x)^(1/2)/c^3/(3-
n)/x^(1/2)-2*(a*x^3+b*x^n)^(1/2)/c/(3-n)/(c*x)^(3/2)
```

3.366.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx = \frac{2x\left(ax^3+bx^n-\sqrt{a}\sqrt{b}x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\,\mathrm{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-3+n)(cx)^{5/2}\sqrt{ax^3+bx^n}}$$

input
```
Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

output
```
(2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))
/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/((-3 + n)*(c*x)^(5/2)*Sqrt[
a*x^3 + b*x^n])
```

3.366.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1934}$$

$$\frac{a\int \frac{\sqrt{cx}}{\sqrt{bx^n+ax^3}}dx}{c^3}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{c^3\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2a\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{a}\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

input
```
Int[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

output
```
(-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Ar
cTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

3.366.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.366.4 Maple [F]

$$\int \frac{\sqrt{a\,x^{3}+b\,x^{n}}}{\left(c x \right)^{\frac{5}{2}}}d x$$

input
```
int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

output
```
int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

3.366.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.366.6 Sympy [F]

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2),x)
```

output
```
Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)
```

3.366.7 Maxima [F]

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

3.366.8 Giac [F]

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^n + a\,x^3}}{(c\,x)^{5/2}}\,dx$$

input
```
int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2),x)
```

output
```
int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2), x)
```

3.367 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$

3.367.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$

output
```
2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))*a^(1/2)/c^2/(2-n)-2*(a*x^2+b*x^n)
^(1/2)/c^2/(2-n)/x
```

3.367.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=\frac{2\left(ax^2+bx^n-\sqrt{a}\sqrt{b}x^{1+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{c^2(-2+n)x\sqrt{ax^2+bx^n}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

output
```
(2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*
ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^
n])
```

3.367.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {27, 1934, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{bx^n+ax^2}}{x^2}dx}{c^2}$$

$$\Big\downarrow \; 1934$$

$$\frac{a\int \frac{1}{\sqrt{bx^n+ax^2}}dx-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}}{c^2}$$

$$\Big\downarrow \; 1914$$

$$\frac{\frac{2a\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}}{c^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}}{c^2}$$

input
```
Int[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

output
```
((-2*Sqrt[a*x^2 + b*x^n])/((2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(2 - n))/c^2
```

3.367.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

3.367.4 Maple [F]

$$\int \frac{\sqrt{a\,x^{2}+b\,x^{n}}}{c^{2}x^{2}}dx$$

input
```
int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

output
```
int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

3.367.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.367.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}$$

input
```
integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)
```

output
```
Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2
```

3.367.7 Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2
```

3.367.8 Giac [F]

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^n}}{c^2x^2} \, dx = \int \frac{\sqrt{b\,x^n + a\,x^2}}{c^2\,x^2} \, dx$$

input
```
int((b*x^n + a*x^2)^(1/2)/(c^2*x^2),x)
```

output
```
int((b*x^n + a*x^2)^(1/2)/(c^2*x^2), x)
```

3.368 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$

3.368.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{2\sqrt{a}\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}$$

output
```
2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/c/(1-n)/(c*x)
^(1/2)-2*(a*x+b*x^n)^(1/2)/c/(1-n)/(c*x)^(1/2)
```

3.368.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \frac{x\left(2ax+2bx^n-2\sqrt{a}\sqrt{b}x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-1+n)(cx)^{3/2}\sqrt{ax+bx^n}}$$

input
```
Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

output
```
(x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n
))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/((-1 + n)*(c*x)^(3/2)*Sqr
t[a*x + b*x^n])
```

3.368.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}dx$$

$$\Big\downarrow \text{ 1934}$$

$$\frac{a\int \frac{1}{\sqrt{cx}\sqrt{bx^n+ax}}dx}{c}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{a\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{c\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2a\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

input
```
Int[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

output
```
(-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[
(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

3.368.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.368.4 Maple [F]

$$\int \frac{\sqrt{ax+b\,x^{n}}}{(cx)^{\frac{3}{2}}}dx$$

input
```
int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

output
```
int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

3.368.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.368.6 Sympy [F]

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2),x)
```

output
```
Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)
```

3.368.7 Maxima [F]

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

3.368.8 Giac [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^n+a\,x}}{(c\,x)^{3/2}}\,dx$$

input
```
int((b*x^n + a*x)^(1/2)/(c*x)^(3/2),x)
```

output
```
int((b*x^n + a*x)^(1/2)/(c*x)^(3/2), x)
```

3.369 $\int \frac{\sqrt{a+bx^n}}{cx} dx$

3.369.1 Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{\sqrt{a+bx^n}}{cx} dx = \frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))*a^(1/2)/c/n+2*(a+b*x^n)^(1/2)/c/n
```

3.369.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^n}}{cx} dx = \frac{2\left(\sqrt{a+bx^n} - \sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{cn}$$

input
```
Integrate[Sqrt[a + b*x^n]/(c*x),x]
```

output
```
(2*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]]))/(c*n)
```

3.369.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {27, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{cx} \, dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\sqrt{bx^n+a}}{x}dx}{c}$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-n}\sqrt{bx^n+a}dx^n}{cn}$$
$$\Big\downarrow \text{60}$$
$$\frac{a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}}{cn}$$
$$\Big\downarrow \text{73}$$
$$\frac{\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}}{cn}$$
$$\Big\downarrow \text{221}$$
$$\frac{2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

input
```
Int[Sqrt[a + b*x^n]/(c*x),x]
```

output
```
(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

3.369.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.369.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
default	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
risch	$\frac{2\sqrt{a+b\,{\rm e}^{n\ln(x)}}}{nc}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\rm e}^{n\ln(x)}}}{\sqrt{a}}\right)}{nc}$	48

input
```
int((a+b*x^n)^(1/2)/c/x,x,method=_RETURNVERBOSE)
```

output
```
1/c/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}}{cn}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)+\sqrt{bx^n+a}\right)}{cn}\right]$$

input
```
integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^
n + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n
 + a))/(c*n)]
```

3.369.6 Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \frac{-\frac{2\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}}{c}$$

input
```
integrate((a+b*x**n)**(1/2)/c/x,x)
```

output
```
(-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/n + 2*a/(sqrt(b)*n*x**(n/2)*
sqrt(a/(b*x**n) + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a/(b*x**n) + 1)))/c
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \frac{\frac{\sqrt{a}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\sqrt{bx^n+a}}{n}}{c}$$

input
```
integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="maxima")
```

output
```
(sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n +
2*sqrt(b*x^n + a)/n)/c
```

3.369.8 Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \int \frac{\sqrt{bx^n+a}}{cx}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/(c*x), x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \int \frac{\sqrt{a+b\,x^n}}{c\,x}\,dx$$

input
```
int((a + b*x^n)^(1/2)/(c*x),x)
```

output
```
int((a + b*x^n)^(1/2)/(c*x), x)
```

3.370 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$

3.370.1 Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{2\sqrt{a}\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

output
```
-2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(1+n)/(c*x)^
(1/2)+2*(c*x)^(1/2)*(a/x+b*x^n)^(1/2)/c/(1+n)
```

3.370.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{2x\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}-\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{(1+n)\sqrt{cx}\sqrt{a+bx^{1+n}}}$$

input
```
Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

output
```
(2*x*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)] - Sqrt[a]*ArcTanh[Sqrt[a + b
*x^(1 + n)]/Sqrt[a]]))/((1 + n)*Sqrt[c*x]*Sqrt[a + b*x^(1 + n)])
```

3.370.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{1934}$$

$$ac\int \frac{1}{(cx)^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{\sqrt{cx}}+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2a\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{(n+1)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

input
```
Int[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

output
```
(2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[S
qrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])
```

3.370.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.370.4 Maple [F]

$$\int \frac{\sqrt{\frac{a}{x}+b\,x^{n}}}{\sqrt{c x}}d x$$

input
```
int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

output
```
int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

3.370.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.370.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$$

input
```
integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)
```

output
```
Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)
```

3.370.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

input
```
integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

3.370.8 Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

input
```
integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{b\,x^n+\frac{a}{x}}}{\sqrt{c\,x}}\,dx$$

input
```
int((b*x^n + a/x)^(1/2)/(c*x)^(1/2),x)
```

output
```
int((b*x^n + a/x)^(1/2)/(c*x)^(1/2), x)
```

3.371 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$

3.371.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

output
```
-2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))*a^(1/2)/(2+n)+2*x*(a/x^2+b*x^n)^
(1/2)/(2+n)
```

3.371.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{(2+n)\sqrt{a+bx^{2+n}}}$$

input
```
Integrate[Sqrt[a/x^2 + b*x^n],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)] - Sqrt[a]*ArcTanh[Sqrt[a +
 b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

3.371.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$$

$$\Big\downarrow \text{1913}$$

$$a\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx+\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2a\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{n+2}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

input
```
Int[Sqrt[a/x^2 + b*x^n],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^n])])/(2 + n)
```

3.371.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.371.4 Maple [F]

$$\int \sqrt{\frac{a}{x^2} + b\,x^n}dx$$

input
```
int((a/x^2+b*x^n)^(1/2),x)
```

output
```
int((a/x^2+b*x^n)^(1/2),x)
```

3.371.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\frac{a}{x^2} + bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.371.6 Sympy [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$$

input
```
integrate((a/x**2+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(a/x**2 + b*x**n), x)
```

3.371.7 Maxima [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^2}}\,dx$$

input
```
integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a/x^2), x)
```

3.371.8 Giac [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^2}}\,dx$$

input
```
integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a/x^2), x)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{x\,\sqrt{b\,x^n+\frac{a}{x^2}}}{\frac{n}{2}+1}+\frac{\sqrt{a}\,x\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}}\right)\,\sqrt{b\,x^n+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}\,\left(\frac{n}{2}+1\right)\,\sqrt{\frac{a}{b\,x^{n+2}}+1}}$$

input
```
int((b*x^n + a/x^2)^(1/2),x)
```

output
```
(x*(b*x^n + a/x^2)^(1/2))/(n/2 + 1) + (a^(1/2)*x*asin((a^(1/2)*1i)/(b^(1/2
)*x^(n/2 + 1)))*(b*x^n + a/x^2)^(1/2)*1i)/(b^(1/2)*x^(n/2 + 1)*(n/2 + 1)*(
a/(b*x^(n + 2)) + 1)^(1/2))
```

3.372 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$

3.372.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)}-\frac{2\sqrt{a}c\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

output
```
-2*c*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(3+n)/(c
*x)^(1/2)+2*(c*x)^(3/2)*(a/x^3+b*x^n)^(1/2)/c/(3+n)
```

3.372.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\frac{2x\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{(3+n)\sqrt{a+bx^{3+n}}}$$

input
```
Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)] - Sqrt[a]*ArcTan
h[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/((3 + n)*Sqrt[a + b*x^(3 + n)])
```

3.372.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

$$\Big\downarrow \text{1934}$$

$$ac^3\int \frac{1}{(cx)^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}$$

$$\Big\downarrow \text{1937}$$

$$\frac{ac\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{\sqrt{cx}}+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2ac\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{(n+3)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}c\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

input
```
Int[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

output
```
(2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*Arc
Tanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

3.372.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.372.4 Maple [F]

$$\int \sqrt{cx}\, \sqrt{\frac{a}{x^{3}}+b\,x^{n}}dx$$

input
```
int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

3.372.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.372.6 Sympy [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx = \int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

input
```
integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)
```

3.372.7 Maxima [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx = \int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

3.372.8 Giac [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{c\,x}\,\sqrt{b\,x^n+\frac{a}{x^3}}\,dx$$

input
```
int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2),x)
```

output
```
int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2), x)
```

3.373 $\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx$

3.373.1 Optimal result

Integrand size = 27, antiderivative size = 141

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

output
```
-2/3*(a*x^j+b*x^n)^(3/2)/c/(j-n)/((c*x)^(3/2*j))+2*a^(3/2)*x^(3/2*j)*arcta
nh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/((c*x)^(3/2*j))-2*a*x^j*
(a*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(3/2*j))
```

3.373.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = -\frac{2(cx)^{-3j/2}\left(4a^2x^{2j}+b^2x^{2n}+5abx^{j+n}-3a^{3/2}\sqrt{b}x^{\frac{1}{2}(3j+n)}\sqrt{1+\frac{ax^{j-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{3c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(-2*(4*a^2*x^(2*j) + b^2*x^(2*n) + 5*a*b*x^(j + n) - 3*a^(3/2)*Sqrt[b]*x^(
(3*j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqr
t[b]]))/(3*c*(j - n)*(c*x)^((3*j)/2)*Sqrt[a*x^j + b*x^n])
```

3.373.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1937, 1934, 1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-\frac{3j}{2}-1}\left(ax^j+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\int x^{-\frac{3j}{2}-1}\left(ax^j+bx^n\right)^{3/2}dx}{c}$$

$$\Big\downarrow \text{1934}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}dx-\frac{2x^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3(j-n)}\right)}{c}$$

$$\Big\downarrow \text{1934}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\left(a\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)-\frac{2x^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3(j-n)}\right)}{c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\left(\frac{2a\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)-\frac{2x^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3(j-n)}\right)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{3j/2}(cx)^{-3j/2}\left(a\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}\right)-\frac{2x^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3(j-n)}\right)}{c}$$

input
```
Int[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(x^((3*j)/2)*((-2*(a*x^j + b*x^n)^(3/2))/(3*(j - n)*x^((3*j)/2)) + a*((-2*
Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^(j/
2))/Sqrt[a*x^j + b*x^n]])/(j - n))))/(c*(c*x)^((3*j)/2))
```

3.373.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.373.4 Maple [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}dx$$

input
```
int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

3.373.5 Fricas [F(-2)]

Exception generated.

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.373.6 Sympy [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = \int (cx)^{-\frac{3j}{2}-1} \left(ax^j+bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**(-3*j/2 - 1)*(a*x**j + b*x**n)**(3/2), x)
```

3.373.7 Maxima [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} dx$$

input
```
integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

3.373.8 Giac [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}\,j-1} \, dx$$

input
```
integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = \int \frac{\left(a\,x^j+b\,x^n\right)^{3/2}}{(c\,x)^{\frac{3\,j}{2}+1}} \, dx$$

input
```
int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1),x)
```

output
```
int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1), x)
```

3.374 $\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}} dx$

3.374.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}} dx = -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{2a^{3/2}\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$

output
```
-2/3*(a*x^3+b*x^n)^(3/2)/c/(3-n)/(c*x)^(9/2)+2*a^(3/2)*arctanh(x^(3/2)*a^(
1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/c^6/(3-n)/x^(1/2)-2*a*(a*x^3+b*x^n)^
(1/2)/c^4/(3-n)/(c*x)^(3/2)
```

3.374.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}} dx = \frac{2\sqrt{cx}\left(4a^2x^6+b^2x^{2n}+5abx^{3+n}-3a^{3/2}\sqrt{b}x^{\frac{9+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^6(-3+n)x^5\sqrt{ax^3+bx^n}}$$

input
```
Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

output
```
(2*Sqrt[c*x]*(4*a^2*x^6 + b^2*x^(2*n) + 5*a*b*x^(3 + n) - 3*a^(3/2)*Sqrt[b
]*x^((9 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/
Sqrt[b]]))/(3*c^6*(-3 + n)*x^5*Sqrt[a*x^3 + b*x^n])
```

3.374.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}dx$$

$$\Big\downarrow \text{ 1934}$$

$$\frac{a\int \frac{\sqrt{bx^n+ax^3}}{(cx)^{5/2}}dx}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{ 1934}$$

$$\frac{a\left(\frac{a\int \frac{\sqrt{cx}}{\sqrt{bx^n+ax^3}}dx}{c^3}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{a\left(\frac{a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{c^3\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{a\left(\frac{2a\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{a\left(\frac{2\sqrt{a}\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}\right)}{c^3}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

input
```
Int[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

output
```
(-2*(a*x^3 + b*x^n)^(3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (a*((-2*Sqrt[a*x^3
+ b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*ArcTanh[(Sqrt[a]*
x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])))/c^3
```

3.374.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.374.4 Maple [F]

$$\int \frac{(a\,x^{3}+b\,x^{n})^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}dx$$

input
```
int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

output
```
int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

3.374.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.374.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}}\, dx = \text{Timed out}$$

input
```
integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2),x)
```

output
```
Timed out
```

3.374.7 Maxima [F]

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}}\, dx = \int \frac{(ax^3 + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\, dx$$

input
```
integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

3.374.8 Giac [F]

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(ax^3 + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

output
```
integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^3 + bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(b\,x^n + a\,x^3)^{3/2}}{(c\,x)^{11/2}}\,dx$$

input
```
int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2),x)
```

output
```
int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2), x)
```

3.375 $\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx$

3.375.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}+\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}$$

output
```
-2/3*(a*x^2+b*x^n)^(3/2)/c^4/(2-n)/x^3+2*a^(3/2)*arctanh(x*a^(1/2)/(a*x^2+
b*x^n)^(1/2))/c^4/(2-n)-2*a*(a*x^2+b*x^n)^(1/2)/c^4/(2-n)/x
```

3.375.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \frac{2\left(4a^2x^4+b^2x^{2n}+5abx^{2+n}-3a^{3/2}\sqrt{b}x^{3+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^4(-2+n)x^3\sqrt{ax^2+bx^n}}$$

input
```
Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

output
```
(2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n
/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(3*
c^4*(-2 + n)*x^3*Sqrt[a*x^2 + b*x^n])
```

3.375.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {27, 1934, 1934, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4} \, dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(bx^n+ax^2\right)^{3/2}}{x^4}dx}{c^4}$$

$$\Big\downarrow \; 1934$$

$$\frac{a\int \frac{\sqrt{bx^n+ax^2}}{x^2}dx-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3(2-n)x^3}}{c^4}$$

$$\Big\downarrow \; 1934$$

$$\frac{a\left(a\int \frac{1}{\sqrt{bx^n+ax^2}}dx-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}\right)-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3(2-n)x^3}}{c^4}$$

$$\Big\downarrow \; 1914$$

$$\frac{a\left(\frac{2a\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}\right)-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3(2-n)x^3}}{c^4}$$

$$\Big\downarrow \; 219$$

$$\frac{a\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{2-n}-\frac{2\sqrt{ax^2+bx^n}}{(2-n)x}\right)-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3(2-n)x^3}}{c^4}$$

input
```
Int[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

output
```
((-2*(a*x^2 + b*x^n)^(3/2))/(3*(2 - n)*x^3) + a*((-2*Sqrt[a*x^2 + b*x^n])/
((2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(2 - n)
))/c^4
```

3.375.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

3.375.4 Maple [F]

$$\int \frac{(a\,x^2 + b\,x^n)^{\frac{3}{2}}}{c^4 x^4} dx$$

input
```
int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

output
```
int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

3.375.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^n)^{3/2}}{c^4 x^4} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.375.6 Sympy [F]

$$\int \frac{(ax^2 + bx^n)^{3/2}}{c^4x^4}\,dx = \frac{\int \frac{a\sqrt{ax^2+bx^n}}{x^2}\,dx + \int \frac{bx^n\sqrt{ax^2+bx^n}}{x^4}\,dx}{c^4}$$

input
```
integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)
```

output
```
(Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 +
 b*x**n)/x**4, x))/c**4
```

3.375.7 Maxima [F]

$$\int \frac{(ax^2 + bx^n)^{3/2}}{c^4x^4}\,dx = \int \frac{(ax^2 + bx^n)^{\frac{3}{2}}}{c^4x^4}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4
```

3.375.8 Giac [F]

$$\int \frac{(ax^2 + bx^n)^{3/2}}{c^4x^4}\,dx = \int \frac{(ax^2 + bx^n)^{\frac{3}{2}}}{c^4x^4}\,dx$$

input
```
integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")
```

output
```
integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx=\int \frac{\left(b\,x^n+a\,x^2\right)^{3/2}}{c^4\,x^4}\,dx$$

input
```
int((b*x^n + a*x^2)^(3/2)/(c^4*x^4),x)
```

output
```
int((b*x^n + a*x^2)^(3/2)/(c^4*x^4), x)
```

3.376 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx$

3.376.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{2a^{3/2}\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}$$

output
```
-2/3*(a*x+b*x^n)^(3/2)/c/(1-n)/(c*x)^(3/2)+2*a^(3/2)*arctanh(a^(1/2)*x^(1/
2)/(a*x+b*x^n)^(1/2))*x^(1/2)/c^2/(1-n)/(c*x)^(1/2)-2*a*(a*x+b*x^n)^(1/2)/
c^2/(1-n)/(c*x)^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \frac{x\left(8a^2x^2+2b^2x^{2n}+10abx^{1+n}-6a^{3/2}\sqrt{b}x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3(-1+n)(cx)^{5/2}\sqrt{ax+bx^n}}$$

input
```
Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

output
```
(x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3
 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]
]))/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

3.376.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\int \frac{\sqrt{bx^n+ax}}{(cx)^{3/2}}dx}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{1934}$$

$$\frac{a\left(\frac{a\int \frac{1}{\sqrt{cx}\sqrt{bx^n+ax}}dx}{c}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{a\left(\frac{a\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{c\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{a\left(\frac{2a\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\left(\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}\right)}{c}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

input
```
Int[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

output
```
(-2*(a*x + b*x^n)^(3/2))/(3*c*(1 - n)*(c*x)^(3/2)) + (a*((-2*Sqrt[a*x + b*
x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])))/c
```

3.376.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.376.4 Maple [F]

$$\int \frac{(ax + b\,x^{n})^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}dx$$

input
```
int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

output
```
int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

3.376.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax + bx^n)^{3/2}}{(cx)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.376.6 Sympy [F]

$$\int \frac{(ax + bx^n)^{3/2}}{(cx)^{5/2}} dx = \int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}} dx$$

input
```
integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)
```

output
```
Integral((a*x + b*x**n)**(3/2)/(c*x)**(5/2), x)
```

3.376.7 Maxima [F]

$$\int \frac{(ax + bx^n)^{3/2}}{(cx)^{5/2}} dx = \int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}} dx$$

input
```
integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

3.376.8 Giac [F]

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(b\,x^n+a\,x)^{3/2}}{(c\,x)^{5/2}}\,dx$$

input
```
int((b*x^n + a*x)^(3/2)/(c*x)^(5/2),x)
```

output
```
int((b*x^n + a*x)^(3/2)/(c*x)^(5/2), x)
```

3.377 $\int \frac{(a+bx^n)^{3/2}}{cx} dx$

3.377.1 Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{(a+bx^n)^{3/2}}{cx} dx = \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} - \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

output
```
2/3*(a+b*x^n)^(3/2)/c/n-2*a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n+2*a
*(a+b*x^n)^(1/2)/c/n
```

3.377.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^n)^{3/2}}{cx} dx = \frac{2\sqrt{a+bx^n}(4a+bx^n) - 6a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

input
```
Integrate[(a + b*x^n)^(3/2)/(c*x),x]
```

output
```
(2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[
a]])/(3*c*n)
```

3.377.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {27, 798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}}{cx}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(bx^n+a)^{3/2}}{x}dx}{c}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int x^{-n}(bx^n+a)^{3/2}dx^n}{cn}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a\int x^{-n}\sqrt{bx^n+a}dx^n+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a\left(a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{a\left(2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^n)^{3/2}}{cn}$$

input
```
Int[(a + b*x^n)^(3/2)/(c*x),x]
```

output
```
((2*(a + b*x^n)^(3/2))/3 + a*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x^n]/Sqrt[a]]))/(c*n)
```

3.377.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.377.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	51
default	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	51
risch	$\frac{2\left(b\,{\mathrm e}^{n\ln(x)}+4a\right)\sqrt{a+b\,{\mathrm e}^{n\ln(x)}}}{3nc}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln(x)}}}{\sqrt{a}}\right)}{nc}$	59

input
```
int((a+b*x^n)^(3/2)/c/x,x,method=_RETURNVERBOSE)
```

output
```
1/c/n*(2/3*(a+b*x^n)^(3/2)+2*a*(a+b*x^n)^(1/2)-2*a^(3/2)*arctanh((a+b*x^n)
^(1/2)/a^(1/2)))
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(bx^n+4\,a)\sqrt{bx^n+a}}{3\,cn}, \frac{2\left(3\,\sqrt{-a}a\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-}}{a}\right.\right.}{3\,c}\right.$$

input
```
integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*
x^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a
)*sqrt(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]
```

3.377.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n}-\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n}+\frac{2\sqrt{a}bx^n\sqrt{1+\frac{bx^n}{a}}}{3n}}{c}$$

input
```
integrate((a+b*x**n)**(3/2)/c/x,x)
```

output
```
(8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/
2)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*
n))/c
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n}+\frac{2\left((bx^n+a)^{\frac{3}{2}}+3\sqrt{bx^n+a}a\right)}{n}}{3\,c}$$

input
```
integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="maxima")
```

output
```
1/3*(3*a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a))
)/n + 2*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n)/c
```

3.377.8 Giac [F]

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{cx}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)/(c*x), x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \int \frac{(a+b\,x^n)^{3/2}}{c\,x}\,dx$$

input
```
int((a + b*x^n)^(3/2)/(c*x),x)
```

output
```
int((a + b*x^n)^(3/2)/(c*x), x)
```

3.378 $\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx$

3.378.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{2a^{3/2}c\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

output
```
2/3*(c*x)^(3/2)*(a/x+b*x^n)^(3/2)/c/(1+n)-2*a^(3/2)*c*arctanh(a^(1/2)/x^(1
/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/(1+n)/(c*x)^(1/2)+2*a*(c*x)^(1/2)*(a/x+b*x^
n)^(1/2)/(1+n)
```

3.378.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}(4a+bx^{1+n})-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{3(1+n)\sqrt{a+bx^{1+n}}}$$

input
```
Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

output
```
(2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n))
- 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b
*x^(1 + n)])
```

3.378.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{1934}$$

$$ac\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}dx+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{1934}$$

$$ac\left(ac\int \frac{1}{(cx)^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{1937}$$

$$ac\left(\frac{a\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{\sqrt{cx}}+\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{1935}$$

$$ac\left(\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2a\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{(n+1)\sqrt{cx}}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

$$\Big\downarrow \text{219}$$

$$ac\left(\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}\right)+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

input
```
Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

output
```
(2*(c*x)^(3/2)*(a/x + b*x^n)^(3/2))/(3*c*(1 + n)) + a*c*((2*Sqrt[c*x]*Sqrt
[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*S
qrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x]))
```

3.378.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.378.4 Maple [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+b\,x^{n}\right)^{\frac{3}{2}}dx$$

input
```
int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

3.378.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.378.6 Sympy [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)
```

output
```
Integral(sqrt(c*x)*(a/x + b*x**n)**(3/2), x)
```

3.378.7 Maxima [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

3.378.8 Giac [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2} dx = \int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2} dx = \int \sqrt{c\,x}\,\left(b\,x^n+\frac{a}{x}\right)^{3/2} dx$$

input
```
int((c*x)^(1/2)*(b*x^n + a/x)^(3/2),x)
```

output
```
int((c*x)^(1/2)*(b*x^n + a/x)^(3/2), x)
```

3.379 $\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx$

3.379.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

output
```
2/3*c^2*x^3*(a/x^2+b*x^n)^(3/2)/(2+n)-2*a^(3/2)*c^2*arctanh(a^(1/2)/x/(a/x
^2+b*x^n)^(1/2))/(2+n)+2*a*c^2*x*(a/x^2+b*x^n)^(1/2)/(2+n)
```

3.379.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\frac{2c^2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}(4a+bx^{2+n})-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{3(2+n)\sqrt{a+bx^{2+n}}}$$

input
```
Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

output
```
(2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) -
3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x
^(2 + n)])
```

3.379.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {27, 1934, 1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int x^2\left(bx^n+\frac{a}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{1934}$$

$$c^2\left(a\int \sqrt{bx^n+\frac{a}{x^2}}dx+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

$$\Big\downarrow \text{1913}$$

$$c^2\left(a\left(a\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx+\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}\right)+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

$$\Big\downarrow \text{1935}$$

$$c^2\left(a\left(\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2a\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{n+2}\right)+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

$$\Big\downarrow \text{219}$$

$$c^2\left(a\left(\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}\right)+\frac{2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}\right)$$

input
```
Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

output
```
c^2*((2*x^3*(a/x^2 + b*x^n)^(3/2))/(3*(2 + n)) + a*((2*x*Sqrt[a/x^2 + b*x^
n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2 + n)
))
```

3.379.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.379.4 Maple [F]

$$\int c^2x^2\left(\frac{a}{x^2}+b\,x^n\right)^{\frac{3}{2}}dx$$

input
```
int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

output
```
int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

3.379.5 Fricas [F(-2)]

Exception generated.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.379.6 Sympy [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx = c^2\left(\int a\sqrt{\frac{a}{x^2}+bx^n}\,dx+\int bx^2x^n\sqrt{\frac{a}{x^2}+bx^n}\,dx\right)$$

input
```
integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2),x)
```

output
```
c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x
**2 + b*x**n), x))
```

3.379.7 Maxima [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2} dx = \int \left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}} c^2x^2 \, dx$$

input
```
integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)
```

3.379.8 Giac [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2} dx = \int \left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}} c^2x^2 \, dx$$

input
```
integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2} dx = \int c^2\,x^2\,\left(b\,x^n+\frac{a}{x^2}\right)^{3/2} dx$$

input
```
int(c^2*x^2*(b*x^n + a/x^2)^(3/2),x)
```

output
```
int(c^2*x^2*(b*x^n + a/x^2)^(3/2), x)
```

3.380 $\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx$

3.380.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{2a^{3/2}c^4\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

output
```
2/3*(c*x)^(9/2)*(a/x^3+b*x^n)^(3/2)/c/(3+n)-2*a^(3/2)*c^4*arctanh(a^(1/2)/
x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/(3+n)/(c*x)^(1/2)+2*a*c^2*(c*x)^(3/2)
*(a/x^3+b*x^n)^(1/2)/(3+n)
```

3.380.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\frac{2c^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}(4a+bx^{3+n})-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{3(3+n)\sqrt{a+bx^{3+n}}}$$

input
```
Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

output
```
(2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(
3 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sq
rt[a + b*x^(3 + n)])
```

3.380.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1934, 1934, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1934}$$

$$ac^3 \int \sqrt{cx}\sqrt{bx^n+\frac{a}{x^3}}dx+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{ 1934}$$

$$ac^3\left(ac^3\int \frac{1}{(cx)^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{ 1937}$$

$$ac^3\left(\frac{ac\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{\sqrt{cx}}+\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{ 1935}$$

$$ac^3\left(\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2ac\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{(n+3)\sqrt{cx}}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

$$\Big\downarrow \text{ 219}$$

$$ac^3\left(\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}c\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}\right)+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

input
```
Int[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

output
```
(2*(c*x)^(9/2)*(a/x^3 + b*x^n)^(3/2))/(3*c*(3 + n)) + a*c^3*((2*(c*x)^(3/2
)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcTanh[Sqrt[a]/
(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x]))
```

3.380.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.380.4 Maple [F]

$$\int (cx)^{\frac{7}{2}} \left(\frac{a}{x^3}+b\,x^n\right)^{\frac{3}{2}} dx$$

input
```
int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

3.380.5 Fricas [F(-2)]

Exception generated.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.380.6 Sympy [F(-1)]

Timed out.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)
```

output
```
Timed out
```

3.380.7 Maxima [F]

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

3.380.8 Giac [F]

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int (c\,x)^{7/2}\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}\,dx$$

input
```
int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2),x)
```

output
```
int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2), x)
```

3.381 $\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$

3.381.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$

output
```
2/3*c^5*x^6*(a/x^4+b*x^n)^(3/2)/(4+n)-2*a^(3/2)*c^5*arctanh(a^(1/2)/x^2/(a
/x^4+b*x^n)^(1/2))/(4+n)+2*a*c^5*x^2*(a/x^4+b*x^n)^(1/2)/(4+n)
```

3.381.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\frac{2c^5x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\sqrt{a+bx^{4+n}}\left(4a+bx^{4+n}\right)-3a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{3(4+n)\sqrt{a+bx^{4+n}}}$$

input
```
Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

output
```
(2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n))
- 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]))/(3*(4 + n)*Sqrt[a + b
*x^(4 + n)])
```

3.381.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {27, 1934, 1934, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{27}$$

$$c^5\int x^5\left(bx^n+\frac{a}{x^4}\right)^{3/2}dx$$

$$\Big\downarrow \text{1934}$$

$$c^5\left(a\int x\sqrt{bx^n+\frac{a}{x^4}}dx+\frac{2x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

$$\Big\downarrow \text{1934}$$

$$c^5\left(a\left(a\int \frac{1}{x^3\sqrt{bx^n+\frac{a}{x^4}}}dx+\frac{2x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}\right)+\frac{2x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

$$\Big\downarrow \text{1935}$$

$$c^5\left(a\left(\frac{2x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}-\frac{2a\int \frac{1}{1-\frac{a}{x^4\left(bx^n+\frac{a}{x^4}\right)}}d\frac{1}{x^2\sqrt{bx^n+\frac{a}{x^4}}}}{n+4}\right)+\frac{2x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

$$\Big\downarrow \text{219}$$

$$c^5\left(a\left(\frac{2x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}\right)+\frac{2x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}\right)$$

input
```
Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

output
```
c^5*((2*x^6*(a/x^4 + b*x^n)^(3/2))/(3*(4 + n)) + a*((2*x^2*Sqrt[a/x^4 + b*
x^n])/(4 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])/(4
+ n)))
```

3.381.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1934
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Simp[a/c^j
  Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m
, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
&& (IntegerQ[j] || GtQ[c, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.381.4 Maple [F]

$$\int c^5x^5\left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}dx$$

input
```
int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

output
```
int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

3.381.5 Fricas [F(-2)]

Exception generated.

$$\int c^5 x^5 \left(\frac{a}{x^4} + b x^n\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.381.6 Sympy [F]

$$\int c^5 x^5 \left(\frac{a}{x^4} + b x^n\right)^{3/2} dx = c^5 \left(\int a x \sqrt{\frac{a}{x^4} + b x^n}\, dx + \int b x^5 x^n \sqrt{\frac{a}{x^4} + b x^n}\, dx\right)$$

input
```
integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)
```

output
```
c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a
/x**4 + b*x**n), x))
```

3.381.7 Maxima [F]

$$\int c^5 x^5 \left(\frac{a}{x^4} + b x^n\right)^{3/2} dx = \int \left(b x^n + \frac{a}{x^4}\right)^{\frac{3}{2}} c^5 x^5\, dx$$

input
```
integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)
```

3.381.8 Giac [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^5x^5\,dx$$

input
```
integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int c^5\,x^5\,\left(b\,x^n+\frac{a}{x^4}\right)^{3/2}\,dx$$

input
```
int(c^5*x^5*(b*x^n + a/x^4)^(3/2),x)
```

output
```
int(c^5*x^5*(b*x^n + a/x^4)^(3/2), x)
```

3.382 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$

3.382.1 Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - 2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}\right)$$

output
```
-2*arctanh(a^(1/2)/x/(a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(a/x^2+b/x)^(1/2)
```

3.382.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx}-\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[(a + b*x)/x^2],x]
```

output
```
(2*x*Sqrt[(a + b*x)/x^2]*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]]))/Sqrt[a + b*x]
```

3.382.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2078, 1913, 1919, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{\frac{a}{x^2}+\frac{b}{x}}dx$$

$$\Big\downarrow \text{1913}$$

$$a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x^2}dx+2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}$$

$$\Big\downarrow \text{1919}$$

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{1091}$$

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2a\int \frac{1}{1-\frac{a}{x^2}}d\frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}$$

$$\Big\downarrow \text{219}$$

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

input
```
Int[Sqrt[(a + b*x)/x^2],x]
```

output
```
2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

3.382.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.382.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{2\sqrt{\frac{bx+a}{x^2}}\, x\left(\sqrt{bx+a}-\sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{\sqrt{bx+a}}$	47

input
```
int(((b*x+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((b*x+a)/x^2)^(1/2)*x*((b*x+a)^(1/2)-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/
2)))/(b*x+a)^(1/2)
```

3.382. $\int \sqrt{\frac{a+bx}{x^2}}\, dx$

3.382.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \left[2\,x\sqrt{\frac{bx+a}{x^2}} + \sqrt{a}\log\left(\frac{bx-2\,\sqrt{a}x\sqrt{\frac{bx+a}{x^2}}+2\,a}{x}\right), 2\,x\sqrt{\frac{bx+a}{x^2}} + 2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx+a}{x^2}}}{a}\right)\right]$$

input
```
integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x
^2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt
((b*x + a)/x^2)/a)]
```

3.382.6 Sympy [F]

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \int \sqrt{\frac{a+bx}{x^2}}\,dx$$

input
```
integrate(((b*x+a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x)/x**2), x)
```

3.382.7 Maxima [F]

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \int \sqrt{\frac{bx+a}{x^2}}\,dx$$

input
```
integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x + a)/x^2), x)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + 2\,\sqrt{bx+a}\operatorname{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

input
```
integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x
) - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

3.382.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2\,x\,\sqrt{\frac{a}{x^2}+\frac{b}{x}} + \frac{\sqrt{a}\,\sqrt{x}\,\operatorname{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

input
```
int(((a + b*x)/x^2)^(1/2),x)
```

output
```
2*x*(a/x^2 + b/x)^(1/2) + (a^(1/2)*x^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(1
/2)))*(a/x^2 + b/x)^(1/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

3.383 $\int \sqrt{\frac{a+bx^2}{x^2}}\,dx$

3.383.1 Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx=\sqrt{b+\frac{a}{x^2}}x-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x}\right)$$

output
```
-arctanh(a^(1/2)/x/(b+a/x^2)^(1/2))*a^(1/2)+x*(b+a/x^2)^(1/2)
```

3.383.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx=\sqrt{b+\frac{a}{x^2}}x-\frac{\sqrt{a}\sqrt{b+\frac{a}{x^2}}x\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(a + b*x^2)/x^2],x]
```

output
```
Sqrt[b + a/x^2]*x - (Sqrt[a]*Sqrt[b + a/x^2]*x*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/Sqrt[a + b*x^2]
```

3.383.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2072, 773, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx^2}{x^2}}dx$$

$$\Big\downarrow \text{ 2072}$$

$$\int \sqrt{\frac{a}{x^2}+b}dx$$

$$\Big\downarrow \text{ 773}$$

$$-\int \sqrt{\frac{a}{x^2}+b}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 247}$$

$$x\sqrt{\frac{a}{x^2}+b}-a\int \frac{1}{\sqrt{\frac{a}{x^2}+b}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 224}$$

$$x\sqrt{\frac{a}{x^2}+b}-a\int \frac{1}{1-\frac{a}{x^2}}d\frac{1}{\sqrt{\frac{a}{x^2}+b}x}$$

$$\Big\downarrow \text{ 219}$$

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

input
```
Int[Sqrt[(a + b*x^2)/x^2],x]
```

output
```
Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

3.383.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

3.383.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\sqrt{\frac{b\,x^{2}+a}{x^{2}}}\,x\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{\sqrt{b\,x^{2}+a}}$	61

input
```
int(((b*x^2+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)/x^2)^(1/2)*x/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)-a^(1/2)*ln(2*(a^(
1/2)*(b*x^2+a)^(1/2)+a)/x))
```

3.383.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.57

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \left[x\sqrt{\frac{bx^2+a}{x^2}}+\frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{a}x\sqrt{\frac{bx^2+a}{x^2}}+2\,a}{x^2}\right), x\sqrt{\frac{bx^2+a}{x^2}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^2+a}{x^2}}}{bx^2+a}\right)\right]$$

input
```
integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[x*sqrt((b*x^2 + a)/x^2) + 1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(a)*x*sqrt((b*x
^2 + a)/x^2) + 2*a)/x^2), x*sqrt((b*x^2 + a)/x^2) + sqrt(-a)*arctan(sqrt(-
a)*x*sqrt((b*x^2 + a)/x^2)/(b*x^2 + a))]
```

3.383.6 Sympy [F]

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \int \sqrt{\frac{a+bx^2}{x^2}}\,dx$$

input
```
integrate(((b*x**2+a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**2)/x**2), x)
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \sqrt{b+\frac{a}{x^2}}x+\frac{1}{2}\sqrt{a}\log\left(\frac{\sqrt{b+\frac{a}{x^2}}x-\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x+\sqrt{a}}\right)$$

input
```
integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b + a/x^2)*x + 1/2*sqrt(a)*log((sqrt(b + a/x^2)*x - sqrt(a))/(sqrt(b
+ a/x^2)*x + sqrt(a)))
```

3.383.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \sqrt{bx^2+a}\operatorname{sgn}(x) - \frac{\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

input
```
integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
a*arctan(sqrt(b*x^2 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + sqrt(b*x^2 + a)*sgn(x
) - (a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

3.383.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = x\sqrt{b+\frac{a}{x^2}} + \frac{\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x}\right)\sqrt{b+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\sqrt{\frac{a}{b\,x^2}+1}}$$

input
```
int(((a + b*x^2)/x^2)^(1/2),x)
```

output
```
x*(b + a/x^2)^(1/2) + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x))*(b + a/x^2)^
(1/2)*1i)/(b^(1/2)*(a/(b*x^2) + 1)^(1/2))
```

3.384 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$

3.384.1 Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

output
```
-2/3*arctanh(a^(1/2)/x/(a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(a/x^2+b*x)^(1/2)
```

3.384.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2x\sqrt{\frac{a}{x^2}+bx}\left(\sqrt{a+bx^3}-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}$$

input
```
Integrate[Sqrt[(a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]]))/(3*Sqrt[a + b*x^3])
```

3.384.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2078, 1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx^3}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{\frac{a}{x^2}+bx}dx$$

$$\Big\downarrow \text{1913}$$

$$a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx}}dx+\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}a\int \frac{1}{1-\frac{a}{x^2\left(\frac{a}{x^2}+bx\right)}}d\frac{1}{x\sqrt{\frac{a}{x^2}+bx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

input
```
Int[Sqrt[(a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x
])])/3
```

3.384.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.384.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{\frac{b\,x^{3}+a}{x^{2}}}\,x\left(-\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{a}+\sqrt{b\,x^{3}+a}\right)}{3\sqrt{b\,x^{3}+a}}$	55

input
```
int(((b*x^3+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((b*x^3+a)/x^2)^(1/2)*x*(-arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+(b*
x^3+a)^(1/2))/(b*x^3+a)^(1/2)
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.04

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \left[\frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}} + \frac{1}{3}\,\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{a}x\sqrt{\frac{bx^3+a}{x^2}}+2\,a}{x^3}\right), \frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}} + \frac{2}{3}\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^3+a}{x^2}}}{a}\right)\right]$$

input
```
integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((
b*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*ar
ctan(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/a)]
```

3.384.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)/x**2)**(1/2),x)
```

output
```
Timed out
```

3.384.7 Maxima [F]

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \int \sqrt{\frac{bx^3+a}{x^2}}\,dx$$

input
```
integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^3 + a)/x^2), x)
```

3.384.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{3\sqrt{-a}} + \frac{2}{3}\sqrt{bx^3+a}\mathrm{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}(x)}{3\sqrt{-a}}$$

input
```
integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*sqrt(b*x^3 +
a)*sgn(x) - 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqr
t(-a)
```

3.384.9 Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2\,x\,\sqrt{b\,x+\frac{a}{x^2}}}{3} + \frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x+\frac{a}{x^2}}\,2\mathrm{i}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{b\,x^3}+1}}$$

input
```
int(((a + b*x^3)/x^2)^(1/2),x)
```

output
```
(2*x*(b*x + a/x^2)^(1/2))/3 + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(3/2))
)*(b*x + a/x^2)^(1/2)*2i)/(3*b^(1/2)*x^(1/2)*(a/(b*x^3) + 1)^(1/2))
```

3.385 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$

3.385.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n} - \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

output
```
-2*arctanh(a^(1/2)/x/(a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(a/x^2+b*x^(-
2+n))^(1/2)/n
```

3.385.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \frac{2x\sqrt{\frac{a+bx^n}{x^2}}\left(\sqrt{a+bx^n}-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}$$

input
```
Integrate[Sqrt[(a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[(a + b*x^n)/x^2]*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x
^n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

3.385.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2078, 1913, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx^n}{x^2}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \sqrt{\frac{a}{x^2}+bx^{n-2}}dx$$

$$\Big\downarrow \text{ 1913}$$

$$a\int \frac{1}{x^2\sqrt{bx^{n-2}+\frac{a}{x^2}}}dx+\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2a\int \frac{1}{1-\frac{a}{x^2\left(bx^{n-2}+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^{n-2}+\frac{a}{x^2}}}}{n}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

input
```
Int[Sqrt[(a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/
x^2 + b*x^(-2 + n)])])/n
```

3.385.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.385.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{2\sqrt{\frac{a+b\,{\mathrm e}^{n\ln\left(x\right)}}{x^{2}}}\,x}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{\sqrt{a}}\right)\sqrt{\frac{a+b\,{\mathrm e}^{n\ln\left(x\right)}}{x^{2}}}\,x}{n\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}$	74

input
```
int(((a+b*x^n)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/n*((a+b*exp(n*ln(x)))/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((a+b*exp(n*ln(x))
)^(1/2)/a^(1/2))*((a+b*exp(n*ln(x)))/x^2)^(1/2)/(a+b*exp(n*ln(x)))^(1/2)*x
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.84

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \left[\frac{2\,x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{a}\log\left(\frac{bx^n-2\,\sqrt{a}x\sqrt{\frac{bx^n+a}{x^2}}+2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^n+a}{x^2}}}{a}\right)\right)}{n}\right]$$

input
```
integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n
 + a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sq
rt(-a)*x*sqrt((b*x^n + a)/x^2)/a))/n]
```

3.385.6 Sympy [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{a+bx^n}{x^2}}\,dx$$

input
```
integrate(((a+b*x**n)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**n)/x**2), x)
```

3.385.7 Maxima [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n+a}{x^2}}\,dx$$

input
```
integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^n + a)/x^2), x)
```

3.385.8 Giac [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n+a}{x^2}}\,dx$$

input
```
integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^n + a)/x^2), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{a+b\,x^n}{x^2}}\,dx$$

input
```
int(((a + b*x^n)/x^2)^(1/2),x)
```

output
```
int(((a + b*x^n)/x^2)^(1/2), x)
```

3.386 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$

3.386.1 Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx=2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x+2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x}\right)$$

output
```
2*arctan(a^(1/2)/x/(-a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(-a/x^2+b/x)^(1/2)
```

3.386.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx=\frac{2x\sqrt{\frac{-a+bx}{x^2}}\left(\sqrt{-a+bx}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{-a+bx}}$$

input
```
Integrate[Sqrt[(-a + b*x)/x^2],x]
```

output
```
(2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/
Sqrt[a]]))/Sqrt[-a + b*x]
```

3.386.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2078, 1913, 1919, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx-a}{x^2}}dx$$
$$\Big\downarrow \text{2078}$$
$$\int \sqrt{\frac{b}{x}-\frac{a}{x^2}}dx$$
$$\Big\downarrow \text{1913}$$
$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}-a\int \frac{1}{\sqrt{\frac{b}{x}-\frac{a}{x^2}}x^2}dx$$
$$\Big\downarrow \text{1919}$$
$$a\int \frac{1}{\sqrt{\frac{b}{x}-\frac{a}{x^2}}}d\frac{1}{x}+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$
$$\Big\downarrow \text{1091}$$
$$2a\int \frac{1}{\frac{a}{x^2}+1}d\frac{1}{\sqrt{\frac{b}{x}-\frac{a}{x^2}}x}+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$
$$\Big\downarrow \text{216}$$
$$2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$

input
```
Int[Sqrt[(-a + b*x)/x^2],x]
```

output
```
2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*
x)]
```

3.386.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.386.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{2\sqrt{-\frac{-bx+a}{x^2}}\,x\left(-\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\right)}{\sqrt{bx-a}}$	55

input
```
int(((b*x-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-(-b*x+a)/x^2)^(1/2)*x*(-a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))+(b*x-a)^
(1/2))/(b*x-a)^(1/2)
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.85

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \left[2\,x\sqrt{\frac{bx-a}{x^2}}+\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{-a}x\sqrt{\frac{bx-a}{x^2}}-2\,a}{x}\right), 2\,x\sqrt{\frac{bx-a}{x^2}} - 2\,\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx-a}{x^2}}}{\sqrt{a}}\right)\right]$$

input
```
integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)
/x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) - 2*sqrt(a)*arctan(x*sqrt((b*x -
a)/x^2)/sqrt(a))]
```

3.386.6 Sympy [F]

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \int \sqrt{\frac{-a+bx}{x^2}}\,dx$$

input
```
integrate(((b*x-a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((-a + b*x)/x**2), x)
```

3.386.7 Maxima [F]

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \int \sqrt{\frac{bx-a}{x^2}}\,dx$$

input
```
integrate(((b*x-a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x - a)/x^2), x)
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = -2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\operatorname{sgn}(x) + 2\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right) - \sqrt{-a}\right)\operatorname{sgn}(x) + 2\sqrt{bx-a}\operatorname{sgn}(x)$$

input
```
integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a))*sgn(x) + 2*(sqrt(a)*arctan(sqrt(-
a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2*sqrt(b*x - a)*sgn(x)
```

3.386.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = 2\,x\,\sqrt{\frac{b}{x}-\frac{a}{x^2}} + \frac{2\,\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{b}{x}-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x}}}$$

input
```
int((-(a - b*x)/x^2)^(1/2),x)
```

output
```
2*x*(b/x - a/x^2)^(1/2) + (2*a^(1/2)*x^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(1/2)
))*(b/x - a/x^2)^(1/2))/(b^(1/2)*(1 - a/(b*x))^(1/2))
```

3.387 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$

3.387.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=\sqrt{b-\frac{a}{x^2}}\,x+\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}\,x}\right)$$

output
```
arctan(a^(1/2)/x/(b-a/x^2)^(1/2))*a^(1/2)+x*(b-a/x^2)^(1/2)
```

3.387.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=\sqrt{b-\frac{a}{x^2}}\,x-\frac{\sqrt{a}\sqrt{b-\frac{a}{x^2}}\,x\arctan\left(\frac{\sqrt{-a+bx^2}}{\sqrt{a}}\right)}{\sqrt{-a+bx^2}}$$

input
```
Integrate[Sqrt[(-a + b*x^2)/x^2],x]
```

output
```
Sqrt[b - a/x^2]*x - (Sqrt[a]*Sqrt[b - a/x^2]*x*ArcTan[Sqrt[-a + b*x^2]/Sqr
t[a]])/Sqrt[-a + b*x^2]
```

3.387.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2072, 773, 247, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx^2-a}{x^2}}dx$$

$$\Big\downarrow \text{2072}$$

$$\int \sqrt{b-\frac{a}{x^2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \sqrt{b-\frac{a}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$a\int \frac{1}{\sqrt{b-\frac{a}{x^2}}}d\frac{1}{x}+x\sqrt{b-\frac{a}{x^2}}$$

$$\Big\downarrow \text{224}$$

$$a\int \frac{1}{\frac{a}{x^2}+1}d\frac{1}{\sqrt{b-\frac{a}{x^2}}x}+x\sqrt{b-\frac{a}{x^2}}$$

$$\Big\downarrow \text{216}$$

$$\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)+x\sqrt{b-\frac{a}{x^2}}$$

input
```
Int[Sqrt[(-a + b*x^2)/x^2],x]
```

output
```
Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

3.387.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

3.387.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,x\left(\sqrt{-a}\,\sqrt{b\,x^{2}-a}+a\ln\left(\frac{-2a+2\sqrt{-a}\,\sqrt{b\,x^{2}-a}}{x}\right)\right)}{\sqrt{-a}\,\sqrt{b\,x^{2}-a}}$	81

input
```
int(((b*x^2-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*x^2+a)/x^2)^(1/2)*x*((-a)^(1/2)*(b*x^2-a)^(1/2)+a*ln(2*((-a)^(1/2)*(
b*x^2-a)^(1/2)-a)/x))/(-a)^(1/2)/(b*x^2-a)^(1/2)
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \left[x\sqrt{\frac{bx^2-a}{x^2}} + \frac{1}{2}\sqrt{-a}\log\left(-\frac{bx^2-2\sqrt{-a}x\sqrt{\frac{bx^2-a}{x^2}}-2a}{x^2}\right), x\sqrt{\frac{bx^2-a}{x^2}} + \sqrt{a}\arctan\left(\frac{\sqrt{a}x\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}\right)\right]$$

input
```
integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[x*sqrt((b*x^2 - a)/x^2) + 1/2*sqrt(-a)*log(-(b*x^2 - 2*sqrt(-a)*x*sqrt((b
*x^2 - a)/x^2) - 2*a)/x^2), x*sqrt((b*x^2 - a)/x^2) + sqrt(a)*arctan(sqrt(
a)*x*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a))]
```

3.387.6 Sympy [F]

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$$

input
```
integrate(((b*x**2-a)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((-a + b*x**2)/x**2), x)
```

3.387.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \sqrt{b-\frac{a}{x^2}}x-\sqrt{a}\arctan\left(\frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a}}\right)$$

input
```
integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b - a/x^2)*x - sqrt(a)*arctan(sqrt(b - a/x^2)*x/sqrt(a))
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = -\sqrt{a}\arctan\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)\operatorname{sgn}(x) + \left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x)+\sqrt{bx^2-a}\operatorname{sgn}(x)$$

input
```
integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a))*sgn(x) + (sqrt(a)*arctan(sqrt(-a)
/sqrt(a)) - sqrt(-a))*sgn(x) + sqrt(b*x^2 - a)*sgn(x)
```

3.387.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = x\sqrt{b-\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)\sqrt{b-\frac{a}{x^2}}}{\sqrt{b}\sqrt{1-\frac{a}{b\,x^2}}}$$

input
```
int((-(a - b*x^2)/x^2)^(1/2),x)
```

output
```
x*(b - a/x^2)^(1/2) + (a^(1/2)*asin(a^(1/2)/(b^(1/2)*x))*(b - a/x^2)^(1/2)
)/(b^(1/2)*(1 - a/(b*x^2))^(1/2))
```

3.388 $\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx$

3.388.1 Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=\frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx}+\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)$$

output
```
2/3*arctan(a^(1/2)/x/(-a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(-a/x^2+b*x)^(1/2)
```

3.388.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=\frac{2x\sqrt{-\frac{a}{x^2}+bx}\left(\sqrt{-a+bx^3}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{-a+bx^3}}$$

input
```
Integrate[Sqrt[(-a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x
^3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

3.388.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2078, 1913, 1935, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx^3-a}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{bx-\frac{a}{x^2}}dx$$

$$\Big\downarrow \text{1913}$$

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}-a\int \frac{1}{x^2\sqrt{bx-\frac{a}{x^2}}}dx$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{3}a\int \frac{1}{\frac{a}{x^2\left(bx-\frac{a}{x^2}\right)}+1}d\frac{1}{x\sqrt{bx-\frac{a}{x^2}}}+\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)+\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}$$

input
```
Int[Sqrt[(-a + b*x^3)/x^2],x]
```

output
```
(2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2)
+ b*x])])/3
```

3.388.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.388.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{2\sqrt{-\frac{-b\,x^{3}+a}{x^{2}}}\,x\left(\sqrt{b\,x^{3}-a}\,\sqrt{-a}+a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}-a}}{\sqrt{-a}}\right)\right)}{3\sqrt{b\,x^{3}-a}\,\sqrt{-a}}$	73

input
```
int(((b*x^3-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-(-b*x^3+a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3
-a)^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)
```

3.388.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.06

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \left[\frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}} + \frac{1}{3}\,\sqrt{-a}\log\left(\frac{bx^3-2\,\sqrt{-a}x\sqrt{\frac{bx^3-a}{x^2}}-2\,a}{x^3}\right), \frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}} - \frac{2}{3}\,\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^3-a}{x^2}}}{\sqrt{a}}\right)\right]$$

input
```
integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt
((b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) - 2/3*sqrt(a)*a
rctan(x*sqrt((b*x^3 - a)/x^2)/sqrt(a))]
```

3.388.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3-a)/x**2)**(1/2),x)
```

output
```
Timed out
```

3.388.7 Maxima [F]

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \int \sqrt{\frac{bx^3-a}{x^2}}\,dx$$

input
```
integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^3 - a)/x^2), x)
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = -\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\operatorname{sgn}(x) + \frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x) + \frac{2}{3}\sqrt{bx^3-a}\operatorname{sgn}(x)$$

input
```
integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a))*sgn(x) + 2/3*(sqrt(a)*arctan(
sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2/3*sqrt(b*x^3 - a)*sgn(x)
```

3.388.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \frac{2\,x\,\sqrt{b\,x-\frac{a}{x^2}}}{3} + \frac{2\,\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x-\frac{a}{x^2}}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{1-\frac{a}{b\,x^3}}}$$

input
```
int((-(a - b*x^3)/x^2)^(1/2),x)
```

output
```
(2*x*(b*x - a/x^2)^(1/2))/3 + (2*a^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(3/2)))*(
b*x - a/x^2)^(1/2))/(3*b^(1/2)*x^(1/2)*(1 - a/(b*x^3))^(1/2))
```

3.389 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$

3.389.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

output
```
2*arctan(a^(1/2)/x/(-a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(-a/x^2+b*x^(-
2+n))^(1/2)/n
```

3.389.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{\frac{-a+bx^n}{x^2}}\left(\sqrt{-a+bx^n}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{-a+bx^n}}$$

input
```
Integrate[Sqrt[(-a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[(-a + b*x^n)/x^2]*(Sqrt[-a + b*x^n] - Sqrt[a]*ArcTan[Sqrt[-a + b
*x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

3.389.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2078, 1913, 1935, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{bx^n - a}{x^2}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{bx^{n-2} - \frac{a}{x^2}}dx$$

$$\Big\downarrow \text{1913}$$

$$\frac{2x\sqrt{bx^{n-2} - \frac{a}{x^2}}}{n} - a\int \frac{1}{x^2\sqrt{bx^{n-2} - \frac{a}{x^2}}}dx$$

$$\Big\downarrow \text{1935}$$

$$\frac{2a\int \frac{1}{\frac{a}{x^2\left(bx^{n-2}-\frac{a}{x^2}\right)}+1}d\frac{1}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}}{n} + \frac{2x\sqrt{bx^{n-2} - \frac{a}{x^2}}}{n}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n} + \frac{2x\sqrt{bx^{n-2} - \frac{a}{x^2}}}{n}$$

input
```
Int[Sqrt[(-a + b*x^n)/x^2],x]
```

output
```
(2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[
-(a/x^2) + b*x^(-2 + n)])])/n
```

3.389.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1913
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Simp[a   Int[x^j*(a*x^j + b*x^n)^(p - 1), x]
, x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simp
lify[j*p + 1], 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.389.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

method	result	size
risch	$-\frac{2\left(a-b\,\mathrm{e}^{n\ln\left(x\right)}\right)\sqrt{\frac{b\,\mathrm{e}^{n\ln\left(x\right)}-a}{x^{2}}}\,x}{n\left(b\,\mathrm{e}^{n\ln\left(x\right)}-a\right)}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{b\,\mathrm{e}^{n\ln\left(x\right)}-a}}{\sqrt{a}}\right)\sqrt{\frac{b\,\mathrm{e}^{n\ln\left(x\right)}-a}{x^{2}}}\,x}{n\sqrt{b\,\mathrm{e}^{n\ln\left(x\right)}-a}}$	105

input
```
int(((b*x^n-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a-b*exp(n*ln(x)))/n/(b*exp(n*ln(x))-a)*((b*exp(n*ln(x))-a)/x^2)^(1/2)*
x-2*a^(1/2)/n*arctan((b*exp(n*ln(x))-a)^(1/2)/a^(1/2))*((b*exp(n*ln(x))-a)
/x^2)^(1/2)/(b*exp(n*ln(x))-a)^(1/2)*x
```

3.389.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$$
$$= \left[\frac{2\,x\sqrt{\frac{bx^n-a}{x^2}}+\sqrt{-a}\log\left(\frac{bx^n-2\,\sqrt{-a}x\sqrt{\frac{bx^n-a}{x^2}}-2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n-a}{x^2}}-\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^n-a}{x^2}}}{\sqrt{a}}\right)\right)}{n}\right]$$

input
```
integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x
^n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) - sqrt(a)*arctan(x
*sqrt((b*x^n - a)/x^2)/sqrt(a)))/n]
```

3.389.6 Sympy [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$$

input
```
integrate(((-a+b*x**n)/x**2)**(1/2),x)
```

output
```
Integral(sqrt((-a + b*x**n)/x**2), x)
```

3.389.7 Maxima [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n-a}{x^2}}\,dx$$

input
```
integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^n - a)/x^2), x)
```

3.389.8 Giac [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n-a}{x^2}}\,dx$$

input
```
integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^n - a)/x^2), x)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \int \sqrt{-\frac{a-b\,x^n}{x^2}}\,dx$$

input
```
int((-(a - b*x^n)/x^2)^(1/2),x)
```

output
```
int((-(a - b*x^n)/x^2)^(1/2), x)
```

3.390 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$

3.390.1 Optimal result

Integrand size = 27, antiderivative size = 62

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{2x^{-j/2}(cx)^{j/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

output
```
2*(c*x)^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/(x^
(1/2*j))/a^(1/2)
```

3.390.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{2\sqrt{b}x^{\frac{1}{2}(-j+n)}(cx)^{j/2}\sqrt{1+\frac{ax^{j-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(S
qrt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

3.390.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j+bx^n}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{x^{-j/2}(cx)^{j/2}\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx}{c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2x^{-j/2}(cx)^{j/2}\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{c(j-n)}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x^{-j/2}(cx)^{j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

input
```
Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*
(j - n)*x^(j/2))
```

3.390.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.390.4 Maple [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{a\,x^{j}+b\,x^{n}}}dx$$

input
```
int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

3.390.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.390.6 Sympy [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j+bx^n}}\,dx$$

input
```
integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)
```

3.390.7 Maxima [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

input
```
integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

3.390.8 Giac [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

input
```
integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(c\,x)^{\frac{j}{2}-1}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

input
```
int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2),x)
```

output
```
int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2), x)
```

3.391 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$

3.391.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=\frac{2\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

output
```
2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/(3-n)/a^(1/2)/x
^(1/2)
```

3.391.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=-\frac{2\sqrt{b}x^{\frac{1}{2}(-1+n)}\sqrt{cx}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-3+n)\sqrt{ax^3+bx^n}}$$

input
```
Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

output
```
(-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sq
rt[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

3.391.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{\sqrt{x}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{(3-n)\sqrt{x}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

input
```
Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

output
```
(2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 -
 n)*Sqrt[x])
```

3.391.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.391.4 Maple [F]

$$\int \frac{\sqrt{cx}}{\sqrt{a\,x^{3}+b\,x^{n}}}dx$$

input
```
int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

3.391.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.391.6 Sympy [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

input
```
integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)
```

3.391.7 Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

3.391.8 Giac [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \int \frac{\sqrt{c\,x}}{\sqrt{b\,x^n+a\,x^3}}\,dx$$

input
```
int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2),x)
```

output
```
int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2), x)
```

3.392 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$

3.392.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

output
```
2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/(2-n)/a^(1/2)
```

3.392.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=-\frac{2\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-2+n)\sqrt{ax^2+bx^n}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^n],x]
```

output
```
(-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2)
)/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

3.392.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^n}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{2-n}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^n],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

3.392.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

3.392.4 Maple [F]

$$\int \frac{1}{\sqrt{a\,x^{2}+b\,x^{n}}}dx$$

input
```
int(1/(a*x^2+b*x^n)^(1/2),x)
```

output
```
int(1/(a*x^2+b*x^n)^(1/2),x)
```

3.392.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.392.6 Sympy [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx = \int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx$$

input
```
integrate(1/(a*x**2+b*x**n)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**n), x)
```

3.392.7 Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$$

input
```
integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x^2 + b*x^n), x)
```

3.392.8 Giac [F]

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$$

input
```
integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*x^2 + b*x^n), x)
```

3.392.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx = \frac{\sqrt{b}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{\frac{a\,x^{2-n}}{b}+1}\,1\mathrm{i}}{\sqrt{a}\,\left(\frac{n}{2}-1\right)\,\sqrt{b\,x^n+a\,x^2}}$$

input
```
int(1/(b*x^n + a*x^2)^(1/2),x)
```

output
```
(b^(1/2)*x^(n/2)*asin((a^(1/2)*x^(1 - n/2)*1i)/b^(1/2))*((a*x^(2 - n))/b +
 1)^(1/2)*1i)/(a^(1/2)*(n/2 - 1)*(b*x^n + a*x^2)^(1/2))
```

3.393 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$

3.393.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

output
```
2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/(1-n)/a^(1/2)/(c*x)^(
1/2)
```

3.393.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = -\frac{2\sqrt{b}x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-1+n)\sqrt{cx}\sqrt{ax+bx^n}}$$

input
```
Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

output
```
(-2*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/
2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-1 + n)*Sqrt[c*x]*Sqrt[a*x + b*x^n])
```

3.393.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}dx$$

$$\Big\downarrow \ 1937$$

$$\frac{\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{\sqrt{cx}}$$

$$\Big\downarrow \ 1935$$

$$\frac{2\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{(1-n)\sqrt{cx}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

input
```
Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

output
```
(2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*
Sqrt[c*x])
```

3.393.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.393.4 Maple [F]

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax + b\,x^n}}dx$$

input
```
int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

output
```
int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

3.393.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax + bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.393.6 Sympy [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax + bx^n}}\,dx = \int \frac{1}{\sqrt{cx}\sqrt{ax + bx^n}}\,dx$$

input
```
integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)
```

3.393.7 Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

3.393.8 Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^n+a\,x}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)), x)
```

3.394 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$

3.394.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

3.394.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

input
```
Integrate[1/(c*x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

3.394.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{cx\sqrt{a+bx^n}}dx$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{1}{x\sqrt{bx^n+a}}dx}{c}$$
$$\Big\downarrow \ 798$$
$$\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{cn}$$
$$\Big\downarrow \ 73$$
$$\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{bcn}$$
$$\Big\downarrow \ 221$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

input
```
Int[1/(c*x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

3.394.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.394.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{c n \sqrt{a}}$	26
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{c n \sqrt{a}}$	26

input
```
int(1/c/x/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}cn}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{acn}\right]$$

input
```
integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(
-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*c*n)]
```

3.394.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}cn}$$

input
```
integrate(1/c/x/(a+b*x**n)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(sqrt(a)*c*n)
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}cn}$$

input
```
integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*c*n)
```

3.394.8 Giac [F]

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}cx}\,dx$$

input
```
integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*c*x), x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \int \frac{1}{c\,x\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(c*x*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(c*x*(a + b*x^n)^(1/2)), x)
```

3.395 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$

3.395.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=-\frac{2\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(1+n)\sqrt{cx}}$$

output
```
-2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/c/(1+n)/a^(1/2)/(c*x
)^(1/2)
```

3.395.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=-\frac{2x\sqrt{a+bx^{1+n}}\,\text{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)}{\sqrt{a}(1+n)(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}$$

input
```
Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

output
```
(-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[
a]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])
```

3.395.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{c\sqrt{cx}}$$

$$\Big\downarrow \text{1935}$$

$$-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{c(n+1)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(n+1)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

output
```
(-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 +
n)*Sqrt[c*x])
```

3.395.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.395.4 Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{2}} \sqrt{\frac{a}{x} + b\, x^{n}}}dx$$

input
```
int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

output
```
int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

3.395.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x} + bx^n}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.395.6 Sympy [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{\frac{a}{x}+bx^n}}\,dx$$

input
```
integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)
```

output
```
Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)
```

3.395.7 Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

3.395.8 Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = \int \frac{1}{{\left(c\,x\right)}^{3/2}\,\sqrt{b\,x^n+\frac{a}{x}}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)), x)
```

3.396 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$

3.396.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(2+n)}$$

output
```
-2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/c^2/(2+n)/a^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = -\frac{2\sqrt{a+bx^{2+n}}\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)}{\sqrt{a}c^2(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

input
```
Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

output
```
(-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]
*c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

3.396.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {27, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx}{c^2}$$
$$\Big\downarrow \text{1935}$$
$$-\frac{2\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{c^2(n+2)}$$
$$\Big\downarrow \text{219}$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(n+2)}$$

input
```
Int[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))
```

3.396.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.396.4 Maple [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+b\,x^n}}dx$$

input
```
int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
```

output
```
int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
```

3.396.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.396.6 Sympy [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

input
```
integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2
```

3.396.7 Maxima [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}c^2x^2}\,dx$$

input
```
integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2
```

3.396.8 Giac [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}c^2x^2}\,dx$$

input
```
integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \int \frac{1}{c^2\,x^2\,\sqrt{b\,x^n+\frac{a}{x^2}}}\,dx$$

input
```
int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)),x)
```

output
```
int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)), x)
```

3.397 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$

3.397.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=-\frac{2\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(3+n)\sqrt{cx}}$$

output
```
-2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/c^2/(3+n)/a^(1/2)/
(c*x)^(1/2)
```

3.397.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=-\frac{2x\sqrt{a+bx^{3+n}}\,\text{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)}{\sqrt{a}(3+n)(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

input
```
Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

output
```
(-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[
a]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])
```

3.397.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}dx$$
$$\Big\downarrow \text{ 1937}$$
$$\frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{c^2\sqrt{cx}}$$
$$\Big\downarrow \text{ 1935}$$
$$-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{c^2(n+3)\sqrt{cx}}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(n+3)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

output
```
(-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(
3 + n)*Sqrt[c*x])
```

3.397.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.397.4 Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}} \sqrt{\frac{a}{x^3}+b\,x^n}}dx$$

input
```
int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

output
```
int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

3.397.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.397.6 Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=\int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$$

input
```
integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)
```

output
```
Integral(1/((c*x)**(5/2)*sqrt(a/x**3 + b*x**n)), x)
```

3.397.7 Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

3.397.8 Giac [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \int \frac{1}{(c\,x)^{5/2}\,\sqrt{b\,x^n+\frac{a}{x^3}}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)), x)
```

3.398 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx$

3.398.1 Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{2x^{-3j/2}(cx)^{3j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}$$

output
```
2*(c*x)^(3/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/a^(3/2)/c/(
j-n)/(x^(3/2*j))-2*(c*x)^(3/2*j)/a/c/(j-n)/(x^j)/(a*x^j+b*x^n)^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=-\frac{2x^{-3j/2}(cx)^{3j/2}\left(\sqrt{a}x^{j/2}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

output
```
(-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j -
n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(a^(3/2)*c*(j - n)*x^((3
*j)/2)*Sqrt[a*x^j + b*x^n])
```

3.398.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1937, 1936, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{3j}{2}-1}}{(ax^j+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{1937}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\int \frac{x^{\frac{3j}{2}-1}}{(ax^j+bx^n)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{1936}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\left(\frac{\int \frac{x^{\frac{j-2}{2}}}{\sqrt{ax^j+bx^n}}dx}{a}-\frac{2x^{j/2}}{a(j-n)\sqrt{ax^j+bx^n}}\right)}{c}$$

$$\Big\downarrow \text{1935}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\left(\frac{2\int \frac{1}{1-\frac{ax^j}{ax^j+bx^n}}d\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}}{a(j-n)}-\frac{2x^{j/2}}{a(j-n)\sqrt{ax^j+bx^n}}\right)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{-3j/2}(cx)^{3j/2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}(j-n)}-\frac{2x^{j/2}}{a(j-n)\sqrt{ax^j+bx^n}}\right)}{c}$$

input
```
Int[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

output
```
((c*x)^((3*j)/2)*((-2*x^(j/2))/(a*(j - n)*Sqrt[a*x^j + b*x^n]) + (2*ArcTan
h[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*(j - n))))/(c*x^((3*j)/
2))
```

3.398. $\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}dx$

3.398.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.398.4 Maple [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(a\,x^{j}+b\,x^{n})^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

3.398.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.398.6 Sympy [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3j}{2}-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**(3*j/2 - 1)/(a*x**j + b*x**n)**(3/2), x)
```

3.398.7 Maxima [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}j-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

3.398.8 Giac [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{3}{2}j-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(c\,x)^{\frac{3\,j}{2}-1}}{(a\,x^j+b\,x^n)^{3/2}}\,dx$$

input
```
int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2),x)
```

output
```
int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2), x)
```

3.399 $\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx$

3.399.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{2c^3\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}$$

output
```
2*c^3*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/a^(3/2)/(3-
n)/x^(1/2)-2*c^2*(c*x)^(3/2)/a/(3-n)/(a*x^3+b*x^n)^(1/2)
```

3.399.2 Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=\frac{2c^3\sqrt{cx}\left(\sqrt{a}x^{3/2}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{3-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-3+n)\sqrt{x}\sqrt{ax^3+bx^n}}$$

input
```
Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

output
```
(2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))
/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-3 + n)*Sqrt[x]*S
qrt[a*x^3 + b*x^n])
```

3.399.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{1936}$$

$$\frac{c^3\int \frac{\sqrt{cx}}{\sqrt{bx^n+ax^3}}dx}{a}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{c^3\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{bx^n+ax^3}}dx}{a\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2c^3\sqrt{cx}\int \frac{1}{1-\frac{ax^3}{bx^n+ax^3}}d\frac{x^{3/2}}{\sqrt{bx^n+ax^3}}}{a(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2c^3\sqrt{cx}\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

input
```
Int[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Ar
cTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

3.399.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.399.4 Maple [F]

$$\int \frac{(cx)^{\frac{7}{2}}}{(a\,x^3+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

3.399.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{7/2}}{(ax^3 + bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^3 + bx^n)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2),x)
```

output
```
Timed out
```

3.399.7 Maxima [F]

$$\int \frac{(cx)^{7/2}}{(ax^3 + bx^n)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(ax^3 + bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

3.399.8 Giac [F]

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(ax^3+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = \int \frac{(c\,x)^{7/2}}{(b\,x^n+a\,x^3)^{3/2}}\,dx$$

input
```
int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2),x)
```

output
```
int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2), x)
```

3.400 $\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx$

3.400.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}$$

output
```
2*c^2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/a^(3/2)/(2-n)-2*c^2*x/a/(2-n)
/(a*x^2+b*x^n)^(1/2)
```

3.400.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \frac{2c^2\left(\sqrt{a}x-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-2+n)\sqrt{ax^2+bx^n}}$$

input
```
Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

output
```
(2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqr
t[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

3.400.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {27, 1936, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int \frac{x^2}{(bx^n+ax^2)^{3/2}}dx$$

$$\Big\downarrow \text{1936}$$

$$c^2\left(\frac{\int \frac{1}{\sqrt{bx^n+ax^2}}dx}{a}-\frac{2x}{a(2-n)\sqrt{ax^2+bx^n}}\right)$$

$$\Big\downarrow \text{1914}$$

$$c^2\left(\frac{2\int \frac{1}{1-\frac{ax^2}{bx^n+ax^2}}d\frac{x}{\sqrt{bx^n+ax^2}}}{a(2-n)}-\frac{2x}{a(2-n)\sqrt{ax^2+bx^n}}\right)$$

$$\Big\downarrow \text{219}$$

$$c^2\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2x}{a(2-n)\sqrt{ax^2+bx^n}}\right)$$

input
```
Int[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

output
```
c^2*((-2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*ArcTanh[(Sqrt[a]*x)/Sqrt[
a*x^2 + b*x^n]])/(a^(3/2)*(2 - n)))
```

3.400.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

3.400.4 Maple [F]

$$\int \frac{c^2x^2}{\left(a\,x^2+b\,x^n\right)^{\frac{3}{2}}}dx$$

input
```
int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

output
```
int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

3.400.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.400.6 Sympy [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = c^2\int \frac{x^2}{ax^2\sqrt{ax^2+bx^n}+bx^n\sqrt{ax^2+bx^n}}\,dx$$

input
```
integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)
```

output
```
c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*
x**n)), x)
```

3.400.7 Maxima [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2x^2}{(ax^2+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)
```

3.400.8 Giac [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2x^2}{(ax^2+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2\,x^2}{(b\,x^n+a\,x^2)^{3/2}}\,dx$$

input
```
int((c^2*x^2)/(b*x^n + a*x^2)^(3/2),x)
```

output
```
int((c^2*x^2)/(b*x^n + a*x^2)^(3/2), x)
```

3.401 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$

3.401.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{2c\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}$$

output
```
2*c*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/(1-n)/(c*x)
^(1/2)-2*(c*x)^(1/2)/a/(1-n)/(a*x+b*x^n)^(1/2)
```

3.401.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \frac{2\sqrt{cx}\left(\sqrt{a}\sqrt{x}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{1-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-1+n)\sqrt{x}\sqrt{ax+bx^n}}$$

input
```
Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

output
```
(2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*
ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[
a*x + b*x^n])
```

3.401.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}dx$$
$$\Big\downarrow \text{ 1936}$$
$$\frac{c\int \frac{1}{\sqrt{cx}\sqrt{bx^n+ax}}dx}{a}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$
$$\Big\downarrow \text{ 1937}$$
$$\frac{c\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{bx^n+ax}}dx}{a\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$
$$\Big\downarrow \text{ 1935}$$
$$\frac{2c\sqrt{x}\int \frac{1}{1-\frac{ax}{bx^n+ax}}d\frac{\sqrt{x}}{\sqrt{bx^n+ax}}}{a(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2c\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

input
```
Int[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

output
```
(-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[
a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

3.401.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.401.4 Maple [F]

$$\int \frac{\sqrt{cx}}{(ax + b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

output
```
int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

3.401.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.401.6 Sympy [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)
```

output
```
Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)
```

3.401.7 Maxima [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

3.401.8 Giac [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^n+a\,x)^{3/2}}\,dx$$

input
```
int((c*x)^(1/2)/(b*x^n + a*x)^(3/2),x)
```

output
```
int((c*x)^(1/2)/(b*x^n + a*x)^(3/2), x)
```

3.402 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$

3.402.1 Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{1}{cx\,(a+bx^n)^{3/2}}\,dx=\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/c/n+2/a/c/n/(a+b*x^n)^(1/2)
```

3.402.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{cx\,(a+bx^n)^{3/2}}\,dx=\frac{\frac{2}{an\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}}{c}$$

input
```
Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]
```

output
```
(2/(a*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*n
))/c
```

3.402.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {27, 798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{cx(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x(bx^n+a)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{(bx^n+a)^{3/2}}dx^n}{cn}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{a}+\frac{2}{a\sqrt{a+bx^n}}}{cn}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{ab}+\frac{2}{a\sqrt{a+bx^n}}}{cn}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2}{a\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}}{cn}$$

input
```
Int[1/(c*x*(a + b*x^n)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2))/(c*
n)
```

3.402.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.402.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{cn}$	42
default	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{cn}$	42

input
```
int(1/c/x/(a+b*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/n*(2/a/(a+b*x^n)^(1/2)-2/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \left[\frac{\left(\sqrt{a}bx^n+a^{\frac{3}{2}}\right)\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}a}{a^2bcnx^n+a^3cn}, \frac{2\left(\left(\sqrt{-a}bx^n+\sqrt{-a}a\right)\text{ar}\right.}{a^2bc}\right.$$

input
```
integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
[((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/
x^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*c*n*x^n + a^3*c*n), 2*((sqrt(-a)*b*x^n
+ sqrt(-a)*a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a)*a)/(a^2
*b*c*n*x^n + a^3*c*n)]
```

3.402.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(42) = 84$.

Time = 1.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.43

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \frac{\frac{2a^3\sqrt{1+\frac{bx^n}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^3\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^2bx^n\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}-\frac{2a^2bx^n\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}}{c}$$

input
```
integrate(1/c/x/(a+b*x**n)**(3/2),x)
```

output
```
(2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x
**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) +
1)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*
n + a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/
2)*n + a**(7/2)*b*n*x**n))/c
```

3.402.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{1}{cx\,(a+bx^n)^{3/2}}\,dx = \frac{\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n}+\frac{2}{\sqrt{bx^n+a}an}}{c}$$

input
```
integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
(log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n)
+ 2/(sqrt(b*x^n + a)*a*n))/c
```

3.402.8 Giac [F]

$$\int \frac{1}{cx\,(a+bx^n)^{3/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}cx}\,dx$$

input
```
integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*c*x), x)
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{cx\,(a+bx^n)^{3/2}}\,dx = \int \frac{1}{c\,x\,(a+b\,x^n)^{3/2}}\,dx$$

input
```
int(1/(c*x*(a + b*x^n)^(3/2)),x)
```

output
```
int(1/(c*x*(a + b*x^n)^(3/2)), x)
```

3.403 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$

3.403.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}$$

output
```
-2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^2/(1+n)/(c
*x)^(1/2)+2/a/c^2/(1+n)/(c*x)^(1/2)/(a/x+b*x^n)^(1/2)
```

3.403.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{1+n}}\,\text{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]
))/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n])
```

3.403.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2} \left(\frac{a}{x}+bx^n\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1936}$$

$$\frac{\int \frac{1}{(cx)^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{ac}+\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x}}}dx}{ac^2\sqrt{cx}}+\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x\left(bx^n+\frac{a}{x}\right)}}d\frac{1}{\sqrt{x}\sqrt{bx^n+\frac{a}{x}}}}{ac^2(n+1)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

output
```
2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]
/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

3.403.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.403.4 Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}} \left(\frac{a}{x}+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

output
```
int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

3.403.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.403.6 Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{(cx)^{\frac{5}{2}}\left(\frac{a}{x}+bx^n\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)
```

output
```
Integral(1/((c*x)**(5/2)*(a/x + b*x**n)**(3/2)), x)
```

3.403.7 Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

3.403.8 Giac [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{5/2}\left(b\,x^n+\frac{a}{x}\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)), x)
```

3.404 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$

3.404.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

output
```
-2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/a^(3/2)/c^4/(2+n)+2/a/c^4/(2+n)/
x/(a/x^2+b*x^n)^(1/2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{2+n}}\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

input
```
Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]
))/(a^(3/2)*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

3.404.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {27, 1936, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^4\left(bx^n+\frac{a}{x^2}\right)^{3/2}}dx}{c^4}$$

$$\Big\downarrow \text{1936}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{bx^n+\frac{a}{x^2}}}dx}{a}+\frac{2}{a(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}}{c^4}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\frac{2}{a(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\int \frac{1}{1-\frac{a}{x^2\left(bx^n+\frac{a}{x^2}\right)}}d\frac{1}{x\sqrt{bx^n+\frac{a}{x^2}}}}{a(n+2)}}{c^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2}{a(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}(n+2)}}{c^4}$$

input
```
Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

output
```
(2/(a*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 +
b*x^n])])/(a^(3/2)*(2 + n)))/c^4
```

3.404.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

3.404.4 Maple [F]

$$\int \frac{1}{c^4 x^4 \left(\frac{a}{x^2} + b\, x^n\right)^{\frac{3}{2}}} dx$$

input
```
int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

output
```
int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

3.404.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.404.6 Sympy [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{ax^2\sqrt{\frac{a}{x^2}+bx^n}+bx^4x^n\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^4}$$

input
```
integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)
```

output
```
Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**
n)), x)/c**4
```

3.404.7 Maxima [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

input
```
integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4
```

3.404.8 Giac [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

input
```
integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)
```

3.404.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c^4\,x^4\,\left(b\,x^n+\frac{a}{x^2}\right)^{3/2}}\,dx$$

input
```
int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)),x)
```

output
```
int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)), x)
```

3.405 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$

3.405.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\,\mathrm{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}$$

output
```
-2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^5/(3+n)/
(c*x)^(1/2)+2/a/c^4/(3+n)/(c*x)^(3/2)/(a/x^3+b*x^n)^(1/2)
```

3.405.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{3+n}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]
))/(a^(3/2)*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

3.405.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1936, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{1936}$$

$$\frac{\int \frac{1}{(cx)^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{ac^3}+\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{bx^n+\frac{a}{x^3}}}dx}{ac^5\sqrt{cx}}+\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\int \frac{1}{1-\frac{a}{x^3\left(bx^n+\frac{a}{x^3}\right)}}d\frac{1}{x^{3/2}\sqrt{bx^n+\frac{a}{x^3}}}}{ac^5(n+3)\sqrt{cx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

input
```
Int[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

output
```
2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqr
t[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

3.405.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

3.405.4 Maple [F]

$$\int \frac{1}{(cx)^{\frac{11}{2}} \left(\frac{a}{x^3} + b\,x^n\right)^{\frac{3}{2}}} dx$$

input
```
int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

output
```
int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

3.405.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)
```

output
```
Timed out
```

3.405.7 Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

3.405.8 Giac [F]

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{11/2}\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)),x)
```

output
```
int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)), x)
```

3.406 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$

3.406.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}$$

output
```
-2*arctanh(a^(1/2)/x^2/(a/x^4+b*x^n)^(1/2))/a^(3/2)/c^7/(4+n)+2/a/c^7/(4+n
)/x^2/(a/x^4+b*x^n)^(1/2)
```

3.406.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{4+n}}\text{arctanh}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}$$

input
```
Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

output
```
(2*(Sqrt[a] - Sqrt[a + b*x^(4 + n)]*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]
))/(a^(3/2)*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n])
```

3.406.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {27, 1936, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x^7\left(bx^n+\frac{a}{x^4}\right)^{3/2}}dx}{c^7}$$

$$\Big\downarrow \text{ 1936}$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{bx^n+\frac{a}{x^4}}}dx}{a}+\frac{2}{a(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}}{c^7}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{\frac{2}{a(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\int \frac{1}{1-\frac{a}{x^4\left(bx^n+\frac{a}{x^4}\right)}}d\frac{1}{x^2\sqrt{bx^n+\frac{a}{x^4}}}}{a(n+4)}}{c^7}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{2}{a(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}(n+4)}}{c^7}$$

input
```
Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

output
```
(2/(a*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^
4 + b*x^n])])/(a^(3/2)*(4 + n)))/c^7
```

3.406.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1936
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (
IntegerQ[j] || GtQ[c, 0])
```

3.406.4 Maple [F]

$$\int \frac{1}{c^7 x^7 \left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}}dx$$

input
```
int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

output
```
int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

3.406.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.406.6 Sympy [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{ax^3\sqrt{\frac{a}{x^4}+bx^n}+bx^7x^n\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{c^7}$$

input
```
integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)
```

output
```
Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**
n)), x)/c**7
```

3.406.7 Maxima [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

input
```
integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a/x^4)^(3/2)*x^7), x)/c^7
```

3.406.8 Giac [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

input
```
integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c^7\,x^7\,\left(b\,x^n+\frac{a}{x^4}\right)^{3/2}}\,dx$$

input
```
int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)),x)
```

output
```
int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)), x)
```

3.407 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$

3.407.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctanh(x*b^(1/2)/(a/x+b*x^2)^(1/2))/b^(1/2)
```

3.407.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx=\frac{2\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx^3}{x}}}$$

input
```
Integrate[1/Sqrt[(a + b*x^3)/x],x]
```

output
```
(2*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqrt
[x]*Sqrt[(a + b*x^3)/x])
```

3.407.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x}+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2}{3}\int \frac{1}{1-\frac{bx^2}{bx^2+\frac{a}{x}}}d\frac{x}{\sqrt{bx^2+\frac{a}{x}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

input
```
Int[1/Sqrt[(a + b*x^3)/x],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])
```

3.407.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.407.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

method	result	size
default	$\frac{2(b\,x^{3}+a)\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{\frac{b\,x^{3}+a}{x}}\,\sqrt{x(b\,x^{3}+a)}\,\sqrt{b}}$	56

input
```
int(1/((b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((b*x^3+a)/x)^(1/2)*(b*x^3+a)/(x*(b*x^3+a))^(1/2)/b^(1/2)*arctanh(1/x^
2*(x*(b*x^3+a))^(1/2)/b^(1/2))
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\,(2\,bx^5+ax^2)\sqrt{b}\sqrt{\frac{bx^3+a}{x}}\right)}{6\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}x^2\sqrt{\frac{bx^3+a}{x}}}{2\,bx^3+a}\right)}{3\,b}\right]$$

input
```
integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((
b*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(-b)*x^2*sqrt((b*x^3 +
a)/x)/(2*b*x^3 + a))/b]
```

3.407.6 Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$$

input
```
integrate(1/((b*x**3+a)/x)**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**3)/x), x)
```

3.407.7 Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^3+a}{x}}}\,dx$$

input
```
integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^3 + a)/x), x)
```

3.407.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^3+a}{x}}}\,dx$$

input
```
int(1/((a + b*x^3)/x)^(1/2),x)
```

output
```
int(1/((a + b*x^3)/x)^(1/2), x)
```

3.408 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}} dx$

3.408.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*arctanh(x*b^(1/2)/(a/x^2+b*x^2)^(1/2))/b^(1/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}} dx = \frac{\sqrt{a+bx^4}\log\left(\sqrt{b}x^2+\sqrt{a+bx^4}\right)}{2\sqrt{b}x\sqrt{\frac{a+bx^4}{x^2}}}$$

input
```
Integrate[1/Sqrt[(a + b*x^4)/x^2],x]
```

output
```
(Sqrt[a + b*x^4]*Log[Sqrt[b]*x^2 + Sqrt[a + b*x^4]])/(2*Sqrt[b]*x*Sqrt[(a
+ b*x^4)/x^2])
```

3.408.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^2}+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{bx^2}{bx^2+\frac{a}{x^2}}}d\frac{x}{\sqrt{bx^2+\frac{a}{x^2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

input
```
Int[1/Sqrt[(a + b*x^4)/x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])
```

3.408.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.408.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{\sqrt{b\,x^{4}+a}\,\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2\sqrt{\frac{b\,x^{4}+a}{x^{2}}}\,x\sqrt{b}}$	49

input
```
int(1/((b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((b*x^4+a)/x^2)^(1/2)/x*(b*x^4+a)^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)
)/b^(1/2)
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \left[\frac{\log\left(-2\,bx^4-2\,\sqrt{b}x^3\sqrt{\frac{bx^4+a}{x^2}}-a\right)}{4\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^3\sqrt{\frac{bx^4+a}{x^2}}}{bx^4+a}\right)}{2\,b}\right]$$

input
```
integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2
*sqrt(-b)*arctan(sqrt(-b)*x^3*sqrt((b*x^4 + a)/x^2)/(b*x^4 + a))/b]
```

3.408.6 Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$$

input
```
integrate(1/((b*x**4+a)/x**2)**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**4)/x**2), x)
```

3.408.7 Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^4+a}{x^2}}}\,dx$$

input
```
integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)
```

3.408.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \frac{\log(|a|)\,\mathrm{sgn}(x)}{4\sqrt{b}} - \frac{\log\left(\left|-\sqrt{b}x^2+\sqrt{bx^4+a}\right|\right)}{2\sqrt{b}\mathrm{sgn}\,(x)}$$

input
```
integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*log(abs(a))*sgn(x)/sqrt(b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a
)))/(sqrt(b)*sgn(x))
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^4+a}{x^2}}}\,dx$$

input
```
int(1/((a + b*x^4)/x^2)^(1/2),x)
```

output
```
int(1/((a + b*x^4)/x^2)^(1/2), x)
```

3.409 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx$

3.409.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

output
```
2/5*arctanh(x*b^(1/2)/(a/x^3+b*x^2)^(1/2))/b^(1/2)
```

3.409.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \frac{2\sqrt{a+bx^5}\log\left(\sqrt{b}x^{5/2}+\sqrt{a+bx^5}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}}$$

input
```
Integrate[1/Sqrt[(a + b*x^5)/x^3],x]
```

output
```
(2*Sqrt[a + b*x^5]*Log[Sqrt[b]*x^(5/2) + Sqrt[a + b*x^5]])/(5*Sqrt[b]*x^(3
/2)*Sqrt[(a + b*x^5)/x^3])
```

3.409.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^3}+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2}{5}\int \frac{1}{1-\frac{bx^2}{bx^2+\frac{a}{x^3}}}d\frac{x}{\sqrt{bx^2+\frac{a}{x^3}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

input
```
Int[1/Sqrt[(a + b*x^5)/x^3],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])
```

3.409.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.409.4 Maple [F]

$$\int \frac{1}{\sqrt{\frac{b\,x^{5}+a}{x^{3}}}}dx$$

input
```
int(1/((b*x^5+a)/x^3)^(1/2),x)
```

output
```
int(1/((b*x^5+a)/x^3)^(1/2),x)
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \left[\frac{\log\left(-8\,b^2x^{10} - 8\,abx^5 - a^2 - 4\,(2\,bx^9 + ax^4)\sqrt{b}\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}x^4\sqrt{\frac{bx^5+a}{x^3}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

input
```
integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt
((b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^
5 + a)/x^3)/(2*b*x^5 + a))/b]
```

3.409.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((b*x**5+a)/x**3)**(1/2),x)
```

output
```
Timed out
```

3.409.7 Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}}\,dx$$

input
```
integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^5 + a)/x^3), x)
```

3.409.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}\,dx$$

input
```
int(1/((a + b*x^5)/x^3)^(1/2),x)
```

output
```
int(1/((a + b*x^5)/x^3)^(1/2), x)
```

3.410 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}} \, dx$

3.410.1 Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^{2-n}\,(a+bx^n)}} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

3.410.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x^{2-n}\,(a+bx^n)}} \, dx = \frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}\,(a+bx^n)}}$$

input
```
Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

output
```
(2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[
a]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])
```

3.410.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{ax^{2-n}+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^{2-n}+bx^2}}d\frac{x}{\sqrt{ax^{2-n}+bx^2}}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}n}$$

input
```
Int[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

3.410.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.410.4 Maple [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+b\,x^{n}\right)}}dx$$

input
```
int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^{n}\right)}}\,dx = \left[\frac{\log\left(\frac{2\,bxx^{n}+ax+2\,\sqrt{b}x^{n}\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{x}\right)}{\sqrt{b}n}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{bx}\right)}{bn}\right]$$

input
```
integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

output
```
[log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(s
qrt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^2*x^n + a*x^2)/x^n)/(b*x)
)/(b*n)]
```

3.410.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx$$

input
```
integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(x**(2 - n)*(a + b*x**n)), x)
```

3.410.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\int \frac{1}{\sqrt{(bx^n+a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

3.410.8 Giac [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\int \frac{1}{\sqrt{(bx^n+a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx = \int \frac{1}{\sqrt{x^{2-n}\,(a+b\,x^n)}}\,dx$$

input
```
int(1/(x^(2 - n)*(a + b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2 - n)*(a + b*x^n))^(1/2), x)
```

3.411 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$

3.411.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

output
```
2/3*arctan(x*b^(1/2)/(a/x-b*x^2)^(1/2))/b^(1/2)
```

3.411.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx=-\frac{2i\sqrt{a-bx^3}\log\left(i\sqrt{b}x^{3/2}+\sqrt{a-bx^3}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a-bx^3}{x}}}$$

input
```
Integrate[1/Sqrt[(a - b*x^3)/x],x]
```

output
```
(((-2*I)/3)*Sqrt[a - b*x^3]*Log[I*Sqrt[b]*x^(3/2) + Sqrt[a - b*x^3]])/(Sqr
t[b]*Sqrt[x]*Sqrt[(a - b*x^3)/x])
```

3.411.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x}-bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2}{3}\int \frac{1}{\frac{bx^2}{\frac{a}{x}-bx^2}+1}d\frac{x}{\sqrt{\frac{a}{x}-bx^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

input
```
Int[1/Sqrt[(a - b*x^3)/x],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

3.411.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.411.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.

Time = 2.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

method	result	size
default	$-\frac{2(-b\,x^3+a)\arctan\left(\frac{\sqrt{x(-b\,x^3+a)}}{x^2\sqrt{b}}\right)}{3\sqrt{\frac{-b\,x^3+a}{x}}\,\sqrt{x(-b\,x^3+a)}\,\sqrt{b}}$	60

input
```
int(1/((-b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/((-b*x^3+a)/x)^(1/2)*(-b*x^3+a)/(x*(-b*x^3+a))^(1/2)/b^(1/2)*arctan((
x*(-b*x^3+a))^(1/2)/x^2/b^(1/2))
```

3.411.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^6+8\,abx^3-a^2+4\,(2\,bx^5-ax^2)\sqrt{-b}\sqrt{-\frac{bx^3-a}{x}}\right)}{6\,b}, -\frac{\arctan\left(\frac{2\sqrt{b}x^2\sqrt{-\frac{bx^3-a}{x}}}{2\,bx^3-a}\right)}{3\sqrt{b}}\right]$$

input
```
integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt
(-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(2*sqrt(b)*x^2*sqrt(-(b*x^3 - a)/
x)/(2*b*x^3 - a))/sqrt(b)]
```

3.411.6 Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$$

input
```
integrate(1/((-b*x**3+a)/x)**(1/2),x)
```

output
```
Integral(1/sqrt((a - b*x**3)/x), x)
```

3.411.7 Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^3-a}{x}}}\,dx$$

input
```
integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

3.411.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^3}{x}}}\,dx$$

input
```
int(1/((a - b*x^3)/x)^(1/2),x)
```

output
```
int(1/((a - b*x^3)/x)^(1/2), x)
```

3.412 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$

3.412.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*arctan(x*b^(1/2)/(a/x^2-b*x^2)^(1/2))/b^(1/2)
```

3.412.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx=-\frac{i\sqrt{a-bx^4}\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{2\sqrt{b}x\sqrt{\frac{a-bx^4}{x^2}}}$$

input
```
Integrate[1/Sqrt[(a - b*x^4)/x^2],x]
```

output
```
((-1/2*I)*Sqrt[a - b*x^4]*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/(Sqrt[b]*x
*Sqrt[(a - b*x^4)/x^2])
```

3.412.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^2}-bx^2}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{1}{2}\int \frac{1}{\frac{bx^2}{\frac{a}{x^2}-bx^2}+1}d\frac{x}{\sqrt{\frac{a}{x^2}-bx^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

input
```
Int[1/Sqrt[(a - b*x^4)/x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

3.412.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.412.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\sqrt{-b\,x^{4}+a}\,\arctan\left(\frac{x^{2}\sqrt{b}}{\sqrt{-b\,x^{4}+a}}\right)}{2\sqrt{\frac{-b\,x^{4}+a}{x^{2}}}\,x\sqrt{b}}$	51

input
```
int(1/((-b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((-b*x^4+a)/x^2)^(1/2)/x*(-b*x^4+a)^(1/2)*arctan(x^2*b^(1/2)/(-b*x^4+a
)^(1/2))/b^(1/2)
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-b}x^3\sqrt{-\frac{bx^4-a}{x^2}}-a\right)}{4\,b}, -\frac{\arctan\left(\frac{\sqrt{b}x^3\sqrt{-\frac{bx^4-a}{x^2}}}{bx^4-a}\right)}{2\,\sqrt{b}}\right]$$

input
```
integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b,
 -1/2*arctan(sqrt(b)*x^3*sqrt(-(b*x^4 - a)/x^2)/(b*x^4 - a))/sqrt(b)]
```

3.412.6 Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$$

input
```
integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
```

output
```
Integral(1/sqrt((a - b*x**4)/x**2), x)
```

3.412.7 Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^4-a}{x^2}}}\,dx$$

input
```
integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 +
 a)
```

3.412.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \frac{\log(|a|)\operatorname{sgn}(x)}{4\sqrt{-b}} - \frac{\log\left(\left|-\sqrt{-b}x^2 + \sqrt{-bx^4+a}\right|\right)}{2\sqrt{-b}\operatorname{sgn}(x)}$$

input
```
integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*log(abs(a))*sgn(x)/sqrt(-b) - 1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4
+ a)))/(sqrt(-b)*sgn(x))
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^4}{x^2}}}\,dx$$

input
```
int(1/((a - b*x^4)/x^2)^(1/2),x)
```

output
```
int(1/((a - b*x^4)/x^2)^(1/2), x)
```

3.413 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx$

3.413.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

output
```
2/5*arctan(x*b^(1/2)/(a/x^3-b*x^2)^(1/2))/b^(1/2)
```

3.413.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx = -\frac{2i\sqrt{a-bx^5}\log\left(i\sqrt{b}x^{5/2}+\sqrt{a-bx^5}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a-bx^5}{x^3}}}$$

input
```
Integrate[1/Sqrt[(a - b*x^5)/x^3],x]
```

output
```
(((-2*I)/5)*Sqrt[a - b*x^5]*Log[I*Sqrt[b]*x^(5/2) + Sqrt[a - b*x^5]])/(Sqr
t[b]*x^(3/2)*Sqrt[(a - b*x^5)/x^3])
```

3.413.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{\frac{a}{x^3}-bx^2}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2}{5}\int \frac{1}{\frac{bx^2}{\frac{a}{x^3}-bx^2}+1}d\frac{x}{\sqrt{\frac{a}{x^3}-bx^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

input
```
Int[1/Sqrt[(a - b*x^5)/x^3],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

3.413.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.413.4 Maple [F]

$$\int \frac{1}{\sqrt{\frac{-b\,x^{5}+a}{x^{3}}}}dx$$

input
```
int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

output
```
int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^{10}+8\,abx^5-a^2+4\,(2\,bx^9-ax^4)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10\,b},\ -\frac{\arctan\left(\frac{2\,\sqrt{b}x^4\sqrt{-\frac{bx^5-a}{x^3}}}{2\,bx^5-a}\right)}{5\,\sqrt{b}}\right]$$

input
```
integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sq
rt(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 -
 a)/x^3)/(2*b*x^5 - a))/sqrt(b)]
```

3.413.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((-b*x**5+a)/x**3)**(1/2),x)
```

output
```
Timed out
```

3.413.7 Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^5-a}{x^3}}}\,dx$$

input
```
integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^5 - a)/x^3), x)
```

3.413.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^5}{x^3}}}\,dx$$

input
```
int(1/((a - b*x^5)/x^3)^(1/2),x)
```

output
```
int(1/((a - b*x^5)/x^3)^(1/2), x)
```

3.414 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$

3.414.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}$$

output

```
2*arctan(x*b^(1/2)/(-b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

3.414.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(38) = 76.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\arcsin\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}(a-bx^n)}}$$

input

```
Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

output

```
(2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])
```

3.414.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}dx$$
$$\Big\downarrow \text{ 2078}$$
$$\int \frac{1}{\sqrt{ax^{2-n}-bx^2}}dx$$
$$\Big\downarrow \text{ 1914}$$
$$\frac{2\int \frac{1}{\frac{bx^2}{ax^{2-n}-bx^2}+1}d\frac{x}{\sqrt{ax^{2-n}-bx^2}}}{n}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}n}$$

input
```
Int[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

3.414.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.414.4 Maple [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-b\,x^{n}\right)}}dx$$

input
```
int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{2\,bxx^n-ax-2\,\sqrt{-b}x^n\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{x}\right)}{bn}, -\frac{2\,\arctan\left(\frac{\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{\sqrt{b}x}\right)}{\sqrt{b}n}\right]$$

input
```
integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2
)/x^n))/x)/(b*n), -2*arctan(sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(sqrt(b)*x))/(s
qrt(b)*n)]
```

3.414.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx=\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx$$

input
```
integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(x**(2 - n)*(a - b*x**n)), x)
```

3.414.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx=\int \frac{1}{\sqrt{-(bx^{n}-a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

3.414.8 Giac [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx=\int \frac{1}{\sqrt{-(bx^{n}-a)x^{-n+2}}}\,dx$$

input
```
integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^n\right)}}\,dx=\int \frac{1}{\sqrt{x^{2-n}\,\left(a-b\,x^n\right)}} \,dx$$

input
```
int(1/(x^(2 - n)*(a - b*x^n))^(1/2),x)
```

output
```
int(1/(x^(2 - n)*(a - b*x^n))^(1/2), x)
```

3.415 $\int \frac{1}{\sqrt{x^n(a+bx^{2-n})}}\,dx$

3.415.1 Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

3.415.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2)
)/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

3.415.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^n (a+bx^{2-n})}} \, dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{ax^n+bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^n+bx^2}}d\frac{x}{\sqrt{ax^n+bx^2}}}{2-n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

3.415.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.415.4 Maple [F]

$$\int \frac{1}{\sqrt{x^{n} \left(a+b\,x^{2-n}\right)}}dx$$

input
```
int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

output
```
int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

3.415.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x^n (a+bx^{2-n})}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.415.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^n (a+bx^{2-n})}}\,dx = \int \frac{1}{\sqrt{x^n (a+bx^{2-n})}}\,dx$$

input
```
integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**n*(a + b*x**(2 - n))), x)
```

3.415.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^n (a + bx^{2-n})}} \, dx = \int \frac{1}{\sqrt{(bx^{-n+2} + a)x^n}} \, dx$$

input
```
integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

3.415.8 Giac [F]

$$\int \frac{1}{\sqrt{x^n (a + bx^{2-n})}} \, dx = \int \frac{1}{\sqrt{(bx^{-n+2} + a)x^n}} \, dx$$

input
```
integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

3.415.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x^n (a + bx^{2-n})}} \, dx = \frac{\sqrt{a} \, x^{n/2} \operatorname{asin}\left(\frac{\sqrt{b} \, x^{1-\frac{n}{2}} \, 1i}{\sqrt{a}}\right) \sqrt{\frac{b \, x^{2-n}}{a} + 1} \, 1i}{\sqrt{b} \, \left(\frac{n}{2} - 1\right) \sqrt{a \, x^n + b \, x^2}}$$

input
```
int(1/(x^n*(a + b*x^(2 - n)))^(1/2),x)
```

output
```
(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
 1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.416 $\int \frac{1}{\sqrt{x^2(b+ax^{-2+n})}}\,dx$

3.416.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

3.416.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2)
)/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

3.416.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (ax^{n-2}+b)}} dx$$
$$\Big\downarrow \text{ 2078}$$
$$\int \frac{1}{\sqrt{ax^n+bx^2}}dx$$
$$\Big\downarrow \text{ 1914}$$
$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^n+bx^2}}d\frac{x}{\sqrt{ax^n+bx^2}}}{2-n}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

3.416.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.416.4 Maple [F]

$$\int \frac{1}{\sqrt{x^2 (b + a\, x^{-2+n})}} dx$$

input
```
int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

output
```
int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.95

$$\int \frac{1}{\sqrt{x^2 (b + ax^{-2+n})}} dx = \left[\frac{\sqrt{b} \log \left(\frac{axx^{n-2} + 2\,bx - 2\,\sqrt{ax^2x^{n-2} + bx^2}\sqrt{b}}{xx^{n-2}} \right)}{bn - 2\,b}, \frac{2\,\sqrt{-b} \arctan \left(\frac{\sqrt{ax^2x^{n-2} + bx^2}\sqrt{-b}}{bx} \right)}{bn - 2\,b} \right]$$

input
```
integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt
(b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) +
b*x^2)*sqrt(-b)/(b*x))/(b*n - 2*b)]
```

3.416.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^2 (b + ax^{-2+n})}} dx = \int \frac{1}{\sqrt{x^2 (ax^{n-2} + b)}} dx$$

input
```
integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(a*x**(n - 2) + b)), x)
```

3.416.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-2}+b)x^2}}\,dx$$

input
```
integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

3.416.8 Giac [F]

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-2}+b)x^2}}\,dx$$

input
```
integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

3.416.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

input
```
int(1/(x^2*(b + a*x^(n - 2)))^(1/2),x)
```

output
```
(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
 1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.417 $\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$

3.417.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

3.417.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2)
)/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

3.417.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2078, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}+bx\right)}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{ax^n+bx^2}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2\int \frac{1}{1-\frac{bx^2}{ax^n+bx^2}}d\frac{x}{\sqrt{ax^n+bx^2}}}{2-n}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

3.417.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.417.4 Maple [F]

$$\int \frac{1}{\sqrt{x\,(bx+a\,x^{-1+n})}}dx$$

input
```
int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

output
```
int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

3.417.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x\,(bx+ax^{-1+n})}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.417.6 Sympy [F]

$$\int \frac{1}{\sqrt{x\,(bx+ax^{-1+n})}}\,dx = \int \frac{1}{\sqrt{x\,(ax^{n-1}+bx)}}\,dx$$

input
```
integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x*(a*x**(n - 1) + b*x)), x)
```

3.417.7 Maxima [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-1}+bx)x}}\,dx$$

input
```
integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

3.417.8 Giac [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-1}+bx)x}}\,dx$$

input
```
integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

3.417.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

input
```
int(1/(x*(b*x + a*x^(n - 1)))^(1/2),x)
```

output
```
(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
 1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.418 $\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$

3.418.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

3.418.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

3.418.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}dx$$
$$\Big\downarrow \text{ 2078}$$
$$\int \frac{1}{\sqrt{ax^n-bx^2}}dx$$
$$\Big\downarrow \text{ 1914}$$
$$\frac{2\int \frac{1}{\frac{bx^2}{ax^n-bx^2}+1}d\frac{x}{\sqrt{ax^n-bx^2}}}{2-n}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

3.418.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.418.4 Maple [F]

$$\int \frac{1}{\sqrt{x^n \left(a - b\, x^{2-n}\right)}} dx$$

input
```
int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

output
```
int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

3.418.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.418.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}}\, dx = \int \frac{1}{\sqrt{x^n (a - bx^{2-n})}}\, dx$$

input
```
integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**n*(a - b*x**(2 - n))), x)
```

3.418.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}}\,dx = \int \frac{1}{\sqrt{-(bx^{-n+2} - a)x^n}}\,dx$$

input
```
integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

3.418.8 Giac [F]

$$\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}}\,dx = \int \frac{1}{\sqrt{-(bx^{-n+2} - a)x^n}}\,dx$$

input
```
integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

3.418.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}}\,dx = -\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\sqrt{a\,x^n-b\,x^2}}$$

input
```
int(1/(x^n*(a - b*x^(2 - n)))^(1/2),x)
```

output
```
-(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a
)^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.419 $\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx$

3.419.1 Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

3.419.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. $2(38) = 76$.

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

3.419.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (a x^{n-2}-b)}} \, dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{a x^n-b x^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2 \int \frac{1}{\frac{b x^2}{a x^n-b x^2}+1}d\frac{x}{\sqrt{a x^n-b x^2}}}{2-n}$$

$$\Big\downarrow \text{216}$$

$$\frac{2 \arctan \left(\frac{\sqrt{b} x}{\sqrt{a x^n-b x^2}}\right)}{\sqrt{b} (2-n)}$$

input
```
Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

3.419.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.419.4 Maple [F]

$$\int \frac{1}{\sqrt{x^{2} \left(-b+a\,x^{-2+n}\right)}}dx$$

input
```
int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

output
```
int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int \frac{1}{\sqrt{x^2 (-b+ax^{-2+n})}}\,dx = \left[-\frac{\sqrt{-b}\log\left(\frac{axx^{n-2}-2\,bx-2\,\sqrt{ax^2x^{n-2}-bx^2}\sqrt{-b}}{xx^{n-2}}\right)}{bn-2\,b}, \frac{2\,\sqrt{b}\arctan\left(\frac{\sqrt{ax^2x^{n-2}-bx^2}}{\sqrt{b}x}\right)}{bn-2\,b}\right]$$

input
```
integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sq
rt(-b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2)
- b*x^2)/(sqrt(b)*x))/(b*n - 2*b)]
```

3.419.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^2 (-b+ax^{-2+n})}}\,dx = \int \frac{1}{\sqrt{x^2 (ax^{n-2}-b)}}\,dx$$

input
```
integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(a*x**(n - 2) - b)), x)
```

3.419.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = \int \frac{1}{\sqrt{(ax^{n-2}-b)x^2}}\,dx$$

input
```
integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

3.419.8 Giac [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = \int \frac{1}{\sqrt{(ax^{n-2}-b)x^2}}\,dx$$

input
```
integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

3.419.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = -\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

input
```
int(1/(-x^2*(b - a*x^(n - 2)))^(1/2),x)
```

output
```
-(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a
)^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.420 $\int \frac{1}{\sqrt{x(-bx+ax^{-1+n})}} dx$

3.420.1 Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}} dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

output
```
2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

3.420.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}} dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

input
```
Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

output
```
(-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

3.420.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2078, 1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}-bx\right)}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{ax^n-bx^2}}dx$$

$$\Big\downarrow \text{1914}$$

$$\frac{2\int \frac{1}{\frac{bx^2}{ax^n-bx^2}+1}d\frac{x}{\sqrt{ax^n-bx^2}}}{2-n}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

input
```
Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

3.420.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.420.4 Maple [F]

$$\int \frac{1}{\sqrt{x\left(-bx+a\,x^{-1+n}\right)}}dx$$

input
```
int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

output
```
int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

3.420.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.420.6 Sympy [F]

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \int \frac{1}{\sqrt{x\left(ax^{n-1}-bx\right)}}\,dx$$

input
```
integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)
```

output
```
Integral(1/sqrt(x*(a*x**(n - 1) - b*x)), x)
```

3.420.7 Maxima [F]

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \int \frac{1}{\sqrt{(ax^{n-1}-bx)x}}\,dx$$

input
```
integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

3.420.8 Giac [F]

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \int \frac{1}{\sqrt{(ax^{n-1}-bx)x}}\,dx$$

input
```
integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

3.420.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = -\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

input
```
int(1/(-x*(b*x - a*x^(n - 1)))^(1/2),x)
```

output
```
-(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a
)^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.421 $\int (cx)^m \left(ax^j+bx^n\right)^{3/2} \, dx$

3.421.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} \, dx=\frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n},1+\frac{1+m+\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

output
```
2*b*x^(1+n)*(c*x)^m*hypergeom([-3/2, (1+m+3/2*n)/(j-n)],[1+(1+m+3/2*n)/(j-
n)],-a*x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

3.421.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(107) = 214.

Time = 0.57 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.04

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} \, dx=\frac{2(cx)^m\left((2+4j+2m-n)x^{-m}(ax^j+bx^n)\left(a(2-j+2m+4n)x^{1+j+m}+b(2+2j+2m+\right.\right.}{(2+4j+2m-n)(2+2j+}$$

input
```
Integrate[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*(c*x)^m*(((2 + 4*j + 2*m - n)*(a*x^j + b*x^n)*(a*(2 - j + 2*m + 4*n)*x^
(1 + j + m) + b*(2 + 2*j + 2*m + n)*x^(1 + m + n)))/x^m + 3*a^2*(j - n)^2*
x^(1 + 2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 4*j + 2*
m - n)/(2*j - 2*n), (2 + 6*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]
))/((2 + 4*j + 2*m - n)*(2 + 2*j + 2*m + n)*(2 + 2*m + 3*n)*Sqrt[a*x^j + b
*x^n])
```

3.421.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}} \sqrt{ax^j+bx^n} \int x^{m+\frac{3n}{2}} \left(ax^{j-n}+b\right)^{3/2} dx}{\sqrt{ax^{j-n}+b}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{b(cx)^m x^{-m-\frac{n}{2}} \sqrt{ax^j+bx^n} \int x^{m+\frac{3n}{2}} \left(\frac{ax^{j-n}}{b}+1\right)^{3/2} dx}{\sqrt{\frac{ax^{j-n}}{b}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2bx^{n+1}(cx)^m \sqrt{ax^j+bx^n} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n},\frac{m+\frac{3n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

input
```
Int[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*b*x^(1 + n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + m
+ (3*n)/2)/(j - n), 1 + (1 + m + (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/((
2 + 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b])
```

3.421.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.421.4 Maple [F]

$$\int (cx)^m \left(a\,x^j + b\,x^n\right)^{\frac{3}{2}} dx$$

input
```
int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

3.421.5 Fricas [F(-2)]

Exception generated.

$$\int (cx)^m \left(ax^j + bx^n\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.421.6 Sympy [F]

$$\int (cx)^m \left(ax^j + bx^n\right)^{3/2} dx = \int (cx)^m \left(ax^j + bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x)**m*(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**m*(a*x**j + b*x**n)**(3/2), x)
```

3.421.7 Maxima [F]

$$\int (cx)^m \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^m dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

3.421.8 Giac [F]

$$\int (cx)^m \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^m dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} \, dx=\int (c\,x)^m\,{\left(a\,x^j+b\,x^n\right)}^{3/2} \,d x$$

input
```
int((c*x)^m*(a*x^j + b*x^n)^(3/2),x)
```

output
```
int((c*x)^m*(a*x^j + b*x^n)^(3/2), x)
```

3.422 $\int (cx)^m \sqrt{ax^j + bx^n}\, dx$

3.422.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx = \frac{2x(cx)^m \sqrt{ax^j + bx^n}\, \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1+m+\frac{n}{2}}{j-n}, 1 + \frac{2+2m+n}{2j-2n}, -\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

output
```
2*x*(c*x)^m*hypergeom([-1/2, (1+m+1/2*n)/(j-n)],[1+(2+2*m+n)/(2*j-2*n)],-a
*x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+n)/(1+a*x^(j-n)/b)^(1/2)
```

3.422.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.56

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx = \frac{2x(cx)^m \left((2+2j+2m-n)(ax^j+bx^n) - a(j-n)x^j \sqrt{1+\frac{ax^{j-n}}{b}}\, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+2j+2m-n}{2j-2n}, 2+\right.\right.}{(2+2j+2m-n)(2+2m+n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(c*x)^m*((2 + 2*j + 2*m - n)*(a*x^j + b*x^n) - a*(j - n)*x^j*Sqrt[1 +
 (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*j + 2*m - n)/(2*j - 2*n),
(2 + 4*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + 2*j + 2*m -
 n)*(2 + 2*m + n)*Sqrt[a*x^j + b*x^n])
```

3.422.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}}\sqrt{ax^j+bx^n}\int x^{m+\frac{n}{2}}\sqrt{ax^{j-n}+b}dx}{\sqrt{ax^{j-n}+b}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(cx)^m x^{-m-\frac{n}{2}}\sqrt{ax^j+bx^n}\int x^{m+\frac{n}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}dx}{\sqrt{\frac{ax^{j-n}}{b}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+\frac{n}{2}+1}{j-n},\frac{2m+n+2}{2j-2n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

input
```
Int[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (1 + m + n/2)/(j
- n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*S
qrt[1 + (a*x^(j - n))/b])
```

3.422.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.422.4 Maple [F]

$$\int (cx)^m \sqrt{a\,x^j + b\,x^n}dx$$

input
```
int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

3.422.5 Fricas [F(-2)]

Exception generated.

$$\int (cx)^m\sqrt{ax^j + bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.422.6 Sympy [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int (cx)^m\,\sqrt{ax^j+bx^n}\,dx$$

input
```
integrate((c*x)**m*(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**m*sqrt(a*x**j + b*x**n), x)
```

3.422.7 Maxima [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

3.422.8 Giac [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx = \int (c\, x)^m\, \sqrt{a\, x^j + b\, x^n}\, dx$$

input
```
int((c*x)^m*(a*x^j + b*x^n)^(1/2),x)
```

output
```
int((c*x)^m*(a*x^j + b*x^n)^(1/2), x)
```

3.423 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$

3.423.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n},1+\frac{1+m-\frac{n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

output
```
2*x*(c*x)^m*hypergeom([1/2, (1+m-1/2*n)/(j-n)],[1+(1+m-1/2*n)/(j-n)],-a*x^
(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/(2+2*m-n)/(a*x^j+b*x^n)^(1/2)
```

3.423.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+2m-n}{2j-2n},1+\frac{2+2m-n}{2j-2n},-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*m - n
)/(2*j - 2*n), 1 + (2 + 2*m - n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2
*m - n)*Sqrt[a*x^j + b*x^n])
```

3.423.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$$
$$\Big\downarrow \text{1938}$$
$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{ax^{j-n}+b}\int \frac{x^{m-\frac{n}{2}}}{\sqrt{ax^{j-n}+b}}dx}{\sqrt{ax^j+bx^n}}$$
$$\Big\downarrow \text{889}$$
$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{m-\frac{n}{2}}}{\sqrt{\frac{ax^{j-n}}{b}+1}}dx}{\sqrt{ax^j+bx^n}}$$
$$\Big\downarrow \text{888}$$
$$\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{n}{2}+1}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n},\frac{m-\frac{n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

input
```
Int[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x^(1 + m - n/2 + (-2*m + n)/2)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hyperg
eometric2F1[1/2, (1 + m - n/2)/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^
(j - n))/b)])/((2 + 2*m - n)*Sqrt[a*x^j + b*x^n])
```

3.423.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.423.4 Maple [F]

$$\int \frac{(cx)^m}{\sqrt{a\,x^j+b\,x^n}}dx$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

output
```
int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

3.423.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.423.6 Sympy [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} \, dx = \int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} \, dx$$

input
```
integrate((c*x)**m/(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral((c*x)**m/sqrt(a*x**j + b*x**n), x)
```

3.423.7 Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} \, dx = \int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} \, dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

3.423.8 Giac [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} \, dx = \int \frac{(cx)^m}{\sqrt{ax^j + bx^n}} \, dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx=\int \frac{{\left(c\,x\right)}^m}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

input
```
int((c*x)^m/(a*x^j + b*x^n)^(1/2),x)
```

output
```
int((c*x)^m/(a*x^j + b*x^n)^(1/2), x)
```

3.424 $\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx$

3.424.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx=\frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n},1+\frac{1+m-\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-n)*(c*x)^m*hypergeom([3/2, (1+m-3/2*n)/(j-n)],[1+(1+m-3/2*n)/(j-n)]
,-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2+2*m-3*n)/(a*x^j+b*x^n)^(1/2)
```

3.424.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx=\frac{2x^{1-j}(cx)^m\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-2j+2m-n}{2j-2n},\frac{2+2m-3n}{2j-2n},-\frac{ax^{j-n}}{b}\right.\right.}{a(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*x^(1 - j)*(c*x)^m*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2
, (2 - 2*j + 2*m - n)/(2*j - 2*n), (2 + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j
- n))/b)]))/(a*(j - n)*Sqrt[a*x^j + b*x^n])
```

3.424.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{1938}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{ax^{j-n}+b}\int \frac{x^{m-\frac{3n}{2}}}{(ax^{j-n}+b)^{3/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{m-\frac{3n}{2}}}{\left(\frac{ax^{j-n}}{b}+1\right)^{3/2}}dx}{b\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{3n}{2}+1}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n},\frac{m-\frac{3n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

input
```
Int[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*x^(1 + m - (3*n)/2 + (-2*m + n)/2)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hy
pergeometric2F1[3/2, (1 + m - (3*n)/2)/(j - n), 1 + (1 + m - (3*n)/2)/(j -
 n), -((a*x^(j - n))/b)])/(b*(2 + 2*m - 3*n)*Sqrt[a*x^j + b*x^n])
```

3.424.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.424.4 Maple [F]

$$\int \frac{(cx)^m}{(a\,x^j+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

output
```
int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

3.424.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.424.6 Sympy [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)**m/(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((c*x)**m/(a*x**j + b*x**n)**(3/2), x)
```

3.424.7 Maxima [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

3.424.8 Giac [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(c\,x)^m}{(a\,x^j+b\,x^n)^{3/2}}\,dx$$

input
```
int((c*x)^m/(a*x^j + b*x^n)^(3/2),x)
```

output
```
int((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

3.425 $\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx$

3.425.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n},1+\frac{1+m-\frac{5n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-2*n)*(c*x)^m*hypergeom([5/2, (1+m-5/2*n)/(j-n)],[1+(1+m-5/2*n)/(j-n
)],-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2+2*m-5*n)/(a*x^j+b*x^n)^(1/2)
```

3.425.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.50

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \frac{2x^{1-2j}(cx)^m\left(-2+2j-2m+3n-\frac{a(j-n)x^j}{ax^j+bx^n}-(-2+2j-2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hy}\right.}{3a^2(j-n)^2\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

output
```
(2*x^(1 - 2*j)*(c*x)^m*(-2 + 2*j - 2*m + 3*n - (a*(j - n)*x^j)/(a*x^j + b*
x^n) - (-2 + 2*j - 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[
1/2, (2 - 4*j + 2*m - n)/(2*j - 2*n), (2 - 2*j + 2*m - 3*n)/(2*j - 2*n), -
((a*x^(j - n))/b)]))/(3*a^2*(j - n)^2*Sqrt[a*x^j + b*x^n])
```

3.425.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{ax^{j-n}+b}\int \frac{x^{m-\frac{5n}{2}}}{(ax^{j-n}+b)^{5/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(cx)^m x^{\frac{1}{2}(n-2m)}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{m-\frac{5n}{2}}}{\left(\frac{ax^{j-n}}{b}+1\right)^{5/2}}dx}{b^2\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2(cx)^m x^{\frac{1}{2}(n-2m)+m-\frac{5n}{2}+1}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n},\frac{m-\frac{5n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

input
```
Int[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

output
```
(2*x^(1 + m - (5*n)/2 + (-2*m + n)/2)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hy
pergeometric2F1[5/2, (1 + m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j -
 n), -((a*x^(j - n))/b)])/(b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])
```

3.425.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.425.4 Maple [F]

$$\int \frac{(cx)^m}{(a\,x^j+b\,x^n)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

output
```
int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

3.425.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.425.6 Sympy [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{5/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)**m/(a*x**j+b*x**n)**(5/2),x)
```

output
```
Integral((c*x)**m/(a*x**j + b*x**n)**(5/2), x)
```

3.425.7 Maxima [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{5/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

3.425.8 Giac [F]

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{5/2}}\,dx = \int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \int \frac{(c\,x)^m}{(a\,x^j+b\,x^n)^{5/2}}\,dx$$

input
```
int((c*x)^m/(a*x^j + b*x^n)^(5/2),x)
```

output
```
int((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

3.426 $\int \left(ax^j+bx^n\right)^{3/2}\,dx$

3.426.1 Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{2bx^{1+n}\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n},\frac{2+2j+n}{2(j-n)},-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

output
```
2*b*x^(1+n)*hypergeom([-3/2, (1+3/2*n)/(j-n)],[1/2*(2+2*j+n)/(j-n)],-a*x^(
j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{2x\left(\left(2+4j-n\right)\left(ax^j+bx^n\right)\left(a(2-j+4n)x^j+b(2+2j+n)x^n\right)+3a^2(j-n)^2x^{2j}\sqrt{1+\frac{ax\cdots}{\cdots}}\cdots\right.}{(2+4j-n)(2+2j+n)(2+3n)\sqrt{ax^j+\cdots}}$$

input
```
Integrate[(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*x*((2 + 4*j - n)*(a*x^j + b*x^n)*(a*(2 - j + 4*n)*x^j + b*(2 + 2*j + n)
*x^n) + 3*a^2*(j - n)^2*x^(2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F
1[1/2, (2 + 4*j - n)/(2*j - 2*n), (2 + 6*j - 3*n)/(2*j - 2*n), -((a*x^(j -
 n))/b)]))/((2 + 4*j - n)*(2 + 2*j + n)*(2 + 3*n)*Sqrt[a*x^j + b*x^n])
```

3.426.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^j+bx^n\right)^{3/2}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{-n/2}\sqrt{ax^j+bx^n}\int x^{3n/2}\left(ax^{j-n}+b\right)^{3/2}dx}{\sqrt{ax^{j-n}+b}}$$

$$\Big\downarrow \text{889}$$

$$\frac{bx^{-n/2}\sqrt{ax^j+bx^n}\int x^{3n/2}\left(\frac{ax^{j-n}}{b}+1\right)^{3/2}dx}{\sqrt{\frac{ax^{j-n}}{b}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n},\frac{2j+n+2}{2(j-n)},-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

input
```
Int[(a*x^j + b*x^n)^(3/2),x]
```

output
```
(2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n)/2)/(
j - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)])/((2 + 3*n)*Sqrt[1
+ (a*x^(j - n))/b])
```

3.426.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.426.4 Maple [F]

$$\int \left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}d x$$

input
```
int((a*x^j+b*x^n)^(3/2),x)
```

output
```
int((a*x^j+b*x^n)^(3/2),x)
```

3.426.5 Fricas [F(-2)]

Exception generated.

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.426.6 Sympy [F]

$$\int \left(ax^j+bx^n\right)^{3/2} dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((a*x**j + b*x**n)**(3/2), x)
```

3.426.7 Maxima [F]

$$\int \left(ax^j+bx^n\right)^{3/2} dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(3/2), x)
```

3.426.8 Giac [F]

$$\int \left(ax^j+bx^n\right)^{3/2} dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(3/2), x)
```

3.426.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{x\left(a\,x^j+b\,x^n\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{\frac{3\,n}{2}+1}{j-n};\ \frac{\frac{3\,n}{2}+1}{j-n}+1;\ -\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{3\,n}{2}+1\right)\,\left(\frac{a\,x^{j-n}}{b}+1\right)^{3/2}}$$

input
```
int((a*x^j + b*x^n)^(3/2),x)
```

output
```
(x*(a*x^j + b*x^n)^(3/2)*hypergeom([-3/2, ((3*n)/2 + 1)/(j - n)], ((3*n)/2
 + 1)/(j - n) + 1, -(a*x^(j - n))/b))/(((3*n)/2 + 1)*((a*x^(j - n))/b + 1)
^(3/2))
```

3.427 $\int \sqrt{ax^j + bx^n}\,dx$

3.427.1 Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \sqrt{ax^j + bx^n}\,dx = \frac{2x\sqrt{ax^j + bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{2+n}{2(j-n)}, 1 + \frac{2+n}{2j-2n}, -\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1 + \frac{ax^{j-n}}{b}}}$$

output
```
2*x*hypergeom([-1/2, 1/2*(2+n)/(j-n)],[1+(2+n)/(2*j-2*n)],-a*x^(j-n)/b)*(a
*x^j+b*x^n)^(1/2)/(2+n)/(1+a*x^(j-n)/b)^(1/2)
```

3.427.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int \sqrt{ax^j + bx^n}\,dx$$
$$= \frac{2x\left(-((2+2j-n)(ax^j+bx^n)) + a(j-n)x^j\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+2j-n}{2j-2n}, \frac{2+4j-3n}{2j-2n}, -\frac{ax^{j-}}{b}\right.\right.}{(2+n)(-2-2j+n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*(-((2 + 2*j - n)*(a*x^j + b*x^n)) + a*(j - n)*x^j*Sqrt[1 + (a*x^(j -
n))/b]*Hypergeometric2F1[1/2, (2 + 2*j - n)/(2*j - 2*n), (2 + 4*j - 3*n)/(
2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + n)*(-2 - 2*j + n)*Sqrt[a*x^j + b*x
^n])
```

3.427.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^j+bx^n}\,dx$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{x^{-n/2}\sqrt{ax^j+bx^n}\int x^{n/2}\sqrt{ax^{j-n}+b}dx}{\sqrt{ax^{j-n}+b}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{x^{-n/2}\sqrt{ax^j+bx^n}\int x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}dx}{\sqrt{\frac{ax^{j-n}}{b}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x^{\frac{n+2}{2}-\frac{n}{2}}\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+2}{2(j-n)},\frac{n+2}{2j-2n}+1,-\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

input
```
Int[Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x^(-1/2*n + (2 + n)/2)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 +
 n)/(2*(j - n)), 1 + (2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + n)*Sq
rt[1 + (a*x^(j - n))/b])
```

3.427.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.427.4 Maple [F]

$$\int \sqrt{a\,x^{j}+b\,x^{n}}dx$$

input
```
int((a*x^j+b*x^n)^(1/2),x)
```

output
```
int((a*x^j+b*x^n)^(1/2),x)
```

3.427.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{ax^j+bx^n}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.427.6 Sympy [F]

$$\int \sqrt{ax^j + bx^n}\, dx = \int \sqrt{ax^j + bx^n}\, dx$$

input
```
integrate((a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**j + b*x**n), x)
```

3.427.7 Maxima [F]

$$\int \sqrt{ax^j + bx^n}\, dx = \int \sqrt{ax^j + bx^n}\, dx$$

input
```
integrate((a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^j + b*x^n), x)
```

3.427.8 Giac [F]

$$\int \sqrt{ax^j + bx^n}\, dx = \int \sqrt{ax^j + bx^n}\, dx$$

input
```
integrate((a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^j + b*x^n), x)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \sqrt{ax^j + bx^n}\,dx = \frac{x\,\sqrt{a\,x^j + b\,x^n}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\,\frac{\frac{n}{2}+1}{j-n};\;\frac{\frac{n}{2}+1}{j-n}+1;\;-\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{n}{2}+1\right)\,\sqrt{\frac{a\,x^{j-n}}{b}+1}}$$

input
```
int((a*x^j + b*x^n)^(1/2),x)
```

output
```
(x*(a*x^j + b*x^n)^(1/2)*hypergeom([-1/2, (n/2 + 1)/(j - n)], (n/2 + 1)/(j
 - n) + 1, -(a*x^(j - n))/b))/((n/2 + 1)*((a*x^(j - n))/b + 1)^(1/2))
```

3.428 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$

3.428.1 Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2(j-n)},1+\frac{1-\frac{n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

output
```
2*x*hypergeom([1/2, 1/2*(2-n)/(j-n)],[1+1/2*(2-n)/(j-n)],-a*x^(j-n)/b)*(1+
a*x^(j-n)/b)^(1/2)/(2-n)/(a*x^j+b*x^n)^(1/2)
```

3.428.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx = -\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-2+n}{2(-j+n)},1+\frac{-2+n}{2(-j+n)},-\frac{ax^{j-n}}{b}\right)}{(-2+n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[1/Sqrt[a*x^j + b*x^n],x]
```

output
```
(-2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (-2 + n)/(2*(-j + n
)), 1 + (-2 + n)/(2*(-j + n)), -((a*x^(j - n))/b)])/((-2 + n)*Sqrt[a*x^j +
 b*x^n])
```

3.428.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{n/2}\sqrt{ax^{j-n}+b}\int \frac{x^{-n/2}}{\sqrt{ax^{j-n}+b}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{-n/2}}{\sqrt{\frac{ax^{j-n}}{b}+1}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2(j-n)},\frac{1-\frac{n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

input
```
Int[1/Sqrt[a*x^j + b*x^n],x]
```

output
```
(2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)),
 1 + (1 - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 - n)*Sqrt[a*x^j + b*x^n])
```

3.428.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.428.4 Maple [F]

$$\int \frac{1}{\sqrt{a\,x^{j}+b\,x^{n}}}dx$$

input
```
int(1/(a*x^j+b*x^n)^(1/2),x)
```

output
```
int(1/(a*x^j+b*x^n)^(1/2),x)
```

3.428.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.428.6 Sympy [F]

$$\int \frac{1}{\sqrt{a x^j + b x^n}}\, dx = \int \frac{1}{\sqrt{a x^j + b x^n}}\, dx$$

input
```
integrate(1/(a*x**j+b*x**n)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**j + b*x**n), x)
```

3.428.7 Maxima [F]

$$\int \frac{1}{\sqrt{a x^j + b x^n}}\, dx = \int \frac{1}{\sqrt{a x^j + b x^n}}\, dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x^j + b*x^n), x)
```

3.428.8 Giac [F]

$$\int \frac{1}{\sqrt{a x^j + b x^n}}\, dx = \int \frac{1}{\sqrt{a x^j + b x^n}}\, dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*x^j + b*x^n), x)
```

3.428.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\, dx = -\frac{x\sqrt{\frac{b\,x^{n-j}}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{\frac{j}{2}-1}{j-n};\ \frac{\frac{j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{j}{2}-1\right)\sqrt{a\,x^j+b\,x^n}}$$

input
```
int(1/(a*x^j + b*x^n)^(1/2),x)
```

output
```
-(x*((b*x^(n - j))/a + 1)^(1/2)*hypergeom([1/2, (j/2 - 1)/(j - n)], (j/2 -
 1)/(j - n) + 1, -(b*x^(n - j))/a))/((j/2 - 1)*(a*x^j + b*x^n)^(1/2))
```

3.429 $\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx$

3.429.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx = \frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n},1+\frac{1-\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-n)*hypergeom([3/2, (1-3/2*n)/(j-n)],[1+(2-3*n)/(2*j-2*n)],-a*x^(j-n
)/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2-3*n)/(a*x^j+b*x^n)^(1/2)
```

3.429.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx = \frac{2x^{1-j}\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{-2+2j+n}{2(j-n)},\frac{2-3n}{2j-2n},-\frac{ax^{j-n}}{b}\right)\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

input
```
Integrate[(a*x^j + b*x^n)^(-3/2),x]
```

output
```
(2*x^(1 - j)*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, -1/2*(
-2 + 2*j + n)/(j - n), (2 - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(a*(j
- n)*Sqrt[a*x^j + b*x^n])
```

3.429.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{n/2}\sqrt{ax^{j-n}+b}\int \frac{x^{-3n/2}}{(ax^{j-n}+b)^{3/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{-3n/2}}{\left(\frac{ax^{j-n}}{b}+1\right)^{3/2}}dx}{b\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n},\frac{1-\frac{3n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

input
```
Int[(a*x^j + b*x^n)^(-3/2),x]
```

output
```
(2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n)/2
)/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b*(2 - 3*n)*Sq
rt[a*x^j + b*x^n])
```

3.429.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.429.4 Maple [F]

$$\int \frac{1}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a*x^j+b*x^n)^(3/2),x)
```

output
```
int(1/(a*x^j+b*x^n)^(3/2),x)
```

3.429.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.429.6 Sympy [F]

$$\int \frac{1}{(ax^j + bx^n)^{3/2}}\, dx = \int \frac{1}{(ax^j + bx^n)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a*x**j+b*x**n)**(3/2),x)
```

output
```
Integral((a*x**j + b*x**n)**(-3/2), x)
```

3.429.7 Maxima [F]

$$\int \frac{1}{(ax^j + bx^n)^{3/2}}\, dx = \int \frac{1}{(ax^j + bx^n)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(-3/2), x)
```

3.429.8 Giac [F]

$$\int \frac{1}{(ax^j + bx^n)^{3/2}}\, dx = \int \frac{1}{(ax^j + bx^n)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(-3/2), x)
```

3.429.9 Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^j + bx^n)^{3/2}} \, dx = -\frac{x \left(\frac{b\,x^{n-j}}{a} + 1\right)^{3/2} {}_2\mathrm{F}_1\left(\frac{3}{2}, \frac{\frac{3\,j}{2}-1}{j-n};\ \frac{\frac{3\,j}{2}-1}{j-n} + 1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{3\,j}{2} - 1\right)\,\left(a\,x^j + b\,x^n\right)^{3/2}}$$

input
```
int(1/(a*x^j + b*x^n)^(3/2),x)
```

output
```
-(x*((b*x^(n - j))/a + 1)^(3/2)*hypergeom([3/2, ((3*j)/2 - 1)/(j - n)], ((
3*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/((((3*j)/2 - 1)*(a*x^j + b*x^n)
^(3/2))
```

3.430 $\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx$

3.430.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx = \frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n},1+\frac{1-\frac{5n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

output
```
2*x^(1-2*n)*hypergeom([5/2, (1-5/2*n)/(j-n)],[1+(2-5*n)/(2*j-2*n)],-a*x^(j
-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2-5*n)/(a*x^j+b*x^n)^(1/2)
```

3.430.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.83

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx = \frac{2x^{1-2j}\Big(-((-2+4j+n)\,(a(-2+j+4n)x^j+b(-2+2j+3n)x^n))+(4+8j^2-8\ldots}{3a^2(2-4\ldots}$$

input
```
Integrate[(a*x^j + b*x^n)^(-5/2),x]
```

output
```
(2*x^(1 - 2*j)*(-((-2 + 4*j + n)*(a*(-2 + j + 4*n)*x^j + b*(-2 + 2*j + 3*n
)*x^n)) + (4 + 8*j^2 - 8*n + 3*n^2 + 2*j*(-6 + 7*n))*Sqrt[1 + (a*x^(j - n)
)/b]*(a*x^j + b*x^n)*Hypergeometric2F1[1/2, -1/2*(-2 + 4*j + n)/(j - n), (
2 - 2*j - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(3*a^2*(2 - 4*j - n)*(j
- n)^2*(a*x^j + b*x^n)^(3/2))
```

3.430.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1917, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{n/2}\sqrt{ax^{j-n}+b}\int \frac{x^{-5n/2}}{(ax^{j-n}+b)^{5/2}}dx}{\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\int \frac{x^{-5n/2}}{\left(\frac{ax^{j-n}}{b}+1\right)^{5/2}}dx}{b^2\sqrt{ax^j+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n},\frac{1-\frac{5n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

input
```
Int[(a*x^j + b*x^n)^(-5/2),x]
```

output
```
(2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)
/2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n
)*Sqrt[a*x^j + b*x^n])
```

3.430.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.430.4 Maple [F]

$$\int \frac{1}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a*x^j+b*x^n)^(5/2),x)
```

output
```
int(1/(a*x^j+b*x^n)^(5/2),x)
```

3.430.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.430.6 Sympy [F]

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\int \frac{1}{(ax^j+bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x**j+b*x**n)**(5/2),x)
```

output
```
Integral((a*x**j + b*x**n)**(-5/2), x)
```

3.430.7 Maxima [F]

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\int \frac{1}{(ax^j+bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^(-5/2), x)
```

3.430.8 Giac [F]

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\int \frac{1}{(ax^j+bx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^(-5/2), x)
```

3.430.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx = -\frac{x\left(\frac{b\,x^{n-j}}{a}+1\right)^{5/2}{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{\frac{5\,j}{2}-1}{j-n};\ \frac{\frac{5\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{5\,j}{2}-1\right)\left(a\,x^j+b\,x^n\right)^{5/2}}$$

input
```
int(1/(a*x^j + b*x^n)^(5/2),x)
```

output
```
-(x*((b*x^(n - j))/a + 1)^(5/2)*hypergeom([5/2, ((5*j)/2 - 1)/(j - n)], ((
5*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((5*j)/2 - 1)*(a*x^j + b*x^n)
^(5/2))
```

3.431 $\int \sqrt{\frac{1+x}{x^5}} \, dx$

3.431.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \sqrt{\frac{1+x}{x^5}} \, dx = -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2} x^6$$

output
```
-2/3*(1/x^5+1/x^4)^(3/2)*x^6
```

3.431.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \sqrt{\frac{1+x}{x^5}} \, dx = -\frac{2}{3}x(1+x)\sqrt{\frac{1+x}{x^5}}$$

input
```
Integrate[Sqrt[(1 + x)/x^5],x]
```

output
```
(-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3
```

3.431.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x+1}{x^5}}\,dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{\frac{1}{x^5}+\frac{1}{x^4}}dx$$

$$\Big\downarrow \text{1906}$$

$$-\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6$$

input
```
Int[Sqrt[(1 + x)/x^5],x]
```

output
```
(-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

3.431.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.431.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2x(1+x)\sqrt{\frac{1+x}{x^5}}}{3}$	16
trager	$-\frac{2x(1+x)\sqrt{-\frac{-x-1}{x^5}}}{3}$	19
default	$-\frac{2\sqrt{\frac{1+x}{x^5}}\,(x^2+x)^{\frac{3}{2}}}{3\sqrt{x(1+x)}}$	26
risch	$-\frac{2\sqrt{\frac{1+x}{x^5}}\,x(x^2+2x+1)}{3(1+x)}$	26

input
```
int(((1+x)/x^5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(1+x)*((1+x)/x^5)^(1/2)
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}\left(x^2+x\right)\sqrt{\frac{x+1}{x^5}}$$

input
```
integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(x^2 + x)*sqrt((x + 1)/x^5)
```

3.431.6 Sympy [F]

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \int \sqrt{\frac{x+1}{x^5}}\,dx$$

input
```
integrate(((1+x)/x**5)**(1/2),x)
```

output
```
Integral(sqrt((x + 1)/x**5), x)
```

3.431.7 Maxima [F]

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \int \sqrt{\frac{x+1}{x^5}}\,dx$$

input
```
integrate(((1+x)/x^5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x + 1)/x^5), x)
```

3.431.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(14) = 28$.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \frac{2\left(3\left(x-\sqrt{x^2+x}\right)^2\operatorname{sgn}(x)+3\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+\operatorname{sgn}(x)\right)}{3\left(x-\sqrt{x^2+x}\right)^3}$$

input
```
integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x
))/(x - sqrt(x^2 + x))^3
```

3.431.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2\,x\,\sqrt{\frac{x+1}{x^5}}\,(x+1)}{3}$$

input
```
int(((x + 1)/x^5)^(1/2),x)
```

output
```
-(2*x*((x + 1)/x^5)^(1/2)*(x + 1))/3
```

3.432 $\int \sqrt{x+x^{5/2}}\,dx$

3.432.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \sqrt{x+x^{5/2}}\,dx = \frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

output
```
4/9*(x+x^(5/2))^(3/2)/x^(3/2)
```

3.432.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{x+x^{5/2}}\,dx = \frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

input
```
Integrate[Sqrt[x + x^(5/2)],x]
```

output
```
(4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

3.432.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^{5/2}+x}\,dx$$
$$\Big\downarrow \text{1906}$$
$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

input
```
Int[Sqrt[x + x^(5/2)],x]
```

output
```
(4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

3.432.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

3.432.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(1+x^{\frac{3}{2}}\right)}{9\sqrt{x}}$	18
default	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(1+x^{\frac{3}{2}}\right)}{9\sqrt{x}}$	18
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(2+2x^{\frac{3}{2}}\right)\sqrt{1+x^{\frac{3}{2}}}}{3}}{3\sqrt{\pi}}$	31

input
```
int((x+x^(5/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(1+x^(3/2))
```

3.432.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{x+x^{5/2}}\,dx = \frac{4\sqrt{x^{\frac{5}{2}}+x}\left(x^2+\sqrt{x}\right)}{9\,x}$$

input
```
integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")
```

output
```
4/9*sqrt(x^(5/2) + x)*(x^2 + sqrt(x))/x
```

3.432.6 Sympy [F]

$$\int \sqrt{x+x^{5/2}}\,dx = \int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

input
```
integrate((x+x**(5/2))**(1/2),x)
```

output
```
Integral(sqrt(x**(5/2) + x), x)
```

3.432.7 Maxima [F]

$$\int \sqrt{x+x^{5/2}}\,dx = \int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

input
```
integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^(5/2) + x), x)
```

3.432.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \sqrt{x + x^{5/2}}\, dx = \frac{4}{9}\left(x^{\frac{3}{2}} + 1\right)^{\frac{3}{2}} - \frac{4}{9}$$

input
```
integrate((x+x^(5/2))^(1/2),x, algorithm="giac")
```

output
```
4/9*(x^(3/2) + 1)^(3/2) - 4/9
```

3.432.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \sqrt{x + x^{5/2}}\, dx = \frac{2\, x\, \sqrt{x + x^{5/2}}\; {}_2\mathrm{F}_1\left(-\frac{1}{2}, 1;\ 2;\ -x^{3/2}\right)}{3\,\sqrt{x^{3/2} + 1}}$$

input
```
int((x + x^(5/2))^(1/2),x)
```

output
```
(2*x*(x + x^(5/2))^(1/2)*hypergeom([-1/2, 1], 2, -x^(3/2)))/(3*(x^(3/2) +
1)^(1/2))
```

3.433 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$

3.433.1 Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

output
```
2*arctan(x^(1/2))
```

3.433.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[(Sqrt[x] + x^(3/2))^(-1),x]
```

output
```
2*ArcTan[Sqrt[x]]
```

3.433.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2027, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}+\sqrt{x}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{\sqrt{x}(x+1)}dx$$
$$\Big\downarrow \text{73}$$
$$2\int \frac{1}{x+1}d\sqrt{x}$$
$$\Big\downarrow \text{216}$$
$$2\arctan\left(\sqrt{x}\right)$$

input
```
Int[(Sqrt[x] + x^(3/2))^(-1),x]
```

output
```
2*ArcTan[Sqrt[x]]
```

3.433.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.433.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2 \arctan\left(\sqrt{x}\right)$	7
default	$2 \arctan\left(\sqrt{x}\right)$	7
meijerg	$2 \arctan\left(\sqrt{x}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right) \ln\left(\frac{2 \operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{1+x}\right)$	29

input
```
int(1/(x^(3/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(x^(1/2))
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2 \arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(x))
```

3.433.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

input
```
integrate(1/(x**(3/2)+x**(1/2)),x)
```

output
```
2*atan(sqrt(x))
```

3.433.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(x))
```

3.433.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="giac")
```

output
```
2*arctan(sqrt(x))
```

3.433.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

input
```
int(1/(x^(1/2) + x^(3/2)),x)
```

output
```
2*atan(x^(1/2))
```

3.434 $\int x\sqrt{x^2(a+bx^3)}\,dx$

3.434.1 Optimal result

Integrand size = 17, antiderivative size = 25

$$\int x\sqrt{x^2(a+bx^3)}\,dx=\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

output
```
2/9*(x^2*(b*x^3+a))^(3/2)/b/x^3
```

3.434.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\sqrt{x^2(a+bx^3)}\,dx=\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

input
```
Integrate[x*Sqrt[x^2*(a + b*x^3)],x]
```

output
```
(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

3.434.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

$$\big\downarrow \text{2021}$$

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

input
```
Int[x*Sqrt[x^2*(a + b*x^3)],x]
```

output
```
(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

3.434.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.434.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2\left(b\,x^{3}+a\right)\sqrt{x^{2}\left(b\,x^{3}+a\right)}}{9bx}$	29
default	$\frac{2\left(b\,x^{3}+a\right)\sqrt{x^{2}\left(b\,x^{3}+a\right)}}{9bx}$	29
trager	$\frac{2\left(b\,x^{3}+a\right)\sqrt{b\,x^{5}+a\,x^{2}}}{9bx}$	29
risch	$\frac{2\left(b\,x^{3}+a\right)\sqrt{x^{2}\left(b\,x^{3}+a\right)}}{9bx}$	29

input
```
int(x*(x^2*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x
```

3.434.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int x\sqrt{x^2(a+bx^3)}\,dx=\frac{2\sqrt{bx^5+ax^2}(bx^3+a)}{9\,bx}$$

input
```
integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

3.434.6 Sympy [F(-1)]

Timed out.

$$\int x\sqrt{x^2(a+bx^3)}\,dx=\text{Timed out}$$

input
```
integrate(x*(x**2*(b*x**3+a))**(1/2),x)
```

output
```
Timed out
```

3.434.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int x\sqrt{x^2(a+bx^3)}\,dx=\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

3.434.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x\sqrt{x^2\,(a+bx^3)}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}\text{sgn}(x)}{9\,b} - \frac{2\,a^{\frac{3}{2}}\text{sgn}(x)}{9\,b}$$

input
```
integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

3.434.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int x\sqrt{x^2\,(a+bx^3)}\,dx = \frac{2\,(b\,x^3+a)^{3/2}\,\sqrt{x^2}}{9\,b\,x}$$

input
```
int(x*(x^2*(a + b*x^3))^(1/2),x)
```

output
```
(2*(a + b*x^3)^(3/2)*(x^2)^(1/2))/(9*b*x)
```

3.435 $\int x\sqrt{ax^2+bx^5}\,dx$

3.435.1 Optimal result

Integrand size = 17, antiderivative size = 25

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

output
```
2/9*(b*x^5+a*x^2)^(3/2)/b/x^3
```

3.435.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

input
```
Integrate[x*Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

3.435.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^5}\,dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

input
```
Int[x*Sqrt[a*x^2 + b*x^5],x]
```

output
```
(2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

3.435.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.435.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
default	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
trager	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
risch	$\frac{2(b\,x^3+a)\sqrt{x^2(b\,x^3+a)}}{9bx}$	29

input
```
int(x*(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)/b/x*(b*x^5+a*x^2)^(1/2)
```

3.435.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int x\sqrt{ax^2+bx^5}\,dx = \frac{2\sqrt{bx^5+ax^2}(bx^3+a)}{9\,bx}$$

input
```
integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

3.435.6 Sympy [F]

$$\int x\sqrt{ax^2+bx^5}\,dx = \int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

input
```
integrate(x*(b*x**5+a*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

3.435.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int x\sqrt{ax^2+bx^5}\,dx = \frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

3.435.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x\sqrt{ax^2+bx^5}\,dx = \frac{2\left(bx^3+a\right)^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b} - \frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}$$

input
```
integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

3.435.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x\sqrt{ax^2+bx^5}\,dx = \frac{\left(\frac{2\,a}{9\,b}+\frac{2\,x^3}{9}\right)\sqrt{b\,x^5+a\,x^2}}{x}$$

input
```
int(x*(a*x^2 + b*x^5)^(1/2),x)
```

output
```
(((2*a)/(9*b) + (2*x^3)/9)*(a*x^2 + b*x^5)^(1/2))/x
```

3.436 $\int \sqrt{x^4 (a+bx^3)}\,dx$

3.436.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \sqrt{x^4 (a+bx^3)}\,dx = \frac{2(ax^4+bx^7)^{3/2}}{9bx^6}$$

output
```
2/9*(b*x^7+a*x^4)^(3/2)/b/x^6
```

3.436.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{x^4 (a+bx^3)}\,dx = \frac{2(x^4(a+bx^3))^{3/2}}{9bx^6}$$

input
```
Integrate[Sqrt[x^4*(a + b*x^3)],x]
```

output
```
(2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)
```

3.436.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^4 (a+bx^3)} \, dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{ax^4+bx^7}dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{2(ax^4+bx^7)^{3/2}}{9bx^6}$$

input
```
Int[Sqrt[x^4*(a + b*x^3)],x]
```

output
```
(2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

3.436.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.436.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(b\,x^{3}+a)\sqrt{x^{4}(b\,x^{3}+a)}}{9b\,x^{2}}$	29
default	$\frac{2(b\,x^{3}+a)\sqrt{x^{4}(b\,x^{3}+a)}}{9b\,x^{2}}$	29
trager	$\frac{2(b\,x^{3}+a)\sqrt{b\,x^{7}+a\,x^{4}}}{9b\,x^{2}}$	29
risch	$\frac{2(b\,x^{3}+a)\sqrt{x^{4}(b\,x^{3}+a)}}{9b\,x^{2}}$	29

input
```
int((x^4*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2
```

3.436.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \sqrt{x^4\left(a+bx^3\right)}\,dx = \frac{2\sqrt{bx^7+ax^4}(bx^3+a)}{9\,bx^2}$$

input
```
integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)
```

3.436.6 Sympy [F]

$$\int \sqrt{x^4\left(a+bx^3\right)}\,dx = \int \sqrt{x^4\left(a+bx^3\right)}\,dx$$

input
```
integrate((x**4*(b*x**3+a))**(1/2),x)
```

output
```
Integral(sqrt(x**4*(a + b*x**3)), x)
```

3.436.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{x^4 (a + bx^3)}\, dx = \frac{2 (bx^3 + a)^{\frac{3}{2}}}{9\, b}$$

input
```
integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

3.436.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{x^4 (a + bx^3)}\, dx = \frac{2 (bx^3 + a)^{\frac{3}{2}}}{9\, b}$$

input
```
integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

3.436.9 Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{x^4 (a + bx^3)}\, dx = \frac{2 (b\, x^3 + a)^{3/2}\, \sqrt{x^4}}{9\, b\, x^2}$$

input
```
int((x^4*(a + b*x^3))^(1/2),x)
```

output
```
(2*(a + b*x^3)^(3/2)*(x^4)^(1/2))/(9*b*x^2)
```

3.437 $\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx$

3.437.1 Optimal result

Integrand size = 19, antiderivative size = 988

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = -\frac{45a^2\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}}{14\sqrt[3]{2}b^3\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$-\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}}{28\sqrt[3]{2}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}}$$

$$+\frac{15\ 3^{3/4}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}\sqrt[3]{-\frac{b}{}}}{7\ 2^{5/6}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}}$$

output
```
-45/28*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(1/3)+9/7*(a+b*x^
(1/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(1/3)-45/28*a^2*(a+2*b*x^(1/3))*(-b
*(a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*2^(2/3)/b^3/(a*x^(1/3)+b*x^(2/3))^(1/3)/
(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))+15/14*3^(3/4)*a^4
*(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3)
)/a^2)^(1/3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(
1/2))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/
2))*((1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^
(1/3))*x^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)
-3^(1/2))^2)^(1/2)*2^(1/6)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(1/3)
/((-1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*x^
(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)-45/56*3^(1/4)*a^4*(1-2^(2/3)*(
-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*
EllipticE((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(1-2^(2
/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/
3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^(1/3))*x^(1/3
)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x
^(2/3))^(1/3)/((-1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3
)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.437.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\frac{9\sqrt[3]{1+\frac{b\sqrt[3]{x}}{a}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}}$$

input
```
Integrate[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1
/3))/a)])/(8*((a + b*x^(1/3))*x^(1/3))^(1/3))
```

3.437.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1917, 864, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt[9]{x}\sqrt[3]{a+b\sqrt[3]{x}}\int \frac{1}{\sqrt[3]{a+b\sqrt[3]{x}}\sqrt[9]{x}}dx}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$\Big\downarrow \text{864}$$

$$\frac{3\sqrt[9]{x}\sqrt[3]{a+b\sqrt[3]{x}}\int \frac{x^{5/9}}{\sqrt[3]{a+b\sqrt[3]{x}}}d\sqrt[3]{x}}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$\Big\downarrow \text{76}$$

$$\frac{3\sqrt[9]{x}\sqrt[3]{\frac{b\sqrt[3]{x}}{a}+1}\int \frac{x^{5/9}}{\sqrt[3]{\frac{\sqrt[3]{x}b}{a}+1}}d\sqrt[3]{x}}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$\Big\downarrow \text{74}$$

$$\frac{9x\sqrt[3]{\frac{b\sqrt[3]{x}}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

input
```
Int[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1
/3))/a)])/(8*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

3.437.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.437.4 Maple [F]

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

output
```
int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

3.437.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.437.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = \int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/(a*x**(1/3)+b*x**(2/3))**(1/3),x)
```

output
```
Integral((a*x**(1/3) + b*x**(2/3))**(-1/3), x)
```

3.437.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = \int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

3.437.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

3.437.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.04

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\frac{9\,x\,\left(\frac{b\,x^{1/3}}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{8\,\left(a\,x^{1/3}+b\,x^{2/3}\right)^{1/3}}$$

input
```
int(1/(a*x^(1/3) + b*x^(2/3))^(1/3),x)
```

output
```
(9*x*((b*x^(1/3))/a + 1)^(1/3)*hypergeom([1/3, 8/3], 11/3, -(b*x^(1/3))/a)
)/(8*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

3.438 $\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$

3.438.1 Optimal result

Integrand size = 19, antiderivative size = 487

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$+\frac{6\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}\left(}$$

output
```
-18/5*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(2/3)+9/5*(a+b*x^(
1/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(2/3)+6/5*2^(1/3)*3^(3/4)*a^4*(1-2^(
2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^
(2/3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(
1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/
2*6^(1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*
2^(1/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))
*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(
2/3))^(2/3)/((-1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*
(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

3.438.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.13

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\frac{9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2/3}x\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}\right)^{2/3}}$$

input
```
Integrate[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1
/3))/a)])/(7*((a + b*x^(1/3))*x^(1/3))^(2/3))
```

3.438.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1917, 864, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$$

$$\Big\downarrow \; 1917$$

$$\frac{x^{2/9}\left(a+b\sqrt[3]{x}\right)^{2/3}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}}dx}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$\Big\downarrow \; 864$$

$$\frac{3x^{2/9}\left(a+b\sqrt[3]{x}\right)^{2/3}\int \frac{x^{4/9}}{\left(a+b\sqrt[3]{x}\right)^{2/3}}d\sqrt[3]{x}}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$\Big\downarrow \; 76$$

$$\frac{3x^{2/9}\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2/3}\int \frac{x^{4/9}}{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2/3}}d\sqrt[3]{x}}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$\Big\downarrow \; 74$$

$$\frac{9x\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

input
```
Int[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1
/3))/a)])/(7*(a*x^(1/3) + b*x^(2/3))^(2/3))
```

3.438.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

3.438. $\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.438.4 Maple [F]

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

output
```
int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

3.438.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.438.6 Sympy [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{\left(a\sqrt[3]{x}+bx^{\frac{2}{3}}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a*x**(1/3)+b*x**(2/3))**(2/3),x)
```

output
```
Integral((a*x**(1/3) + b*x**(2/3))**(-2/3), x)
```

3.438.7 Maxima [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

3.438.8 Giac [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

3.438.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = \frac{9\,x\,\left(\frac{b\,x^{1/3}}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{7\,\left(a\,x^{1/3}+b\,x^{2/3}\right)^{2/3}}$$

input
```
int(1/(a*x^(1/3) + b*x^(2/3))^(2/3),x)
```

output
```
(9*x*((b*x^(1/3))/a + 1)^(2/3)*hypergeom([2/3, 7/3], 10/3, -(b*x^(1/3))/a)
)/(7*(a*x^(1/3) + b*x^(2/3))^(2/3))
```

3.439 $\int x^m (ax^j + bx^n)^p \, dx$

3.439.1 Optimal result

Integrand size = 17, antiderivative size = 89

$$\int x^m (ax^j + bx^n)^p \, dx = \frac{x^{1+m}(ax^j + bx^n)^p (a + bx^{-j+n}) \text{Hypergeometric2F1}\left(1, 1 + p + \frac{1+m+jp}{-j+n}, 1 + \frac{1+m+jp}{-j+n}, -\frac{bx^{-j+n}}{a}\right)}{a(1 + m + jp)}$$

output
```
x^(1+m)*(a*x^j+b*x^n)^p*(a+b*x^(-j+n))*hypergeom([1, 1+p+(j*p+m+1)/(-j+n)]
,[1+(j*p+m+1)/(-j+n)],-b*x^(-j+n)/a)/a/(j*p+m+1)
```

3.439.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x^m (ax^j + bx^n)^p \, dx = \frac{x^{1+m}\left(1 + \frac{ax^{j-n}}{b}\right)^{-p} (ax^j + bx^n)^p \text{Hypergeometric2F1}\left(-p, \frac{1+m+np}{j-n}, 1 + \frac{1+m+np}{j-n}, -\frac{ax^{j-n}}{b}\right)}{1 + m + np}$$

input
```
Integrate[x^m*(a*x^j + b*x^n)^p,x]
```

output
```
(x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n),
1 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(
j - n))/b)^p)
```

3.439.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1938, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(ax^j+bx^n\right)^p \, dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-np} \left(ax^{j-n}+b\right)^{-p} \left(ax^j+bx^n\right)^p \int x^{m+np} \left(ax^{j-n}+b\right)^p dx$$

$$\Big\downarrow \text{889}$$

$$x^{-np} \left(\frac{ax^{j-n}}{b}+1\right)^{-p} \left(ax^j+bx^n\right)^p \int x^{m+np} \left(\frac{ax^{j-n}}{b}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1} \left(\frac{ax^{j-n}}{b}+1\right)^{-p} \left(ax^j+bx^n\right)^p \text{Hypergeometric2F1}\left(-p,\frac{m+np+1}{j-n},\frac{m+np+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

input
```
Int[x^m*(a*x^j + b*x^n)^p,x]
```

output
```
(x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n),
1 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(
j - n))/b)^p)
```

3.439.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.439.4 Maple [F]

$$\int x^m \left(a\,x^j+b\,x^n\right)^p dx$$

input
```
int(x^m*(a*x^j+b*x^n)^p,x)
```

output
```
int(x^m*(a*x^j+b*x^n)^p,x)
```

3.439.5 Fricas [F]

$$\int x^m \left(ax^j+bx^n\right)^p \, dx = \int \left(ax^j+bx^n\right)^p x^m \, dx$$

input
```
integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((a*x^j + b*x^n)^p*x^m, x)
```

3.439.6 Sympy [F]

$$\int x^m \left(ax^j + bx^n\right)^p \, dx = \int x^m \left(ax^j + bx^n\right)^p \, dx$$

input
```
integrate(x**m*(a*x**j+b*x**n)**p,x)
```

output
```
Integral(x**m*(a*x**j + b*x**n)**p, x)
```

3.439.7 Maxima [F]

$$\int x^m \left(ax^j + bx^n\right)^p \, dx = \int \left(ax^j + bx^n\right)^p x^m \, dx$$

input
```
integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((a*x^j + b*x^n)^p*x^m, x)
```

3.439.8 Giac [F]

$$\int x^m \left(ax^j + bx^n\right)^p \, dx = \int \left(ax^j + bx^n\right)^p x^m \, dx$$

input
```
integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((a*x^j + b*x^n)^p*x^m, x)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(ax^j + bx^n\right)^p \, dx = \int x^m \left(a\,x^j + b\,x^n\right)^p \, dx$$

input
```
int(x^m*(a*x^j + b*x^n)^p,x)
```

output
```
int(x^m*(a*x^j + b*x^n)^p, x)
```

3.440 $\int x^{-1-pq}(bx^n + ax^q)^p \, dx$

3.440.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int x^{-1-pq}(bx^n + ax^q)^p \, dx = -\frac{x^{-pq}(a + bx^{n-q})(bx^n + ax^q)^p \text{Hypergeometric2F1}\left(1, 1 + p, 2 + p, 1 + \frac{bx^{n-q}}{a}\right)}{a(1 + p)(n - q)}$$

output
```
-(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, p+1],[2+p],1+b*x^(n-q)/a)/a/(
p+1)/(n-q)/(x^(p*q))
```

3.440.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int x^{-1-pq}(bx^n + ax^q)^p \, dx = \frac{x^{-pq}(bx^n + ax^q)^p \left(1 + \frac{ax^{-n+q}}{b}\right)^{-p} \text{Hypergeometric2F1}\left(-p, -p, 1 - p, -\frac{ax^{-n+q}}{b}\right)}{p(n - q)}$$

input
```
Integrate[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

output
```
((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((a*x^(-n + q))/b)])/
(p*(n - q)*x^(p*q)*(1 + (a*x^(-n + q))/b)^p)
```

3.440.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1938, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-pq-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int \frac{(bx^{n-q}+a)^p}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{q-n}(bx^{n-q}+a)^p\,dx^{n-q}}{n-q}$$

$$\Big\downarrow \text{75}$$

$$-\frac{x^{-pq}\left(a+bx^{n-q}\right)(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^{n-q}}{a}+1\right)}{a(p+1)(n-q)}$$

input
```
Int[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

output
```
-(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))
```

3.440.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.440.4 Maple [F]

$$\int x^{-pq-1}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

input
```
int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

3.440.5 Fricas [F]

$$\int x^{-1-pq}(bx^n+ax^q)^p\ dx=\int (bx^n+ax^q)^p x^{-pq-1}\,dx$$

input
```
integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

3.440.6 Sympy [F]

$$\int x^{-1-pq}(bx^n+ax^q)^p\ dx=\int x^{-pq-1}(ax^q+bx^n)^p\ dx$$

input
```
integrate(x**(-p*q-1)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-p*q - 1)*(a*x**q + b*x**n)**p, x)
```

3.440.7 Maxima [F]

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-pq-1}\,dx$$

input
```
integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

3.440.8 Giac [F]

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-pq-1}\,dx$$

input
```
integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{p\,q+1}}\,dx$$

input
```
int((b*x^n + a*x^q)^p/x^(p*q + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(p*q + 1), x)
```

3.441 $\int x^{-1-np}(bx^n+ax^q)^p\,dx$

3.441.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx = -\frac{x^{-np}(a+bx^{n-q})\,(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1,1,1-p,-\frac{bx^{n-q}}{a}\right)}{ap(n-q)}$$

output
```
-(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, 1],[1-p],-b*x^(n-q)/a)/a/p/(n
-q)/(x^(n*p))
```

3.441.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx = -\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

input
```
Integrate[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

output
```
-(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])
/(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

3.441.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1938, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{-np+qp-1}\left(bx^{n-q}+a\right)^p dx$$

$$\Big\downarrow \text{882}$$

$$\frac{x^{-p(n-q)-pq}\left(\frac{x^{n-q}}{a+bx^{n-q}}\right)^p(ax^q+bx^n)^p\int \frac{\left(\frac{x^{n-q}}{bx^{n-q}+a}\right)^{-p-1}}{1-\frac{bx^{n-q}}{bx^{n-q}+a}}d\frac{x^{n-q}}{bx^{n-q}+a}}{n-q}$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-p(n-q)-pq}(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{bx^{n-q}}{bx^{n-q}+a}\right)}{p(n-q)}$$

input
```
Int[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

output
```
-((x^(-(p*(n - q)) - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, -p, 1 - p
, (b*x^(n - q))/(a + b*x^(n - q))])/(p*(n - q)))
```

3.441.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.441.4 Maple [F]

$$\int x^{-np-1}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

input
```
int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

3.441.5 Fricas [F]

$$\int x^{-1-np}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

3.441.6 Sympy [F]

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int x^{-np-1}(ax^q+bx^n)^p\,dx$$

input
```
integrate(x**(-n*p-1)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-n*p - 1)*(a*x**q + b*x**n)**p, x)
```

3.441.7 Maxima [F]

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

3.441.8 Giac [F]

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{n\,p+1}}\,dx$$

input
```
int((b*x^n + a*x^q)^p/x^(n*p + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(n*p + 1), x)
```

3.442 $\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx$

3.442.1 Optimal result

Integrand size = 27, antiderivative size = 69

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx = \frac{bx^{-pq}(a+bx^{n-q})(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx^{n-q}}{a}\right)}{a^2(1+p)(n-q)}$$

output
```
b*(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([2, p+1],[2+p],1+b*x^(n-q)/a)/a^
2/(p+1)/(n-q)/(x^(p*q))
```

3.442.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx = \frac{x^{-n+q-pq}(bx^n+ax^q)^p\left(1+\frac{ax^{-n+q}}{b}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{ax^{-n+q}}{b}\right)}{(-1+p)(n-q)}$$

input
```
Integrate[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

output
```
(x^(-n + q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(
(a*x^(-n + q))/b)])/((-1 + p)*(n - q)*(1 + (a*x^(-n + q))/b)^p)
```

3.442.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1938, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-(p-1)q-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{-n+q-1}\left(bx^{n-q}+a\right)^p dx$$

$$\Big\downarrow \text{798}$$

$$\frac{x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{2q-2n}\left(bx^{n-q}+a\right)^p dx^{n-q}}{n-q}$$

$$\Big\downarrow \text{75}$$

$$\frac{bx^{-pq}\left(a+bx^{n-q}\right)(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^{n-q}}{a}+1\right)}{a^2(p+1)(n-q)}$$

input
```
Int[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

output
```
(b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p,
1 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))
```

3.442.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.442.4 Maple [F]

$$\int x^{-1-n-(p-1)q}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

input
```
int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
```

3.442.5 Fricas [F]

$$\int x^{-1-n-(-1+p)q}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-(p-1)q-n-1}\,dx$$

input
```
integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

3.442.6 Sympy [F]

$$\int x^{-1-n-(-1+p)q}(bx^{n}+ax^{q})^{p}\,dx=\int x^{-n-q(p-1)-1}(ax^{q}+bx^{n})^{p}\,dx$$

input
```
integrate(x**(-1-n-(-1+p)*q)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-n - q*(p - 1) - 1)*(a*x**q + b*x**n)**p, x)
```

3.442.7 Maxima [F]

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p \, dx = \int (bx^n+ax^q)^p x^{-(p-1)q-n-1} \, dx$$

input
```
integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

3.442.8 Giac [F]

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p \, dx = \int (bx^n+ax^q)^p x^{-(p-1)q-n-1} \, dx$$

input
```
integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p \, dx = \int \frac{(b\,x^n+a\,x^q)^p}{x^{n+q\,(p-1)+1}} \, dx$$

input
```
int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1), x)
```

3.443 $\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx$

3.443.1 Optimal result

Integrand size = 27, antiderivative size = 84

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx = \frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

output
```
x^(-n*p+n-q)*(b*x^n+a*x^q)^p*hypergeom([-p, 1-p],[2-p],-b*x^(n-q)/a)/(1-p)
/(n-q)/((1+b*x^(n-q)/a)^p)
```

3.443.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx = -\frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^{n-q}}{a}\right)}{(-1+p)(n-q)}$$

input
```
Integrate[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

output
```
-((x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -
((b*x^(n - q))/a)])/((-1 + p)*(n - q)*(1 + (b*x^(n - q))/a)^p))
```

3.443.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1938, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-1)-q-1}(ax^q+bx^n)^p\,dx$$

$$\Big\downarrow \text{1938}$$

$$x^{-pq}\left(a+bx^{n-q}\right)^{-p}(ax^q+bx^n)^p\int x^{-pn+n-(1-p)q-1}\left(bx^{n-q}+a\right)^p dx$$

$$\Big\downarrow \text{882}$$

$$\frac{ax^{-p(n-q)-pq}\left(\frac{x^{n-q}}{a+bx^{n-q}}\right)^p(ax^q+bx^n)^p\int\frac{\left(\frac{x^{n-q}}{bx^{n-q}+a}\right)^{-p}}{\left(1-\frac{bx^{n-q}}{bx^{n-q}+a}\right)^2}d\frac{x^{n-q}}{bx^{n-q}+a}}{n-q}$$

$$\Big\downarrow \text{74}$$

$$\frac{ax^{-p(n-q)+n-pq-q}(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^{n-q}}{bx^{n-q}+a}\right)}{(1-p)(n-q)\left(a+bx^{n-q}\right)}$$

input
```
Int[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

output
```
(a*x^(n - p*(n - q) - q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 -
p, 2 - p, (b*x^(n - q))/(a + b*x^(n - q))])/((1 - p)*(n - q)*(a + b*x^(n -
 q)))
```

3.443.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.443.4 Maple [F]

$$\int x^{-1-n(p-1)-q}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

input
```
int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

output
```
int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

3.443.5 Fricas [F]

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\ dx=\int (bx^n+ax^q)^p x^{-n(p-1)-q-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a*x^q)^p*x^(-n*p + n - q - 1), x)
```

3.443.6 Sympy [F]

$$\int x^{-1-n(-1+p)-q}(bx^n + ax^q)^p\,dx = \int x^{-n(p-1)-q-1}(ax^q + bx^n)^p\,dx$$

input
```
integrate(x**(-1-n*(-1+p)-q)*(b*x**n+a*x**q)**p,x)
```

output
```
Integral(x**(-n*(p - 1) - q - 1)*(a*x**q + b*x**n)**p, x)
```

3.443.7 Maxima [F]

$$\int x^{-1-n(-1+p)-q}(bx^n + ax^q)^p\,dx = \int (bx^n + ax^q)^p x^{-n(p-1)-q-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

3.443.8 Giac [F]

$$\int x^{-1-n(-1+p)-q}(bx^n + ax^q)^p\,dx = \int (bx^n + ax^q)^p x^{-n(p-1)-q-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{q+n\,(p-1)+1}}\,dx$$

input
```
int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1),x)
```

output
```
int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1), x)
```

3.444 $\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx$

3.444.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+1))^(p+1)/b/(p+1)/(m*p+1)/(x^(m*(p+1)))
```

3.444.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+mp}\right)\right)^{1+p}}{b(1+p)(1+mp)}$$

input
```
Integrate[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

output
```
(x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

3.444.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^m+bx^{mp+m+1}\right)^p\,dx$$

$$\Big\downarrow \ 1906$$

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

input
```
Int[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

3.444.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

3.444.4 Maple [F]

$$\int \left(x^m a+b\,x^{mp+m+1}\right)^p dx$$

input
```
int((x^m*a+b*x^(m*p+m+1))^p,x)
```

output
```
int((x^m*a+b*x^(m*p+m+1))^p,x)
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int (ax^m + bx^{1+m+mp})^p \, dx = \frac{(bxx^{mp+m+1} + axx^m)(bx^{mp+m+1} + ax^m)^p}{(bmp^2 + (bm+b)p + b)x^{mp+m+1}}$$

input
```
integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + m + 1) + a*x*x^m)*(b*x^(m*p + m + 1) + a*x^m)^p/((b*m*p^2 +
(b*m + b)*p + b)*x^(m*p + m + 1))
```

3.444.6 Sympy [F]

$$\int (ax^m + bx^{1+m+mp})^p \, dx = \int (ax^m + bx^{mp+m+1})^p \, dx$$

input
```
integrate((a*x**m+b*x**(m*p+m+1))**p,x)
```

output
```
Integral((a*x**m + b*x**(m*p + m + 1))**p, x)
```

3.444.7 Maxima [F]

$$\int (ax^m + bx^{1+m+mp})^p \, dx = \int (bx^{mp+m+1} + ax^m)^p \, dx$$

input
```
integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

3.444.8 Giac [F]

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\int \left(bx^{mp+m+1}+ax^m\right)^p\,dx$$

input
```
integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

3.444.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.73

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\frac{a\,\left(a\,x^m+b\,x^{m+m\,p+1}\right)^p\,\left(\frac{b\,x^{m\,p+1}}{a}-\frac{1}{\left(\frac{b\,x^{m\,p+1}}{a}+1\right)^p}+1\right)}{b\,x^{m\,p}\,\left(m\,p+1\right)\,\left(p+1\right)}$$

input
```
int((a*x^m + b*x^(m + m*p + 1))^p,x)
```

output
```
(a*(a*x^m + b*x^(m + m*p + 1))^p*((b*x^(m*p + 1))/a - 1/((b*x^(m*p + 1))/a
 + 1)^p + 1))/(b*x^(m*p)*(m*p + 1)*(p + 1))
```

3.445 $\int (x^m(a+bx^{1+mp}))^p \, dx$

3.445.1 Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+1))^(p+1)/b/(p+1)/(m*p+1)/(x^(m*(p+1)))
```

3.445.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+mp}\right)\right)^{1+p}}{b(1+p)(1+mp)}$$

input
```
Integrate[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

output
```
(x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

3.445.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^m \left(a+bx^{mp+1}\right)\right)^p \, dx$$

$$\Big\downarrow \text{2078}$$

$$\int \left(ax^m+bx^{mp+m+1}\right)^p dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{x^{-m(p+1)} \left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

input
```
Int[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

3.445.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.445.4 Maple [F]

$$\int \left(x^m \left(a+b\,x^{mp+1}\right)\right)^p dx$$

input
```
int((x^m*(a+b*x^(m*p+1)))^p,x)
```

output
```
int((x^m*(a+b*x^(m*p+1)))^p,x)
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{(bxx^{mp+1}+ax)(bx^{mp+1}x^m+ax^m)^p}{(bmp^2+(bm+b)p+b)x^{mp+1}}$$

input
```
integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b
)*p + b)*x^(m*p + 1))
```

3.445.6 Sympy [F]

$$\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx = \int \left(x^m \left(a+bx^{mp+1}\right)\right)^p \, dx$$

input
```
integrate((x**m*(a+b*x**(m*p+1)))**p,x)
```

output
```
Integral((x**m*(a + b*x**(m*p + 1)))**p, x)
```

3.445.7 Maxima [F]

$$\int \left(x^m \left(a + bx^{1+mp}\right)\right)^p \, dx = \int \left(\left(bx^{mp+1} + a\right)x^m\right)^p \, dx$$

input
```
integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="maxima")
```

output
```
integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

3.445.8 Giac [F]

$$\int \left(x^m \left(a + bx^{1+mp}\right)\right)^p \, dx = \int \left(\left(bx^{mp+1} + a\right)x^m\right)^p \, dx$$

input
```
integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
```

output
```
integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \left(x^m \left(a + bx^{1+mp}\right)\right)^p \, dx = \int \left(x^m \left(a + b\,x^{m\,p+1}\right)\right)^p \, dx$$

input
```
int((x^m*(a + b*x^(m*p + 1)))^p,x)
```

output
```
int((x^m*(a + b*x^(m*p + 1)))^p, x)
```

3.446 $\int x^n(x^m(a+bx^{1+n+mp}))^p\,dx$

3.446.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+n+1))^(p+1)/b/(p+1)/(m*p+n+1)/(x^(m*(p+1)))
```

3.446.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+n+mp}\right)\right)^{1+p}}{b(1+p)(1+n+mp)}$$

input
```
Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

output
```
(x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p
)))
```

3.446.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2079, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n \left(x^m \left(a+b x^{m p+n+1}\right)\right)^p \, dx$$

$$\Big\downarrow \; 2079$$

$$\int x^n \left(a x^m+b x^{m p+m+n+1}\right)^p \, dx$$

$$\Big\downarrow \; 1920$$

$$\frac{x^{-m (p+1)} \left(a x^m+b x^{m p+m+n+1}\right)^{p+1}}{b (p+1) (m p+n+1)}$$

input
```
Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
 p)))
```

3.446.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 2079
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

3.446.4 Maple [F]

$$\int x^n \left(x^m \left(a+b\,x^{mp+n+1}\right)\right)^p dx$$

input
```
int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

output
```
int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p dx = \frac{(bxx^{mp+n+1}x^n + axx^n)(bx^{mp+n+1}x^m + ax^m)^p}{(bmp^2 + bn + (bm + bn + b)p + b)x^{mp+n+1}}$$

input
```
integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*
m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))
```

3.446.6 Sympy [F(-1)]

Timed out.

$$\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p dx = \text{Timed out}$$

input
```
integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)
```

output
```
Timed out
```

3.446.7 Maxima [F]

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\int\left(\left(bx^{mp+n+1}+a\right)x^m\right)^px^n\,dx$$

input
```
integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")
```

output
```
integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

3.446.8 Giac [F]

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\int\left(\left(bx^{mp+n+1}+a\right)x^m\right)^px^n\,dx$$

input
```
integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")
```

output
```
integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\int x^n\,\left(x^m\,\left(a+b\,x^{n+m\,p+1}\right)\right)^p\,dx$$

input
```
int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p,x)
```

output
```
int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p, x)
```

3.447 $\int x^n(ax^m + bx^{1+m+n+mp})^p \, dx$

3.447.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int x^n(ax^m + bx^{1+m+n+mp})^p \, dx = \frac{x^{-m(1+p)}(ax^m + bx^{1+m+n+mp})^{1+p}}{b(1+p)(1+n+mp)}$$

output
```
(a*x^m+b*x^(m*p+m+n+1))^(p+1)/b/(p+1)/(m*p+n+1)/(x^(m*(p+1)))
```

3.447.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x^n(ax^m + bx^{1+m+n+mp})^p \, dx = \frac{x^{-m(1+p)}(x^m(a + bx^{1+n+mp}))^{1+p}}{b(1+p)(1+n+mp)}$$

input
```
Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

output
```
(x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p
)))
```

3.447.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n \left(ax^m + bx^{mp+m+n+1}\right)^p \, dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{x^{-m(p+1)}\left(ax^m + bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

input
```
Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

output
```
(a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
 p)))
```

3.447.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.447.4 Maple [F]

$$\int x^n \left(x^m a + b\, x^{mp+m+n+1}\right)^p dx$$

input
```
int(x^n*(x^m*a+b*x^(m*p+m+n+1))^p,x)
```

output
```
int(x^n*(x^m*a+b*x^(m*p+m+n+1))^p,x)
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \frac{(bxx^{mp+m+n+1}x^n + axx^m x^n)(bx^{mp+m+n+1} + ax^m)^p}{(bmp^2 + bn + (bm + bn + b)p + b)x^{mp+m+n+1}}$$

input
```
integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="fricas")
```

output
```
(b*x*x^(m*p + m + n + 1)*x^n + a*x*x^m*x^n)*(b*x^(m*p + m + n + 1) + a*x^m
)^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + m + n + 1))
```

3.447.6 Sympy [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int x^n \left(ax^m + bx^{mp+m+n+1}\right)^p \, dx$$

input
```
integrate(x**n*(a*x**m+b*x**(m*p+m+n+1))**p,x)
```

output
```
Integral(x**n*(a*x**m + b*x**(m*p + m + n + 1))**p, x)
```

3.447.7 Maxima [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int \left(bx^{mp+m+n+1} + ax^m\right)^p x^n \, dx$$

input
```
integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

3.447.8 Giac [F]

$$\int x^n\left(ax^m+bx^{1+m+n+mp}\right)^p\,dx=\int\left(bx^{mp+m+n+1}+ax^m\right)^p x^n\,dx$$

input
```
integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int x^n\left(ax^m+bx^{1+m+n+mp}\right)^p\,dx=\int x^n\,\left(a\,x^m+b\,x^{m+n+m\,p+1}\right)^p\,dx$$

input
```
int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p,x)
```

output
```
int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p, x)
```

3.448 $\int \sqrt{x^{2(-1+n)}(a+bx^n)}\,dx$

3.448.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt{x^{2(-1+n)}(a+bx^n)}\,dx=\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}$$

output
```
2/3*x^(3-3*n)*(a/(x^(2-2*n))+b*x^(-2+3*n))^(3/2)/b/n
```

3.448.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt{x^{2(-1+n)}(a+bx^n)}\,dx=\frac{2x^{3-3n}\left(x^{-2+2n}(a+bx^n)\right)^{3/2}}{3bn}$$

input
```
Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

output
```
(2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)
```

3.448.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^{2(n-1)}\left(a+bx^n\right)}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \sqrt{ax^{2(n-1)}+bx^{2(n-1)+n}}dx$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

input
```
Int[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

output
```
(2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

3.448.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.448.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{2\sqrt{\frac{x^{2n}(a+b\,x^{n})}{x^{2}}}\,(a+b\,x^{n})x^{-n}x}{3bn}$	40

input
```
int((x^(-2+2*n)*(a+b*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/x^2*(x^n)^2*(a+b*x^n))^(1/2)*(a+b*x^n)/(x^n)*x/b/n
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\frac{2\left(bxx^{n}+ax\right)\sqrt{\frac{bx^{3\,n}+ax^{2\,n}}{x^{2}}}}{3\,bnx^{n}}$$

input
```
integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)
```

3.448.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\text{Timed out}$$

input
```
integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)
```

output
```
Timed out
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx=\frac{2\left(bx^n+a\right)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

3.448.8 Giac [F]

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx=\int \sqrt{(bx^n+a)x^{2\,n-2}}\,dx$$

input
```
integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx=\int \sqrt{x^{2\,n-2}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int((x^(2*n - 2)*(a + b*x^n))^(1/2),x)
```

output
```
int((x^(2*n - 2)*(a + b*x^n))^(1/2), x)
```

3.449 $\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx$

3.449.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}$$

output
```
3/4*x^(4-4*n)*(a/(x^(3-3*n))+b*x^(-3+4*n))^(4/3)/b/n
```

3.449.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3x^{4-4n}\left(x^{-3+3n}(a+bx^n)\right)^{4/3}}{4bn}$$

input
```
Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

output
```
(3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)
```

3.449.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x^{3(n-1)}\left(a+bx^n\right)}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt[3]{ax^{3(n-1)}+bx^{3(n-1)+n}}dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

input
```
Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

output
```
(3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)
```

3.449.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.449.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{3\left(\frac{x^{3n}(a+b\,x^{n})}{x^{3}}\right)^{\frac{1}{3}} x\,x^{-n}(a+b\,x^{n})}{4bn}$	40

input
```
int((x^(-3+3*n)*(a+b*x^n))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*(1/x^3*(x^n)^3*(a+b*x^n))^(1/3)*x/(x^n)*(a+b*x^n)/b/n
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3\left(bxx^n+ax\right)\left(\frac{bx^{4\,n}+ax^{3\,n}}{x^3}\right)^{\frac{1}{3}}}{4\,bnx^n}$$

input
```
integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")
```

output
```
3/4*(b*x*x^n + a*x)*((b*x^(4*n) + a*x^(3*n))/x^3)^(1/3)/(b*n*x^n)
```

3.449.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\text{Timed out}$$

input
```
integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)
```

output
```
Timed out
```

3.449.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx = \frac{3(bx^n+a)^{\frac{4}{3}}}{4\,bn}$$

input
```
integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="maxima")
```

output
```
3/4*(b*x^n + a)^(4/3)/(b*n)
```

3.449.8 Giac [F]

$$\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx = \int \left((bx^n+a)x^{3\,n-3}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx = \int \left(x^{3\,n-3}\,(a+b\,x^n)\right)^{1/3}\,dx$$

input
```
int((x^(3*n - 3)*(a + b*x^n))^(1/3),x)
```

output
```
int((x^(3*n - 3)*(a + b*x^n))^(1/3), x)
```

3.450 $\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx$

3.450.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}$$

output
```
4/5*x^(5-5*n)*(a/(x^(4-4*n))+b*x^(-4+5*n))^(5/4)/b/n
```

3.450.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4x^{5-5n}\left(x^{-4+4n}\left(a+bx^n\right)\right)^{5/4}}{5bn}$$

input
```
Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

output
```
(4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)
```

3.450.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{x^{4(n-1)} (a+bx^n)}\,dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \sqrt[4]{ax^{4(n-1)}+bx^{4(n-1)+n}}dx$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

input
```
Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

output
```
(4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

3.450.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.450.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{4\left(\frac{x^{4n}(a+b\,x^{n})}{x^{4}}\right)^{\frac{1}{4}} x\,x^{-n}(a+b\,x^{n})}{5bn}$	40

input
```
int((x^(-4+4*n)*(a+b*x^n))^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/5*(1/x^4*(x^n)^4*(a+b*x^n))^(1/4)*x/(x^n)*(a+b*x^n)/b/n
```

3.450.5 Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^{n}\right)}\,dx=\frac{4\left(bxx^{n}+ax\right)\left(\frac{bx^{5\,n}+ax^{4\,n}}{x^{4}}\right)^{\frac{1}{4}}}{5\,bnx^{n}}$$

input
```
integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="fricas")
```

output
```
4/5*(b*x*x^n + a*x)*((b*x^(5*n) + a*x^(4*n))/x^4)^(1/4)/(b*n*x^n)
```

3.450.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^{n}\right)}\,dx=\text{Timed out}$$

input
```
integrate((x**(-4+4*n)*(a+b*x**n))**(1/4),x)
```

output
```
Timed out
```

3.450.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx = \frac{4(bx^n+a)^{\frac{5}{4}}}{5\,bn}$$

input
```
integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="maxima")
```

output
```
4/5*(b*x^n + a)^(5/4)/(b*n)
```

3.450.8 Giac [F]

$$\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx = \int \left((bx^n+a)x^{4\,n-4}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx = \int \left(x^{4\,n-4}\,(a+b\,x^n)\right)^{1/4}\,dx$$

input
```
int((x^(4*n - 4)*(a + b*x^n))^(1/4),x)
```

output
```
int((x^(4*n - 4)*(a + b*x^n))^(1/4), x)
```

3.451 $\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx$

3.451.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx = \frac{px^{(1-n)(1+p)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}$$

output
```
p*x^((1-n)*(p+1))*(a/(x^((1-n)*p))+b*x^(n-(1-n)*p))^(1+1/p)/b/n/(p+1)
```

3.451.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx = \frac{x^{1-n}(a+bx^n)\left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}}{bn\left(1+\frac{1}{p}\right)}$$

input
```
Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

output
```
(x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-
1)))
```

3.451.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^{(n-1)p}(a+bx^n)\right)^{\frac{1}{p}} dx$$

$$\Big\downarrow \text{2078}$$

$$\int \left(ax^{(n-1)p}+bx^{(n-1)p+n}\right)^{\frac{1}{p}} dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{px^{(1-n)(p+1)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

input
```
Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

output
```
(p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1)
))/(b*n*(1 + p))
```

3.451.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.451.4 Maple [F]

$$\int \left(x^{(-1+n)p}(a+b\,x^{n})\right)^{\frac{1}{p}}dx$$

input
```
int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

output
```
int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \left(x^{(-1+n)p}(a+bx^{n})\right)^{\frac{1}{p}}\,dx = \frac{(bpxx^{n}+apx)\left((bx^{n}+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}}{(bnp+bn)x^{n}}$$

input
```
integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")
```

output
```
(b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)
```

3.451.6 Sympy [F]

$$\int \left(x^{(-1+n)p}(a+bx^{n})\right)^{\frac{1}{p}}\,dx = \int \left(x^{p(n-1)}(a+bx^{n})\right)^{\frac{1}{p}}\,dx$$

input
```
integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)
```

output
```
Integral((x**(p*(n - 1))*(a + b*x**n))**(1/p), x)
```

3.451.7 Maxima [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx = \int \left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)} dx$$

input
```
integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

3.451.8 Giac [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx = \int \left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)} dx$$

input
```
integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx = \int \left(x^{p\,(n-1)}\,(a+b\,x^n)\right)^{1/p} dx$$

input
```
int((x^(p*(n - 1))*(a + b*x^n))^(1/p),x)
```

output
```
int((x^(p*(n - 1))*(a + b*x^n))^(1/p), x)
```

3.452 $\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx$

3.452.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}$$

output
```
x^((1-n)*(p+1)/p)*(b*x^(n+(-1+n)/p)+a/(x^((1-n)/p)))^(p+1)/b/n/(p+1)
```

3.452.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\frac{x^{1-n}(a+bx^n)\left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p}{bn(1+p)}$$

input
```
Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

output
```
(x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))
```

3.452.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2078, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^{\frac{n-1}{p}}(a+bx^n)\right)^p dx$$

$$\Big\downarrow \text{2078}$$

$$\int \left(ax^{\frac{n-1}{p}}+bx^{\frac{n-1}{p}+n}\right)^p dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

input
```
Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

output
```
(x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/
(b*n*(1 + p))
```

3.452.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.452.4 Maple [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+b\,x^n)\right)^p dx$$

input
```
int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

output
```
int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \frac{(bxx^n+ax)\left(bx^nx^{\frac{n-1}{p}}+ax^{\frac{n-1}{p}}\right)^p}{(bnp+bn)x^n}$$

input
```
integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")
```

output
```
(b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x
^n)
```

3.452.6 Sympy [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p dx = \int \left(x^{\frac{n-1}{p}}(a+bx^n)\right)^p dx$$

input
```
integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)
```

output
```
Integral((x**((n - 1)/p)*(a + b*x**n))**p, x)
```

3.452.7 Maxima [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx = \int \left((bx^n+a)x^{\frac{n-1}{p}}\right)^p\,dx$$

input
```
integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

3.452.8 Giac [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx = \int \left((bx^n+a)x^{\frac{n-1}{p}}\right)^p\,dx$$

input
```
integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")
```

output
```
integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx = \int \left(x^{\frac{n-1}{p}}\,(a+b\,x^n)\right)^p\,dx$$

input
```
int((x^((n - 1)/p)*(a + b*x^n))^p,x)
```

output
```
int((x^((n - 1)/p)*(a + b*x^n))^p, x)
```

3.453 $\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx$

3.453.1 Optimal result

Integrand size = 25, antiderivative size = 39

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(n-p)(1+q)}$$

output
```
(a*x^n+b*x^p)^(1+q)/a/(n-p)/(1+q)/(x^(p*(1+q)))
```

3.453.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=-\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(-n+p)(1+q)}$$

input
```
Integrate[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

output
```
-((a*x^n + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^(p*(1 + q))))
```

3.453.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-p(q+1)-1}(ax^n+bx^p)^q\,dx$$

$$\Big\downarrow \; 1920$$

$$\frac{x^{-p(q+1)}(ax^n+bx^p)^{q+1}}{a(q+1)(n-p)}$$

input
```
Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

output
```
(a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

3.453.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.453.4 Maple [F]

$$\int x^{-1+n-p(q+1)}(a\,x^n+b\,x^p)^q\,dx$$

input
```
int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

output
```
int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{(axx^{-pq+n-p-1}x^n+bxx^{-pq+n-p-1}x^p)(ax^n+bx^p)^q}{(an-ap+(an-ap)q)x^n}$$

input
```
integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="fricas")
```

output
```
(a*x*x^(-p*q + n - p - 1)*x^n + b*x*x^(-p*q + n - p - 1)*x^p)*(a*x^n + b*x
^p)^q/((a*n - a*p + (a*n - a*p)*q)*x^n)
```

3.453.6 Sympy [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int x^{n-p(q+1)-1}(ax^n+bx^p)^q\,dx$$

input
```
integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)
```

output
```
Integral(x**(n - p*(q + 1) - 1)*(a*x**n + b*x**p)**q, x)
```

3.453.7 Maxima [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int (ax^n+bx^p)^q x^{-p(q+1)+n-1}\,dx$$

input
```
integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="maxima")
```

output
```
integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

3.453.8 Giac [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q \, dx = \int (ax^n+bx^p)^q x^{-p(q+1)+n-1} \, dx$$

input
```
integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")
```

output
```
integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q \, dx = \int x^{n-p\,(q+1)-1}\,(a\,x^n+b\,x^p)^q \, dx$$

input
```
int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q,x)
```

output
```
int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q, x)
```

3.454 $\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx$

3.454.1 Optimal result

Integrand size = 28, antiderivative size = 40

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = -\frac{x^{-((n+p)(1+q))}(ax^n+bx^{n+p})^{1+q}}{ap(1+q)}$$

output
```
-(a*x^n+b*x^(n+p))^(1+q)/a/p/(1+q)/(x^((n+p)*(1+q)))
```

3.454.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = -\frac{x^{-((n+p)(1+q))}(x^n(a+bx^p))^{1+q}}{ap(1+q)}$$

input
```
Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

output
```
-((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

3.454.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2079, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-nq-p(q+1)-1}(x^n(a+bx^p))^q\,dx$$

$$\Big\downarrow \text{2079}$$

$$\int x^{-nq-p(q+1)-1}\left(ax^n+bx^{n+p}\right)^q dx$$

$$\Big\downarrow \text{1920}$$

$$-\frac{x^{-((q+1)(n+p))}\left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

input
```
Int[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

output
```
-((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

3.454.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 2079
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

3.454.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(41) = 82.

Time = 2.92 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

method	result	size
parallelrisch	$-\frac{x\,x^{p} x^{-q n-p q-p-1} \left(x^{n} \left(a+b\,x^{p}\right)\right)^{q} b^{2}+x\,x^{-q n-p q-p-1} \left(x^{n} \left(a+b\,x^{p}\right)\right)^{q} a b}{b p \left(q+1\right) a}$	86

input
```
int(x^(-1-q*n-p*(q+1))*(x^n*(a+b*x^p))^q,x,method=_RETURNVERBOSE)
```

output
```
-(x*x^p*x^(-n*q-p*q-p-1)*(x^n*(a+b*x^p))^q*b^2+x*x^(-n*q-p*q-p-1)*(x^n*(a+
b*x^p))^q*a*b)/b/p/(q+1)/a
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = -\frac{\left(bxx^{-(n+p)q-p-1}x^p + axx^{-(n+p)q-p-1}\right)(bx^nx^p + ax^n)^q}{apq + ap}$$

input
```
integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")
```

output
```
-(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p
+ a*x^n)^q/(a*p*q + a*p)
```

3.454.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*q-p*(1+q))*(x***n*(a+b*x**p))**q,x)
```

output
```
Timed out
```

3.454.7 Maxima [F]

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=\int ((bx^p+a)x^n)^q\,x^{-p(q+1)-nq-1}\,dx$$

input
```
integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")
```

output
```
integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

3.454.8 Giac [F]

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=\int ((bx^p+a)x^n)^q\,x^{-p(q+1)-nq-1}\,dx$$

input
```
integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")
```

output
```
integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=\int \frac{(x^n\,(a+b\,x^p))^q}{x^{n\,q+p\,(q+1)+1}}\,dx$$

input
```
int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1),x)
```

output
```
int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```